Mausam, (1995), **46**, 2, 187-192

550.348.436 : 517.93

Large fractal dimension of chaotic attractor for earthquake sequence near Nurek reservoir

S. N. BHATTACHARYA, K. C. SINHA RAY* and H. N. SRIVASTAVA*
Meteorological Office, New Delhi
(Received 12 October 1993)

सार — 1976-87 की अवधि के दौरान रिकार्ड किए गए 22,000 भूकम्पों और दिसंबर 1977 से दिसबर 1987 तक किए गए प्रेक्षणों की पूरी अवधि में न्यूरिक बांध में भूकम्पों के अनुक्रम के कैओटिक अटरेक्टर के फ्रक्टल विस्तार का अध्ययन किया गया है। इस अध्ययन में जलाशय भरने की दूसरी अवस्था के दौरान बढ़ी हुई भूकम्पी गतिविधि शामिल नहीं है। उपर्युक्त अवधियों में क्रमश: 8.3 और 7.3 के कैओटिक अटरेक्टर के बड़े फ्रक्टल विस्तार पाए गए जिनसे यह पता चलता है कि कोयना जलाशय की तुलना में इस क्षेत्र की भूकम्प-गतिकी जटिल है।

ABSTRACT. Fractal dimension of the chaotic attractor for earthquake sequence in Nurek dam based on 22,000 earthquakes detected during the period 1976-87 has been studied for this total period of observations as well as for the period from December 1977 to December 1987. The second period excluded increased seismic activity during second stage of filling the reservoir. Large fractal dimensions of the chaotic attractor of 8.3 and 7.3 were found for the respective period which suggests the complexity of earthquake dynamics in this region as compared to Koyna reservoir.

Key words — Earthquake, Random, Deterministic, Attractor, Thrust, Fault Chaos, Seismicity.

1. Introduction

The distribution of earthquakes for a region gives a complex temporal pattern. This distribution in time and space has been studied by many investigators in order to understand the earthquake generation process. If each earthquake occurrence is totally uncorrelated with previous earthquakes, then the earthquake distribution would be a random process. The random distribution of point events is known as a Poisson process. A variety of statistical methods have been applied in order to quantify deviations from random occurrence (Vere-Jones and Ozaki 1982, Matsumura 1984, Dziewonski and Prozorov 1984). Rikitake (1976) noted quasi-periodical patterns for sequences of great earthquakes at subduction zone. However, Knopoff (1964) and Gardner and Knopoff (1974) showed that although typical catalogues might be non-Poissonian, but main sequence was Poissonian provided that the aftershocks are removed skillfully. Thus, it is necessary to characterize the complexity of earthquake dynamics more precisely and to determine whether observed fluctuations in the data are random or belong to some deterministic behaviour.

Over the last fifteen years, scientists of many disciplines have developed physics of chaos to look at the complexity of nature. A chaotic system is deterministic, *i.e.,* it obeys certain equations, but behaviour of the system is so complicated that it looks random. A primary motivation of physics of chaos is to find out whether there exists a simple explanation for an observation with irregular behaviour and if it exists, how simple is the explanation. The realization that highly irregular and quasi-random behaviour may be generated from simple deterministic dynamics led to the search of deterministic chaos in many fields. These investigations were triggered by the work of Packard *et al.* (1980) and Takens (1981) which showed that a phase portrait underlying a given dynamical system can be associated with a strange attractor. An attractor is a set of points in a state (or, phase) space towards which a time history approaches after transients die out. One of the most distinctive characteristics of a strange attractor is that it is fractal and has non-integer dimension (Mandelbrot 1982). This dimension is a quantitative measure of the degree of chaos. In a random sample this dimension tends to be infinite. Further, the dimension gives minimum number of independent variables needed to model the system. This means that the determination of the fractal dimension of the attractor of the system sets a number constraints that is required to model the system.

*Meteorological Office, Pune.

