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Numerical Solution for 3-D lee wave associated with barotropic mean flow across the Assam-Burma hills

PRASANTA DAS

Department of Mathematics, Ramananda College, Bishnupur, West Bengal – 722 122, India

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e mail : pdas.math1986@gmail.com

सार – भारत में, उत्तर-पूर्व क्षेत्र में, असम-बर्मा हिल्स (ABH) को दो त्रि-आयामी अण्डाकार बाधाओं द्वारा संश्लेषित किया गया है, वे कुछ परिमित दूरी की घाटी द्वारा अलग किए गए हैं। इस पेपर में एबीएच में मेसो-स्केल ड्राई मीन फ्लो से जुड़े 3-डी ली वेव न्यूमेरिकल सॉल्यूशन को प्राप्त करने का प्रयास किया गया है। जहां प्रवाह एक एडियाबेटिक, इनविसिड, लामिनार, स्थिर, बूसिनस्क, गैर-घूर्णी है और मूल प्रवाह में जोनल घटक (यू) और मेरिडियनल घटक (वी) होते हैं, वे अंडाकार के प्रमुख धुरी के सामान्य और समानांतर होते हैं बाधा क्रमशः। सादगी के लिए, मूल प्रवाह के दो घटकों (यू, वी) और उत्प्लावकता आवृत्ति (एन) को ऊंचाई के साथ एक समान माना जाता है और गवर्निंग समीकरणों के लिए गड़बड़ी तकनीक भी लागू की गई है। पर्टर्बेशन वर्टिकल वेलोसिटी (w') और स्ट्रीम लाइन विस्थापन (η') को डबल इंटीग्रल के रूप में व्यक्त किया जाता है, जिसका मूल्यांकन संख्यात्मक विस्तार के रूप में अनुमानित किया गया है। अंत में, गणना किए गए परिणामों की तुलना पहले के जांचकर्ताओं द्वारा प्राप्त किए गए स्पर्शोन्मुख परिणामों से की गई है।

ABSTRACT. In India, in the North-East region, the Assam-Burma Hills (ABH) has been synthesized by two three-dimensional elliptical barriers, they are separated by a valley of some finite distance. In this paper, an attempt has been made to obtain a 3-D lee wave numerical solution associated with a meso-scale dry mean flow across the ABH. Where the flow is an adiabatic, inviscid, laminar, steady, Boussinesq, non-rotational and the basic flow consists of the zonal component (U) and the meridional component (V), they are normal and parallel to the major axis of the elliptical barrier respectively. For simplicity, the two components (U, V) of the basic flow and Buoyancy frequency (N) are assumed to be uniform with height and also the perturbation technique has been applied to the governing equations. The perturbation vertical velocity (w') and stream line displacement (η') are expressed as a double integral, which have been evaluated to approximate as the numerical expansion. Finally, the computed results have compared with the asymptotic results obtained by earlier investigators.

Key words – ABH, Perturbation vertical velocity (w'), Streamline displacement (η').

1. Introduction

Orography is the study of the formation and relief of mountains and can more broadly include hills and any part of a region's elevated terrain. The climate and weather of a place are strongly influenced by the orography. In the past, many aircraft accidents reported in mountainous areas are often attributed to the vertical velocities of large magnitude associated with the lee waves. Hence, the studies on the lee waves are associated with air flow across an orographic barrier, have an important bearing to the safety of aviation.

Theoretical studies on this field can widely be divided by two categories. In one category, the mountain barrier has been assumed the two-dimensional. The two-dimensional mountain wave problem was first addressed by Lyra (1943) and subsequently by Queney (1947); Scorer (1949); Sawyer (1960); Sarker (1965, 1966); De (1973); Sinha Ray (1988); Kumar *et al*. (1998) etc. In another category of theoretical studies on the three-dimensional mountain wave problem was first addressed by Scorer and Wilkinson (1956) and subsequently by Wurtele (1957); Crapper (1959); Sawyer (1962); Das (1964); Smith (1979); Dutta *et al*. (2002); Dutta (2005, 2007); Das *et al*. (2013, 2016) etc.

In India, studies on the effects of an orographic barrier on airflow have been addressed by Das (1964); Sarker (1965, 1966); Sarker *et al.* (1978); De (1973);

Sinha Ray (1988); Dutta *et al*. (2002); Dutta (2005); Dutta and Kumar (2005) etc. In other countries, studies on the orographic effects are associated with airflow have been addressed by Abbs & Pielke (1987); Bischoff-Gauss *et al*. (1989); Leung and Ghan (1995); Lin and Chen (2002); Li *et al*. (2007); Xu *et al*. (2008); Jourdain & Gallee (2010) etc.

In some of the above studies, the wind and stability were assumed to be either constant with height or assumed to be variant with height. Solutions for these studies were essentially obtained by an analytical method or the numerical method. Das *et al*. (2013) developed a three-dimensional mountain waves problem over the Assam-Burma hills (ABH) are associated with idealistic basic flow. They obtained the asymptotic solutions using the perturbation approach and compared with the two-dimensional waves problem of the earlier authors.