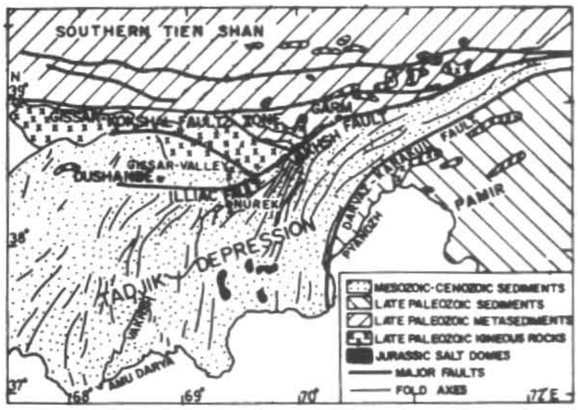


Fig. 1. Generalised geology of the Tadjik depression, southern Tien Shan and north-western Pamir

The earthquake sequence in Nurek dam area is being monitored by a large number of seismological observatories. Over 22,000 earthquakes have been located between 1976 and 1987. This gives an excellent opportunity to find the presence of chaotic attractor and to determine the dimension of this attractor. This, in turn, estimates the minimum number of independent variables necessary to model the system.

2. Seismo-tectonics of Nurek dam area

Nurek dam is located in the northern part of the Tadjik depression which is post-Paleozoic sedimentary basin. The basin is in a gorge of the Vaksh river, a tributary of Amu Dary'a. The Tadjik depression is a structural block involved in the collision between the Indian and Eurasian plate which started 40 million years ago. The depression on the west is bounded by the Turan platform. The other three sides are bounded by major active tectonic features: the Gissar-Kokshal fault in the north, the Darvaz-Karakul fault in the east and Hindukush region in the south (Fig. 1). The Gissar-Kokshal and Vaksh-Illiack faults form a major boundary between the Caledonian and Hercynian-deformed structures of the Tien Shan to the north and Alpine deformed structure to the south which is compressing north-north-westward related to the Indo-Eurasian collision. Many earthquakes of magnitude 8 and above occurred during this century along Gissar-Kakshal fault and Pamir-Hindukush region.

Satellite image mapping of Tadjik depression complemented by field survey shows that the deformation of the subsided margin sequence exposed in the folds and thrusts of the Vaksh belt has resulted from northward protrusion of Pamir into the southern margin of Asia (Leith and Alvarez 1985). The development of folds and thrust belt has included the progressive overlapping of thrusts.

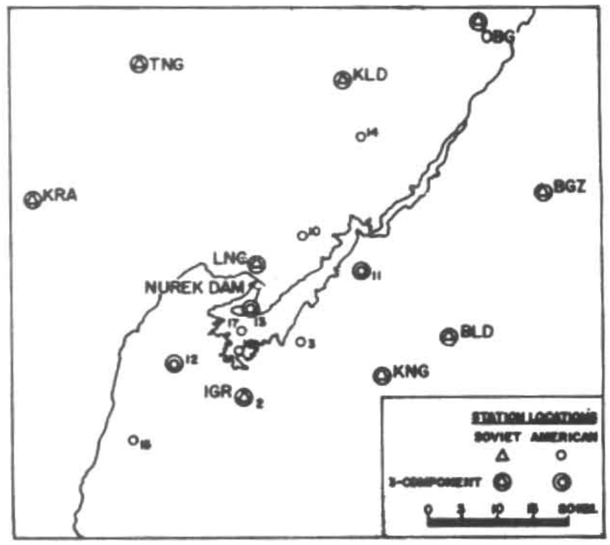


Fig. 2. Seismograph stations in the vicinity of Nurek reservoir

Fault plane solutions for earthquakes in the Tadjik depression show both strike fault and thrust fault (Soboleva and Mamadaliev 1976). Thrust motion is found on tectonic features with north-easterly trend; strike slip motion occurs on east-west features such as the Illiack fault where right-lateral motions take place on the north-south features in the central and western part of the depression where left-lateral motions take place. The induced earthquakes, after second stage of filling of the reservoir in 1976, are confined to depths of 8 km and mainly confined in Ionakhsh thrust, a listric fault 1 km north of the dam (Keith *et al.* 1982). Fault plane solutions for the induced earthquakes show short segment of strike slip and thrust faulting for the most activity near the central basin and normal and thrust faulting along the upstream edges of the reservoir (Keith *et al.* 1982).