To develop this model, here consider the same mountain profile the Assam-Burma Hills (ABH) and obtain the numerical solutions for the perturbation vertical velocity (w') and stream line displacement (η') are associated with 3-D lee wave using the perturbation approach and the computed results have compared with the earlier investigators Das *et al*. (2013).

2. Database

As the Assam-Burma Hills are situated in North-East position of India, the only station to the upstream side is Guwahati (26.19° N Latitude and 91.73° E Longitude). The average of 0000 UTC and 1200 UTC RS/RW data of Guwahati for those dates, which corresponds to the observed lee waves across ABH, as reported by De (1970, 1971); Farooqui and De (1974) and Das *et al*. (2013), has been obtained from the Archive of India Meteorological Department (IMD), Pune.

3. Methodology

In this model, an adiabatic, steady, laminar, inviscid, non-rotating flow of a vertical unbounded, a stratified and Boussinesq fluid across 3-D meso-scale elliptical orographic barrier has been considered. Here, this model has applied on the Assam-Burma Hills. The profile of the Assam-Burma Hills (Fig. 1) is analytically expressed as:

$$h(x,y)=\frac{h_1}{1+\frac{x^2}{a^2}+\frac{y^2}{b^2}}+\frac{h_2}{1+\frac{(x-d)^2}{a^2}+\frac{y^2}{b^2}} \qquad (1)$$

where, *a* and *b* are the half width of the barrier along the zonal wind component (U) and along the meridional component (V) respectively, h_1 and h_2 are the height of the

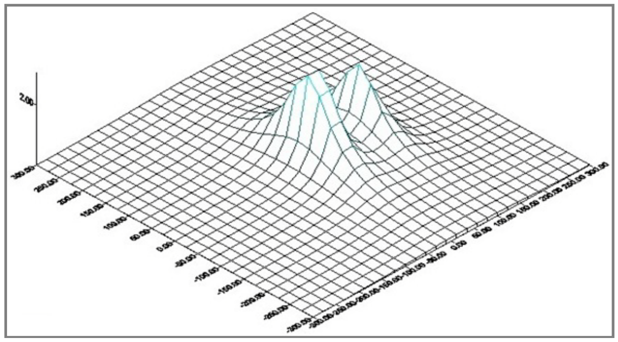


Fig. 1. The profile of the Assam-Burma hills

two ridges of the mountain and *d* be the distance of the valley between two ridges.

We consider a co-ordinate system in which the *x*-axis and the *y*-axis are perpendicular and parallel to the axis of the major ridge of the barrier and the *z*-axis is vertically upwards. The two components *U* and *V* of the basic flow, are normal and parallel to the major ridge of the barrier respectively. It is again simplified by assuming *U*, *V* and the Buoyancy frequency (*N*), to be invariant with height. Under the above assumptions, the linearized governing equations can be written as:

$$U\frac{\partial u'}{\partial x}+V\frac{\partial u'}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \qquad (2)$$

$$U\frac{\partial v'}{\partial x}+V\frac{\partial v'}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial y} \qquad (3)$$

$$U\frac{\partial w'}{\partial x}+V\frac{\partial w'}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial z}+\frac{g\theta'}{\theta_0} \qquad (4)$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0 \qquad (5)$$

$$U\frac{\partial \theta'}{\partial x}+V\frac{\partial \theta'}{\partial y}+w'\frac{d\theta_0}{dz}=0 \qquad (6)$$

where, $\rho_0=\rho_0(z)$, $\theta_0=\theta_0(z)$ are respectively density and potential temperature of the basic flow and u', v', w', p', θ' are respectively the perturbation part of the zonal wind, the meridional wind, the vertical wind, pressure and potential temperature. Since the perturbation quantities u', v', w', p', θ' are all continuous functions of *x, y, z*. Hence, the double Fourier integral is :

$$u'(x,y,z)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{u}(k,l,z)e^{i(kx+ly)}dkdl$$

where, $u'(k,l,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u'(x,y,z)e^{-i(kx+ly)}dxdy$ is the double Fourier transformation of $u'(x, y, z)$. Using 2-D Fourier transformation in the equations (2)-(6) and they are transformed to :

$$i(kU+lV)\hat{u}=-ik\frac{\hat{p}}{\rho_0} \tag{7}$$

$$i(kU+lV)\hat{v}=-il\frac{\hat{p}}{\rho_0} \tag{8}$$

$$i(kU+lV)\hat{w}=-\frac{1}{\rho_0}\frac{\partial\hat{p}}{\partial z}+g\frac{\hat{\theta}}{\theta_0} \tag{9}$$