3. Data

Tadjik Institute of Seismo-Resistant Construction and Seismology has operated 15 seismograph stations within 100 km of Nurek since 1955. Among these there were only 5 stations within 40 km of the reservoir; four additional stations were added before 1976. In 1975, as a part of joint Soviet-American program, a radio-telemetered network of 10 stations was installed around the reservoir. Fig. 2 shows the locations of two sets of stations. As all these stations were installed by 1976, we have considered data from this year for the sake of uniformity. As mentioned earlier over 22,000 earthquakes were located during the period from 1976 to 1987.

4. Strange or chaotic attractors

We may consider the dynamics of system, such as earthquakes, simulated by partial differential equations describing the underlying physical processes. These equations can be transformed to a set of n ordinary differential equations :

$$x'_j = f_j\,(x_1, x_2, \ldots, x_n);\; j = 1, 2, \ldots, n \qquad (1)$$

where the prime denotes differentiation with respect to time t. The time evolution of the system from an initial condition can be described by trajectories in n-dimensional state space with coordinates x_1, $x_2, \ldots, x_n$ which are n independent variables. Normally all trajectories converge and remain on a submanifold of the total available space. The submanifold which attracts the trajectories is called an attractor. The dimension D of the attractor is less than that of the state space, *i.e.*, $D < n$. An attractor of a dynamical system represents the asymptotic limit of the trajectories in a state space spanned by the independent variables which define the dynamics. Thus, if $D = I + p$, where I is an integer and p is $0 < p < 1$, then minimum number of independent variables of the system is $I + 1$ (Moon 1987).

The classic attractors are all associated with classic geometric objects in space, such as, (*i*) the equilibrium state with a point has dimension zero and (*ii*) the periodic motion or limit cycle with closed curve has dimension one. These attractors have integer dimensions which are equal to topological dimensions of submanifold in the state space. The trajectories converging on these attractors do not diverge and maintain a constant distance from each other. Thus, the states of the system at a later time will differ to the same extent that they differed initially. Thus, knowing the evolution of such a system from an initial condition, we can predict the evolution of the system from some other initial condition. Such attractors may be called as non-chaotic attractors.

But there are other dynamical systems where the trajectories remain on an attracting submanifold that is not topological. Such submanifold is strange attractor which has non-integer dimension and associated with a new geometric object called a fractal set (Mandelbrot 1982, Peitgen and Richter 1986). In a three dimensional space, the fractal set of a strange attractor looks like a collection of infinite set of sheet or parallel surfaces, some of which are separated by distances that approach infinitesimal. With such an attractor, initially nearby trajectories diverge; thus the evolution of the system from two slightly different initial conditions will be completely different. Thus, following the equations that describe the system, the state of the system after sometime can be anything despite the fact that the initial conditions were close to each other. This imposes limits on prediction and even if the system is described by equations, the system shows randomness. The randomness generated this way has been termed as chaos. Such systems are chaotic dynamical systems and their attractors are called strange or chaotic attractors.

5. Computation of attractor's dimension

The system (1) can be reduced to a single differential equation of one of the variables $x_j\,(t)$, say $x(t)$, if all others are eliminated by differentiation. This gives an nth order non-linear differential equation as :

$$x^{(n)} = f(x, x', \ldots\ldots x^{(n-1)}) \qquad (2)$$

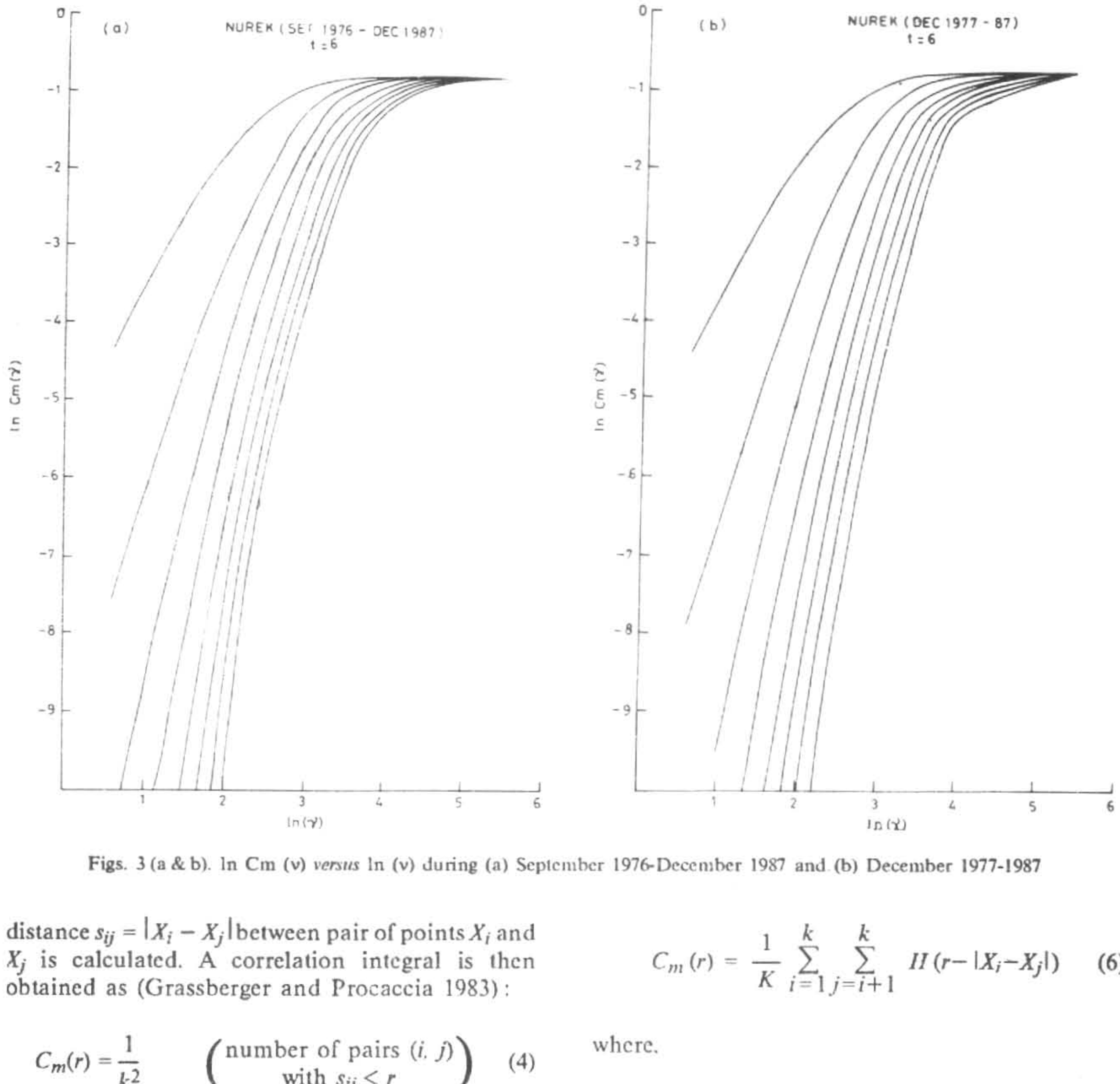
So, we replace the state space with $x, x', \ldots\ldots x^{(n-1)}$ without any loss of information about the dynamics of the system.