$$i(k\hat{u}+l\hat{v})+\frac{\partial\hat{w}}{\partial z}=0 \tag{10}$$

$$i(kU+lV)\hat{\theta}+\hat{w}\frac{d\theta_0}{dz}=0 \tag{11}$$

where, $\hat{u},\hat{v},\hat{w},\hat{p},\hat{\theta}$ are respectively double Fourier transformations of u', v', w', p', θ'. Now, eliminating $\hat{u},\hat{v}$, $\hat{p},\hat{\theta}$ from the equations (7) - (11) we have :

$$\frac{\partial^2\hat{w}}{\partial z^2}+\frac{1}{\rho_0}\frac{d\rho_0}{dz}\frac{\partial\hat{w}}{\partial z}+\left(k^2+l^2\right)\left[\frac{N^2}{(Uk+Vl)^2}-1\right]\hat{w}=0 \tag{12}$$

where, $N=\sqrt{\frac{g}{\theta_0}\frac{d\theta_0}{dz}}$ is the Buoyancy frequency.

Now, by the substitution $\hat{w}(k,l,z)=\left[\frac{\rho_0(0)}{\rho_0(z)}\right]^{1/2}\hat{w}_1(k,l,z)$, the equation (12) is further simplified to :

$$\frac{\partial^2\hat{w}_1}{\partial z^2}+\left[\frac{N^2\left(k^2+l^2\right)}{(Uk+Vl)^2}-\frac{1}{2\rho_0}\frac{d^2\rho_0}{dz^2}+\frac{1}{4\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^2-\left(k^2+l^2\right)\right]\hat{w}_1=0 \tag{13}$$

where, the terms $\left(-\frac{1}{2\rho_0}\frac{d^2\rho_0}{dz^2}\right)$ and $\frac{1}{4\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^2$ in equation (13) are smaller in their magnitude than other terms in the square bracket. So, the equation (13) reduces to :

$$\frac{\partial^2\hat{w}_1}{\partial z^2}+\left(k^2+l^2\right)\left[\frac{N^2}{(Uk+Vl)^2}-1\right]\hat{w}_1=0 \tag{14}$$

If η' (x, y, z) be the perturbation streamline displacement, then we can write :

$$w'(x,y,z)=U\frac{\partial\eta'}{\partial x}+V\frac{\partial\eta'}{\partial y} \tag{15}$$

Using 2-D Fourier transformation in the above equation, then it becomes :

$$\hat{w}(k,l,z)=i(kU+lV)\hat{\eta}$$

Clearly seen that, $\hat{\eta}$ satisfies equation (14). Now, by the substitution $\hat{\eta}(k,l,z)=\sqrt{\frac{\rho_0(0)}{\rho_0(z)}}\hat{\eta}_1(k,l,z)$ we obtain :

$$\frac{\partial^2\hat{\eta}_1}{\partial z^2}+\left(k^2+l^2\right)\left[\frac{N^2}{(Uk+Vl)^2}-1\right]\hat{\eta}_1=0 \tag{16}$$

The equations (14) and (16) will be solved using the following boundary conditions:

(a) At the surface, that is lower boundary streamline pattern follows the contour of the terrine.

(b) At the upper boundary, the mountain waves are permitted to propagate vertically.

The general integral of the equations (14) and (16) are [using boundary condition (b)] :

$$\hat{w}_1(k,l,z)=\mathrm{A}e^{imz} \tag{17}$$

and

$$\hat{\eta}_1(k,l,z)=\mathrm{B}e^{imz} \tag{18}$$

where, A and B are arbitrary constants and m is given:

$$m^2=\left[\frac{N^2}{(kU+lV)^2}-1\right]\left(k^2+l^2\right)$$

Clearly m may be recognized as the vertical wave number of the vertically propagating mountain waves. Now, at the lower boundary, that is, at the surface, the airflow follows the contour of the mountain profile which is given in Eqn. (1).

In the present study, the values of a, d, h_1 and h_2 are the same as those in De (1971) and b = 2.5a as in Dutta (2005), as in Das *et al*. (2013, 2016). Therefore, we take a = 20 km, b = 2.5a, d = 45 km, h_1 = 0.9 km and h_2 = 0.7 km. Now, the 2-D Fourier transformation of the mountain profile (1) is :

$$\hat{h}(k,l) = 2\pi ab\left(h_1 + h_2 e^{-ikd}\right)K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)$$

[See Appendix] (19)

where, $K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)$ is the zero-order second kind Bessel function. Using lower boundary condition, we have :

$$\eta'(x,y,0) = h(x,y)$$

Using double Fourier transformation in the above equation, we get :

$$\hat{\eta}(k,l,0) = \hat{h}(k,l)$$

Hence,

$$B = 2\pi ab\left(h_1 + h_2 e^{-ikd}\right)K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)$$

Again, the linearized lower boundary condition for w', the equation (15) becomes :

$$w'(x,y,0) = U\frac{\partial \eta'(x,y,0)}{\partial x} + V\frac{\partial \eta'(x,y,0)}{\partial y}$$

Using double Fourier transformation in the above equation, we have :

$$\hat{w}(k,l,0) = i(kU + lV)\hat{\eta}(k,l,0) \tag{20}$$

Hence,

$$A = 2\pi iab\left(h_1 + h_2 e^{-ikd}\right)K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)$$

Thus, the solutions of (14) and (16) are obtained by putting the values of A and B respectively, we get :

$$\hat{w}_1(k,l,z) = 2\pi iab\left(h_1 + h_2 e^{-ikd}\right)K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)e^{imz} \tag{21}$$

$$\hat{\eta}_1(k,l,z) = 2\pi ab\left(h_1 + h_2 e^{-ikd}\right)K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)e^{imz} \tag{22}$$

Using inverse Fourier transformation $w_1'(x,y,z)$ can be expressed as :

$$w_1'(x,y,z) = \mathrm{Re}\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{w}_1(k,l,z)e^{-i(kx+ly)}dkdl$$

$$= \frac{ab}{2\pi}\mathrm{Re}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i(kU + lV)\left(h_1 + h_2 e^{-ikd}\right) K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)e^{i(kx+ly+mz)}\,dkdl \tag{23}$$

Similarly, $\eta_1'(x,y,z)$ can also be expressed as :

$$\eta_1'(x,y,z) = \mathrm{Re}\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{\eta}_1(k,l,z)e^{i(kx+ly)}dkdl$$

$$= \frac{ab}{2\pi}\mathrm{Re}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(h_1 + h_2 e^{-ikd}\right)K_0\left(\sqrt{a^2k^2 + b^2l^2}\right) e^{-i(kx+ly+mz)}dkdl \tag{24}$$

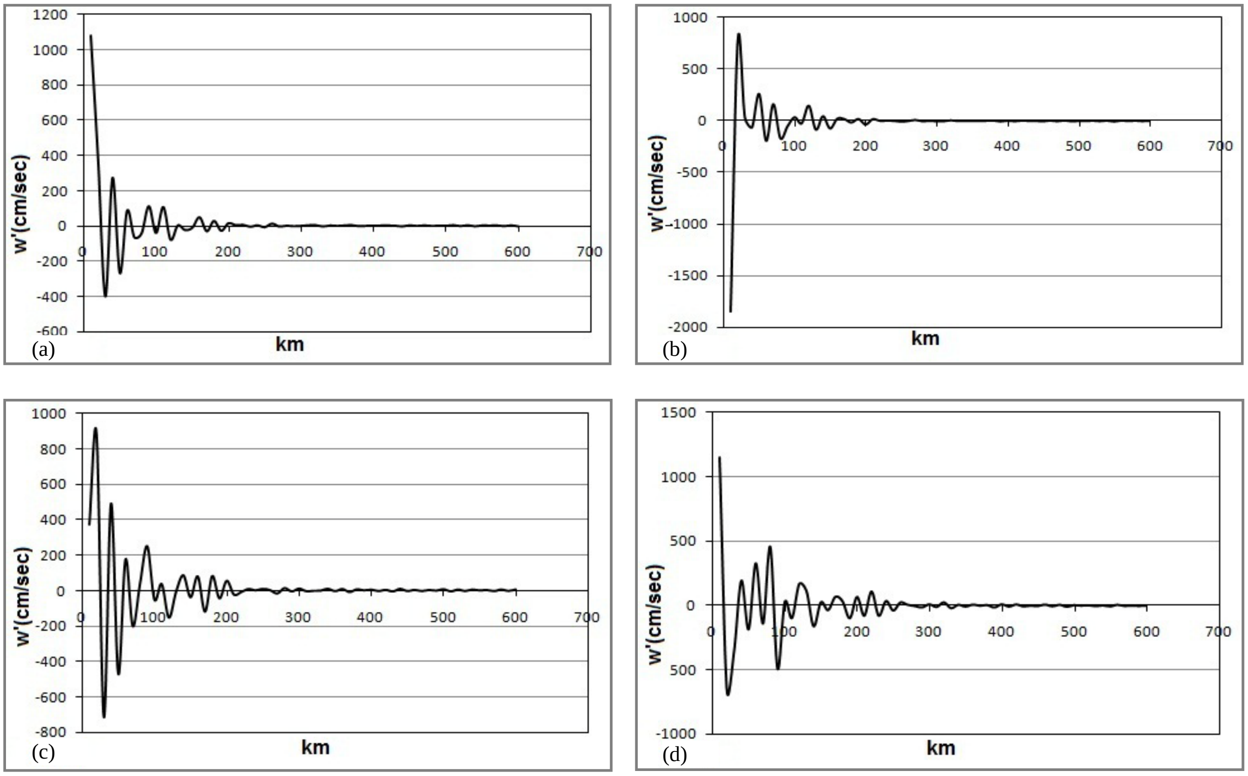
The above two equations (23) and (24) reduce to :

$$w'(x,y,z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}}w_1'(x,y,z) = c \times \mathrm{Re}(I_1) \tag{25}$$