The theorem of Takens (1981) says that D-dimensional manifolds can be embedded into $m = 2D + 1$ dimensional space. Thus, for deriving the dimension of an attractor from a single state variable, it is sufficient to embed them into an m-dimensional space spanned by x and its $(m$-1) derivatives, *i.e.*, $x, x', \ldots\ldots x^{(m-1)}$. Thus, it is not necessary to know the original state space and its dimension n as long as m is chosen large enough. Ruelle (1981) suggested that instead of continuous variables $x(t)$ and its $(m$-1) derivatives, a discrete time series $x(t)$ and its shifts $(m$-1) time lags by a delay parameter τ can be considered.

We may begin computation with a time series of dependent or independent variable $x(t)$ of the system. We construct points X_i in an m-dimensional embedding space :

$$X_i = [x(t_i), x(t_i + \tau), \ldots\ldots, x\{t_i + (m-1)\tau\}] \qquad (3)$$

$i = 1, 2, \ldots, N-m+1$ ($= k$, say), where N is the number of data in the time series of x. Here t_1 is initial time and $t_i = t_1 + (i\text{-}1)\tau$. Thus one discretizes the orbit to a set of k points X_i in the state space. The



Figs. 3 (a & b). ln Cm (v) *versus* ln (v) during (a) September 1976-December 1987 and (b) December 1977-1987

distance $s_{ij} = |X_i - X_j|$ between pair of points X_i and X_j is calculated. A correlation integral is then obtained as (Grassberger and Procaccia 1983):

$$C_m(r) = \frac{1}{k^2} \begin{pmatrix} \text{number of pairs } (i, j) \\ \text{with } s_{ij} < r \end{pmatrix} \quad (4)$$

where r is correlation length. For an attractor with dimension D it has been found:

$$\lim_{m \to \infty} \lim_{r \to 0} C_m(r) = a\, r^D$$

where, a is a constant. Thus, the (correlation) dimension is obtained by

$$D = \lim_{m \to \infty} \lim_{r \to 0} \frac{d\{\ln C_m(r)\}}{d(\ln r)} \quad (5)$$

$C_m(r)$ may be calculated more effectively using the relation (Abraham *et al.* 1986, Theiler 1988)

$$C_m(r) = \frac{1}{K} \sum_{i=1}^{k} \sum_{j=i+1}^{k} H(r - |X_i - X_j|) \quad (6)$$

where,

$$H(x) = 1, \quad \text{for } x > 0$$
$$= 0, \quad \text{for } x < 0$$

and $K = k(k-1)/2$ is number of distinct pairs of points. In practice, ln $C_m(r)$ against ln r is plotted. The $C_m(r)$ saturates at large values of r due to finite N. In the plot we get a scaling region, where ln $C_m(r)$ is linear to ln r. The slope ν of the straight line passing through the points in scaling region is obtained. The value of ν is obtained for increasing sequence of embedding dimension m. If the time series belongs to deterministic dynamics, ν will reach a saturation value D with increasing m and if D is non-integer the system is chaotic. However, if the

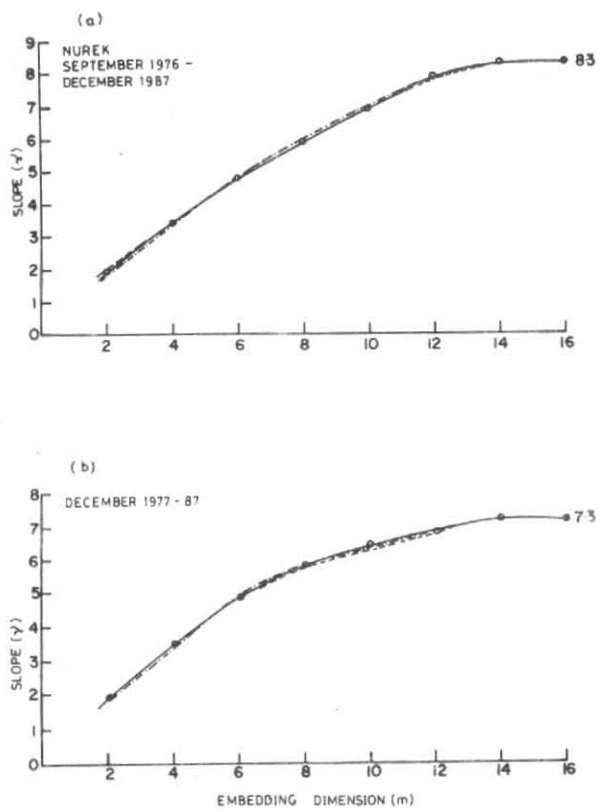


Fig. 4 (a & b). Embedded dimension *versus* ν for time delays 2, 4, 6 days during (a) September 1976-December 1987) and (b) December 1977-1987

time series is random, ν will not saturate with increase of m (Fraedrich 1986).

6. Results and discussion

The second stage of filling in the reservoir of Nurek dam began in July 1976 and level of seismicity began to increase by the end of August 1976. This increased activity continued till November 1977. For this reason we have considered two periods : (*i*) From September 1976 to December 1987 and (*ii*) from December 1977 to December 1987. Thus the former period includes the increased activity that occurred immediately after second stage of filling and the later period excludes this period.

The number of earthquakes in each of two days' duration is considered as observed variable and the time series is formed. For the first period, N = 2069, and for the second period, N = 1840. With this time series and using the method described in previous section, the correlation integral $C_m(r)$ is obtained with τ = 2, 4 and 6 days; $C_m(r)$ is plotted against correlation length r. For each embedding dimension m, the scaling region is determined and the slope ν is calculated by fitting a straight line through the points in the scaling region. Fig. 3 (a) shows such plotting for τ = 6 days and for the first period. Similar plotting for the second period is shown in Fig. 3 (b).

Fig. 4 (a) shows the slope ν as a function of embedding dimension m for τ = 2, 4, 6 days and for the first period. It is seen that ν increases with m until saturation value of ν is reached at m = 14 and the saturation value is 8.3. This gives fractal dimension D = 8.3 of the attractor and since the dimension is non-integer, it is a strange attractor. Similarly, Fig. 4 (b) depicts saturation of ν for the second period. The slope ν saturates to 7.3 at m = 14. Thus the fractal dimension of strange attractor for this period is 7.3. Figs. 4 (a & b) show that the difference in variation of ν with τ = 2, 4, 6 is very small.

It may be mentioned that the strange attractor was also noted for earthquakes in Hindukush region with fractal dimension 6.9 (Bhattacharya and Srivastava 1992). This was almost similar to that reported for Parkfield, California where the underlying structure had only six degrees of freedom. On the other hand, only 5 parameters are needed to model earthquakes in the Koyna region which is one of the unique examples of reservoir associated seismicity in the intra-plate regime, far away from the Indian-Eurasian plate boundary (Srivastava *et al.* 1993). Thus, large fractal dimension in the Nurek dam area could be attributed to the complexity of earthquake dynamics in the region as compared to Koyna region.

7. Conclusions

(*i*) Using the data of September 1976 to December 1987, it has been shown that the earthquakes in Nurek dam area are chaotic and a strange attractor exists.

(*ii*) The strange attractor has a fractal dimension of 8.3. However, the fractal dimension becomes 7.3 if we exclude the increased seismic activity of September 1976 to November 1977.

(*iii*) Minimum 9 independent variables are necessary to model the dynamics of earthquake system in Nurek dam area. However, minimum number of the variables reduces to eight in case we exclude the increased seismic activity of September 1976 to November 1977.

(*iv*) The large number of independent variables for modelling the system is due to complexity of tectonic features leading to better understanding of earthquake dynamics of the region.

Acknowledgements

One of the authors, H. N. Srivastava, is thankful to the Secretary, Department of Science and Technology, Director General of Meteorology and Prof. V. K. Gaur, Area Co-ordinator of the ILTP programme for sponsorship to Russia.