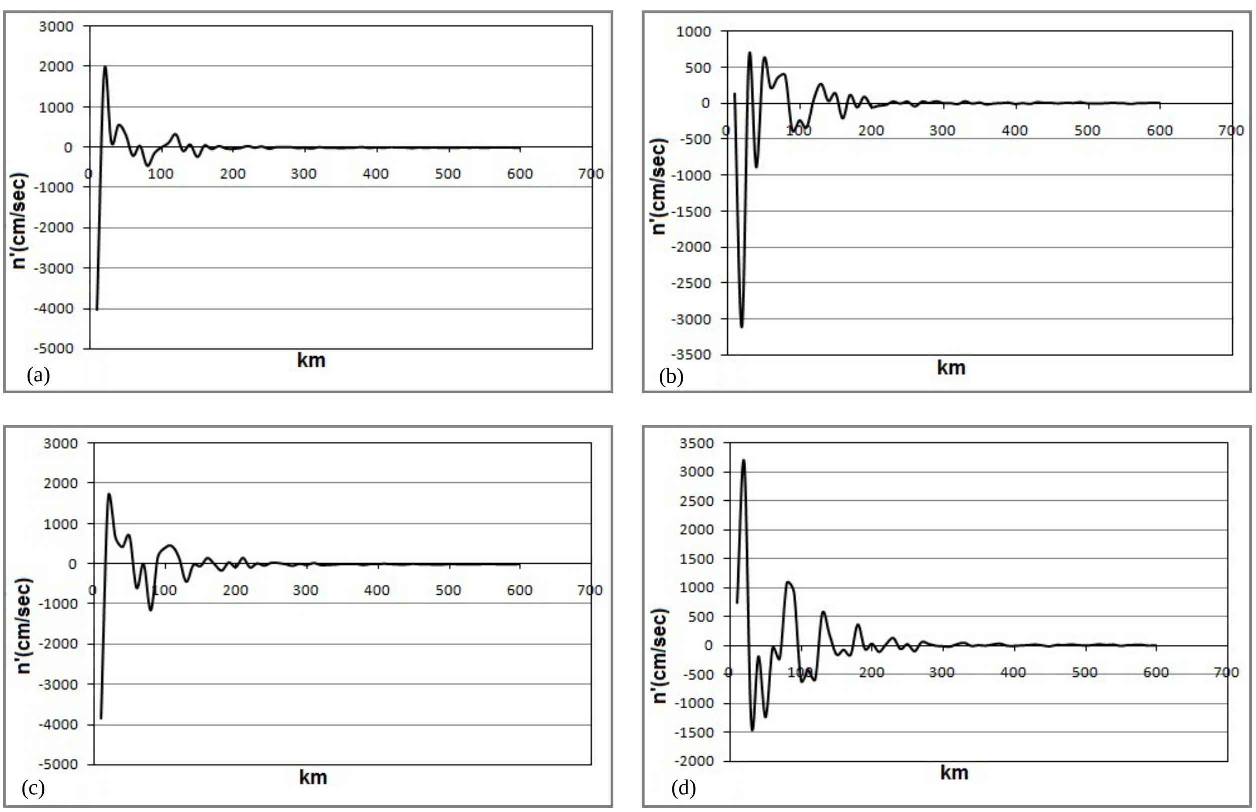
and

$$\eta'(x,y,z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}}\eta_1'(x,y,z) = c \times \mathrm{Re}(I_2) \tag{26}$$

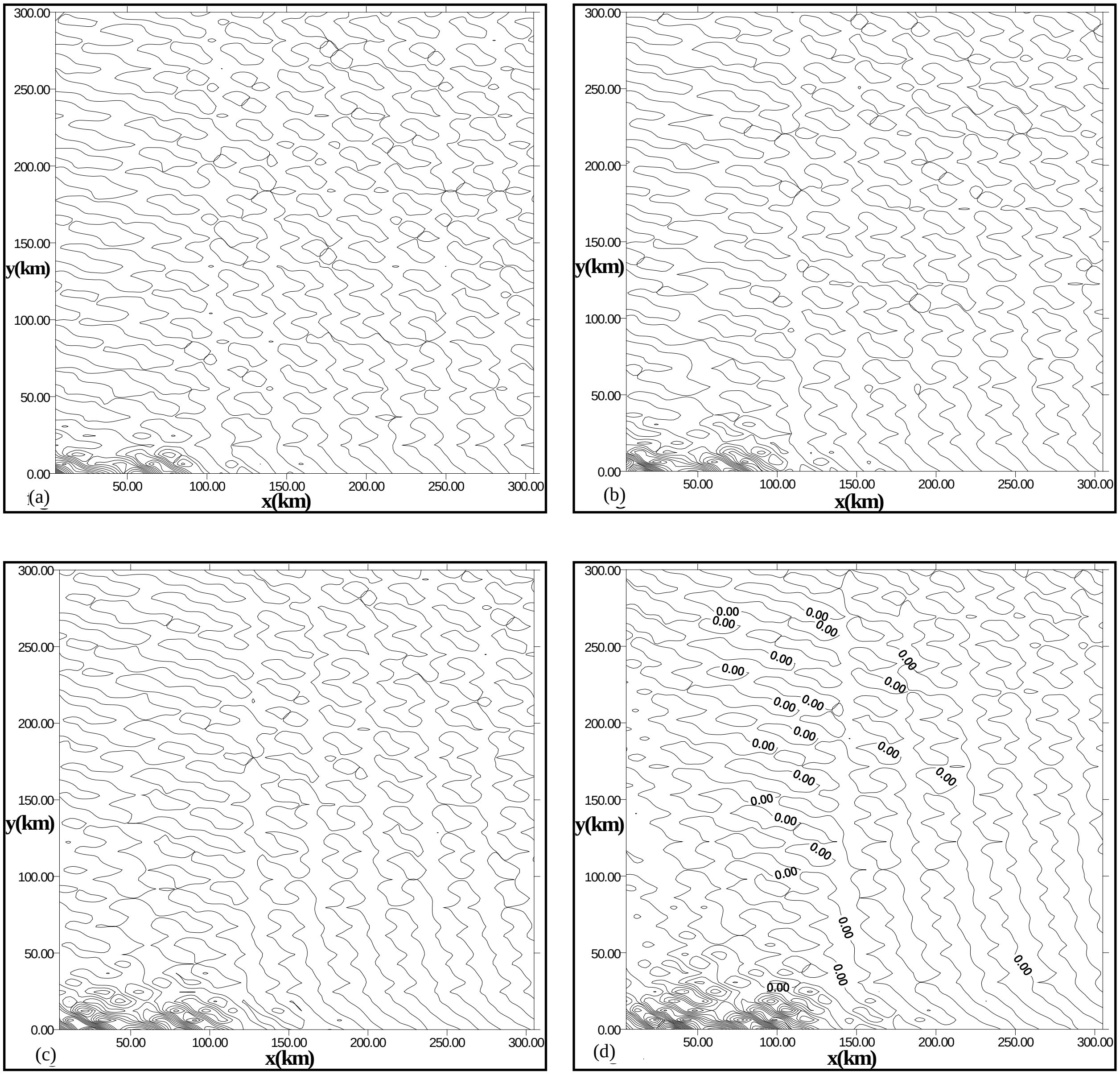
where, $c = \frac{ab}{2\pi}\sqrt{\frac{\rho_0(0)}{\rho_0(z)}}$



Figs. 2(a-d). Down-stream variation of *w*' along the line $Uy – Vx = 0$, at 1.5 km, 3 km, 6 km and 9 km above the mean sea level respectively



Figs. 3(a-d). Down-stream variation of η' along the line $Uy – Vx = 0$, at 1.5 km, 3 km, 6 km and 9 km above the mean sea level respectively



Figs. 4(a-d). Contours of the perturbation vertical velocity (*w*') at 1.5 km, 3 km, 6 km and 9 km above the mean sea level respectively