References

Abraham, N. B., Albano, A. M., Das, B., Guzam, G. De., Yong, S., Goggia, R. S., Puccioni, G. P. and Tredicce, J. R., 1986, "Calculating dimension of attractor from small data sets," *Phys. Lett. A.*, **114**, pp. 217-221.

Bhattacharya, S. N. and Srivastava, H. N., 1992, "Earthquake predictibility in Hindukush region using seismicity pattern and chaos," *Bull. Ind. Soc. Eq. Tech.*, **29**, 4, pp. 23-36.

Dziewonskii, S. M. and Prozorov, A. G., 1984, "Self similar determination of earthquake clustering, *Com. Seism.*, **16**, pp. 7-16.

Fraedrich, K., 1986, "Estimating the dimensions of weather and climate attractors," *Bull. Amer. Met. Soc.*, **43**, pp. 419-432.

Gardener, J. K. and Knopoff, L., 1974, "Is the sequence of earthquake in Southern California, with aftershocks removed, Poissonian ?" *Bull. Seism. Soc. Am.*, **64**, pp. 1363-1367.

Grassberger, P. and Procaccia, I., 1983, "Characterization of strange attractors," *Phys. Rev. Lett.*, **50**, pp. 346-349.

Keith, C. M., Simpson, D. W. and Soboleva, O. V., 1982, "Induced seismicity and style of deformation at Nurek reservoir, Tadjik SSR," *J. Geophys. Res.*, **87**, pp. 4609-4624.

Knopoff, L., 1964, "The statistics of earthquakes in Southern California," *Bull. Seism. Soc. Am.*, **54**, pp. 1871-1873.

Leith, W. and Alvarez, W., 1985, "Structure of the Vaksh fold and thrust belt, Tadjik SSR : Geologic mapping on a Landsat image base", *Geol. Soc. Am. Bull.*, **96**, pp. 875-885.

Mandellbrot, B. B., 1982, "The fractal geometry in nature", W. H. Freeman, San Francisco.

Matsumura, S., 1984, "A one parameter expression of seismicity patterns in space and time," *Bull. Seism. Soc. Am.*, **74**, pp. 2559-2576.

Moon, F. C., 1987, "Chaotic vibrations," John Willy and Sons, NY.

Packard, N. H., Crutchfield, J. P., Farmer, J. D., and Shaw, R. S., 1980, "Geometry of time series," *Phys. Rev. Lett.*, **45**, pp. 712-716.

Peitgen. H. O. and Richter, P. H., 1986, "The beauty of fractals," Springer Verlag, Berlin.

Rikitake, T., 1976, "Recurrence of great earthquakes at subduction zones", *Tectonophys.*, **35**, pp. 335-362.

Ruelle, D., 1981, "Chemical kinetics and differentiable dynamical systems in non-linear phenomenon in chemical dynamics," Eds. A. Pacoult and C. Vidal, Springer-Verlag, pp. 30-37.

Soboleva, O. V. and Mamadaliev, U.A., 1976, "The influence of the Nurek reservoir on local earthquake activity," *Eng. Geol.*, **10**, pp. 293-305.

Srivastava, H. N., Bhattacharya, S. N. and Sinha Ray, K. C., 1993, "Strange attractor dimension as a new measure of seismotectonics around Koyna reservoir" (under publication).

Takens, F., 1981, "Detecting strange attractors in turbulence", Lecture notes in Mathematics, **898**, 366-381,Springer-Verlag, Berlin.

Theiler, J., 1988, "Lacunarity is a best estimator of fractal dimension," *Phys. Lett. A.*, **133**, pp. 195-200.

Vere-Jones, D. and Ozaki, T., 1982, "Some examples of statistical estimation of earthquake data Pt. I. Cyclic Poisson and self exciting models," *Ann. Inst. Statist. Math.*, **34**, pp. 189-207.