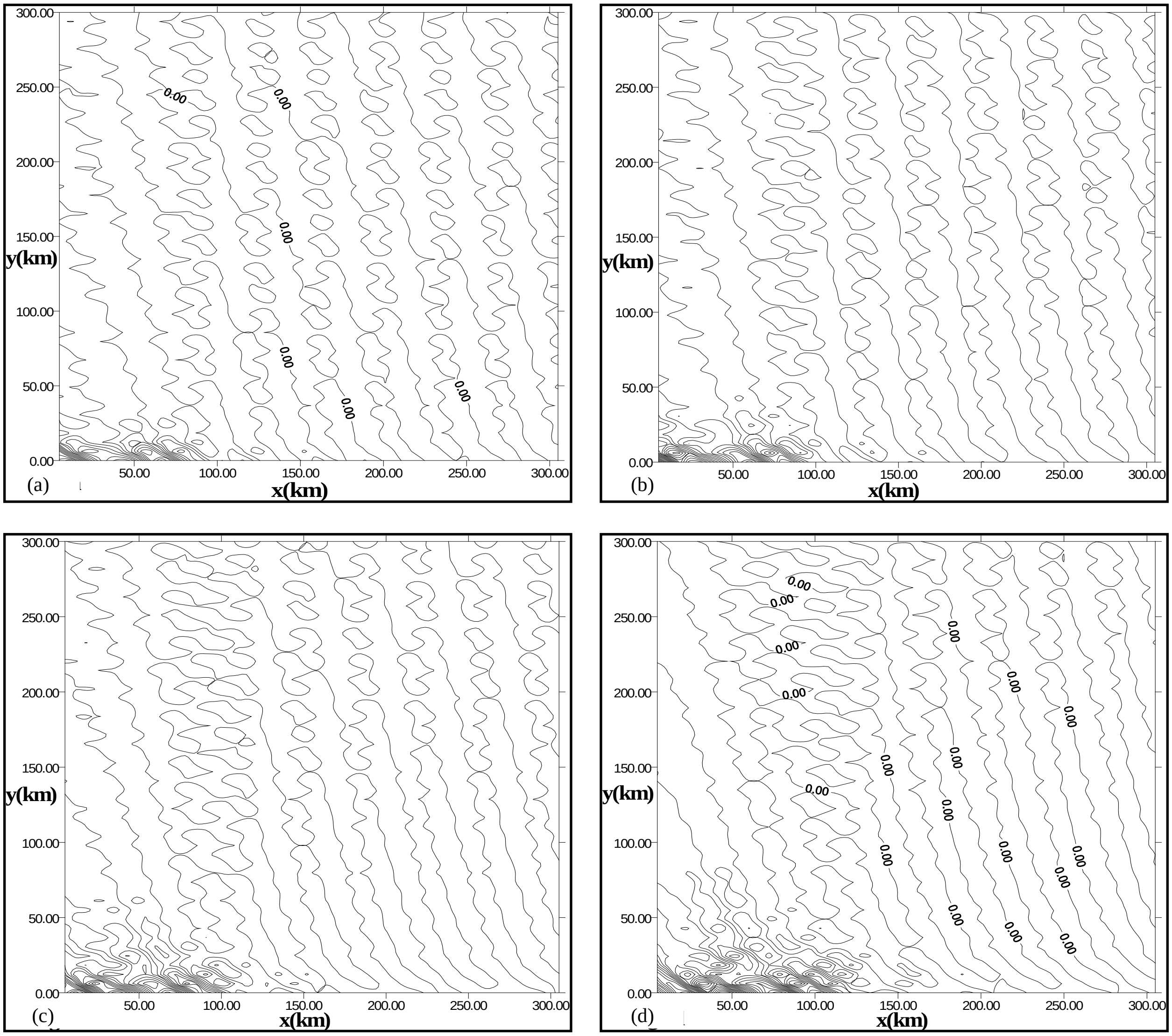
$$I_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i(kU + lV)\left(h_1 + h_2 e^{-ikd}\right) K_0\left(\sqrt{a^2k^2 + b^2l^2}\right) e^{i(kx+ly+mz)}dkdl$$

$$I_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i\left(h_1 + h_2 e^{-ikd}\right) K_0\left(\sqrt{a^2k^2 + b^2l^2}\right) e^{i(kx+ly+mz)}dkdl$$

The integrals (25) and (26) are evaluated numerically, we get :

$$w'(x,y,z) = -c\sum\sum (kU + lV) K_0\left(\sqrt{a^2k^2 + b^2l^2}\right) \{h_1 \sin(kx+ly+mz) + h_2 \sin(kx+ly+mz-kd)\}\delta k\delta l \tag{27}$$

$$\eta'(x,y,z) = c\sum\sum K_0\left(\sqrt{a^2k^2 + b^2l^2}\right) \{h_1 \sin(kx+ly+mz) + h_2 \sin(kx+ly+mz-kd)\}\delta k\delta l \tag{28}$$



Figs. 5(a-d). Contours of stream line displacement (η') at 1.5 km, 3 km, 6 km and 9 km above the mean see level respectively

where, the above summations are extended between those wave numbers determined by the maximum wave length and the minimum wave length of the disturbance and $\delta k = \delta l = \frac{2\pi}{4L_{\max}}$, where L_{max} is the maximum wave length.

4. Results and discussion

The numerical computation for both the vertical velocity (w') and streamline displacement (η') are made using equations (27) and (28) respectively, for all those waves for which $\left(k^2 + l^2\right) < \frac{N^2\left(k^2 + l^2\right)}{(Uk + Vl)^2}$. Scale analysis by Sarker (1965), Dutta *et al*. (2002) and Das *et al*. (2013) exhibited that, by ignore the effects of earth's rotation for the basic flow consists of both the zonal wind (U) and the meridional component (V), scale of the disturbance should not exceed 150 km. Since the horizontal grid size in the present study, has been taken to be 5 km. Hence, the minimum wave length should not be less than 30 km. In the equations (27) and (28) the summations k ranges from 4δk to 20δk and l ranges from $-20\delta k$ to $-4\delta k$ and $4\delta k$ to $20\delta k$. Where, we have been taken to avoid all those wave number vectors (k, l), which are inclined with the basic flow vectors (U, V) at an angle of $90°$ or more, to eliminate the critical level effect.

The down-stream variation of both w' and η' on a horizontal plane along the line $Uy - Vx = 0$ have been shown in Figs. 2(a-d) and Figs. 3(a-d) respectively, at

1.5 km, 3 km, 6 km and 9 km above the mean sea level, which is approximately resemble to 850 hPa, 700 hPa, 500 hPa and 300 hPa respectively.

From these figures, we see that both *w*' and η' decay downwind of the barrier, in qualitative conformity with the asymptotic solutions of Das *et al*. (2013, 2016). But, is no such specific rate of decay like as in the case of the asymptotic solution [Das *et al*. (2013)] could be found in the numerical case. From the expressions (27) and (28) of *w*' and η' respectively, we see that there are only one damping factor Bessel function is present, whereas in Das *et al*. (2013) found two damping factors in the asymptotic solution across the ABH.

The contours of the perturbation vertical velocity *w*' and streamline displacement η' at different horizontal planes above the mean sea level have been shown in Figs. 4(a-d) and Figs. 5(a-d) respectively. These figures show that, the vertical tilt of the wave field is insignificant and the maximum updraft/downdraft regions are no specific shaped, whereas, Das *et al*. (2013) have shown that, the maximum updraft/downdraft regions are crescent shaped in the asymptotic case across the same mountain barrier ABH.

Das *et al*. (2013) found the vertical velocity (*w*') and streamline displacement (η') tilt upstream and spread laterally with the vertical across the ABH. But, in the numerical case, the spreading rate of both *w*' and η' are almost the same at every level, *i.e*., is no spreading laterally with the vertical across the same barrier ABH. The dynamical cause of this situation may be due to the presence of a divergent part in the asymptotic case and the absence of a divergent part in the numerical case.

5. Conclusions

In this model, we have presented the numerical solution of 3-D meso-scale lee wave across the 3-D elliptical mountain barrier following the numerical approach. In the sequel, we have furnished some remarkable results. Moreover,

(*i*) The numerical solution for the vertical velocity (*w*') and streamline displacement (η') along the line $Uy - Vx = 0$ both decay down wind of the barrier. But, is no such specific rate of decay has found across the barrier.

(*ii*) In the horizontal plane, the contours of the vertical velocity (*w*') and streamline displacement (η') have not been seen any specific shaped across the barrier.

(*iii*) Both the vertical velocity (*w*') and streamline displacement (η') across the 3-D mountain barrier are upwind tilt along the line $Uy - Vx = 0$ and not spread laterally with height. The spreading rate almost the same for both *w*' and η' on every level across the barrier.

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Appendix

The Fourier Transform of the function $h(x,y)=\dfrac{h_1}{1+\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}}+\dfrac{h_2}{1+\dfrac{(x-d)^2}{a^2}+\dfrac{y^2}{b^2}}$ is

$$F[h(x,y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(x,y)e^{-i(kx+ly)}dxdy$$

$$\hat{h}(k,l)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{h_1}{1+\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}}+\frac{h_2}{1+\dfrac{(x-d)^2}{a^2}+\dfrac{y^2}{b^2}}\right]e^{-i(kx+ly)}dxdy$$

$$\hat{h}(k,l)=ab\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{h_1e^{-i(kx+ly)}}{1+\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}}dxdy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{h_2e^{-i(kx+ly)}}{1+\dfrac{(x-d)^2}{a^2}+\dfrac{y^2}{b^2}}\right]dxdy$$

Putting $x = aX$, $y = bY$ for the first term and $x - d = aX$, $y = bY$ for the second term

and use $k = \dfrac{k'}{a}, l = \dfrac{l'}{b}$ for the both terms, we have

$$\hat{h}(k,l) = ab\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{h_1 e^{-i(k'X+l'Y)}}{1+X^2+Y^2}dXdY + abe^{-i\frac{k'd}{a}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{h_2 e^{-i(k''X+l'Y)}}{1+X^2+Y^2}dXdY$$

$$\hat{h}(k,l) = ab\left(h_1 + h_2 e^{-ikd}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-i(k'X+l'Y)}}{1+X^2+Y^2}dXdY$$

Putting $X = r\cos\theta$, $Y = r\sin\theta$ and $k' = \kappa\cos\alpha$, $l' = \kappa\sin\alpha$ we get

$$\hat{h}(k,l) = ab\left(h_1 + h_2 e^{-ikd}\right)\int_{0}^{\infty}\int_{0}^{2\pi}\frac{e^{-ir\kappa\cos(\theta-\alpha)}}{1+r^2}rdrd\theta$$

Now, $\displaystyle\int_0^{2\pi} e^{-ir\kappa\cos(\theta-\alpha)}d\theta = 2\pi J_0[r\kappa]$ [Dutta *et al*. (2002)]

and $\displaystyle\int_0^{\infty}\frac{J_0(r\kappa)}{1+r^2}rdr = K_0(\kappa)$ [Dutta *et al*. (2002)]

where $J_0(r\kappa)$ and $K_0(\kappa)$ are Bessel function of 1st and 2nd kind of order zero respectively.

Hence, $\hat{h}(k,l) = 2\pi a\ b\left(h_1 + h_2 e^{-ikd}\right)K_0(\kappa)$, where $\kappa = \sqrt{a^2k^2 + b^2l^2}$

Therefore, $\hat{h}(k,l) = 2\pi a\ b\left(h_1 + h_2 e^{-ikd}\right)K_0\left(\sqrt{a^2k^2 + b^2l^2}\right)$