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Analysis of soil temperature at various depths by Fourier technique

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सार — भारत के शुष्क खेती प्रदेश में से सात स्थानों के 5, 15 और 30 सें. मी. गहराइयों के औसत साप्ताहिक मृदा तापमानों का प्रसंवादी विश्लेषण किया गया है। विभिन्न प्रसंवादों के तदनुरूप आयाम तथा प्रावस्था कोण प्रस्तुत किये गये हैं।

मृदा की सतह के पास (5 सें. मी. गहराई पर) अधिकतम उष्ण मृदा 16 वें और 19 वें सप्ताह के बीच में होती है जबकि उच्चतम अधिकतम मृदा तापमान 20 वें और 26 वें सप्ताह की अवधि में (30 सें. मी. गहराई पर) पाया जाता है।

ABSTRACT. Harmonic analysis of weekly means of soil temperatures at 5, 15 and 30 cm depths have been done for seven stations of India. The corresponding amplitudes and phase angles in respect of different harmonics are presented.

The warmest soil near the soil surface (5 cm depth) occurs during the period 16th to 19th week. While the highest maximum occurs during the period 20th to 26th week (30 cm depth).

Key words — Harmonic analysis, normal soil temp., dry farming soil, amplitude, phase angle, bimodal time series & annual temp. cycle.

1. Introduction

Soil temperatures, particularly at the extremes, influence the germination of seed, the functional activity of roots, the date and duration of plant growth and the occurrence and severity of plant diseases. Extremely high soil temperatures have a harmful effect on roots and may cause destructive lesions on the stems of plants. On the other hand, low temperatures impede the plant mineral nutrient intake and persistently cold soil results in dwarfed growth. The ecological significance of soil temperature is obvious from the fact that an unfavourable value of this parameter during the growing season may retard or even ruin the crops. All these factors adversely affect the crop yield. Therefore, soil temperature is an important agrometeorological parameter and its correct estimate over the march of time is essential (Chang 1964).

Earlier, it was noticed by several scientists that the annual temperature cycle in the soil was fairly well described by first harmonic of soil temperature alone showing a symmetrical pattern with a maximum around mid July at the top layers with progressive lag towards the lower depths. But the actual observed soil temperature variation for various depths in case of several stations of India, indicates that the said pattern is not a symmetrical one. Considerable deviation is produced by the monsoon season from June to end of September. The fall of soil temperatures with the onset of monsoon and later rise during the early postmonsoon season causes the time series of soil temperatures in the said stations to be bimodal and does not allow the use of a simple sine curve to explain the complete cycle. Hence, the effect of higher harmonics have also to be considered for representing the actual observed variation of soil temperatures (Krishnan and Kushwaha 1972). The aim of the present study is to find out the periodicity of the soil temperatures at various depths using Fourier technique.

2. Data

The soil temperature data are recorded by standard soil thermometers installed at the depths of 5 cm, 15 cm and 30 cm in the agromet observatories (Table 2). These stations have been selected so that most of the types of dry farming soil in India could be represented.

The soil at experimental site of these stations is as follows :

Pune	Brown to dark brown calcareous soil with varying depth and texture
Bhubaneshwar	Sandy loamy
Pantnagar	Haldy loamy, fine texture, brownish
Hisar	Sierozem soil, calcareous
Solapur	Medium type, coarse texture, blackish brown
Pattambi	Laterite type, reddish
Rajahmundry	Laterite (Alexander 1972)

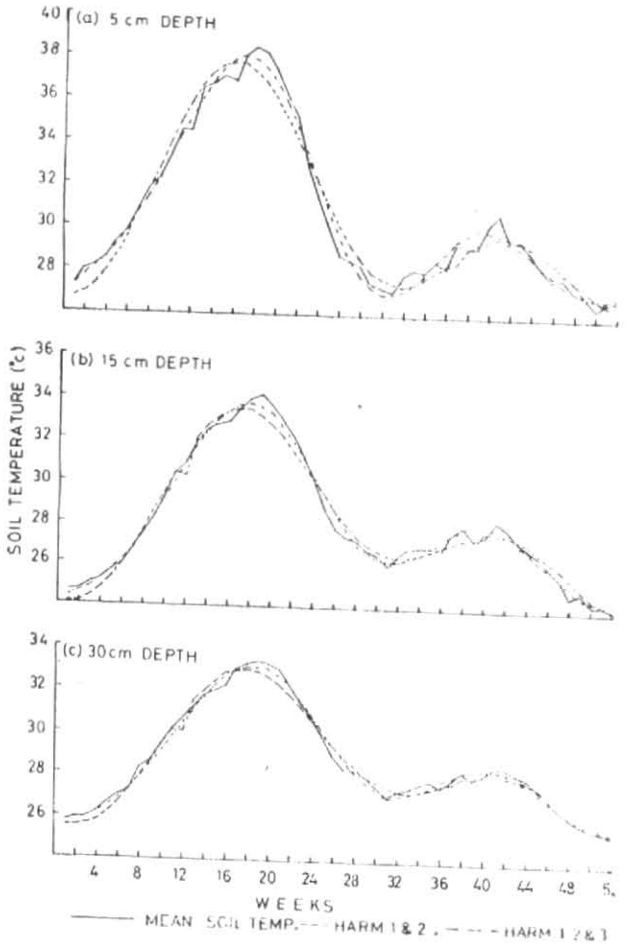

Fig. 1. Weekly mean observed soil temperature at Solapur at (a) 5 cm, (b) 15 cm & (c) 30 cm depths and those estimated by first two and first three harmonics

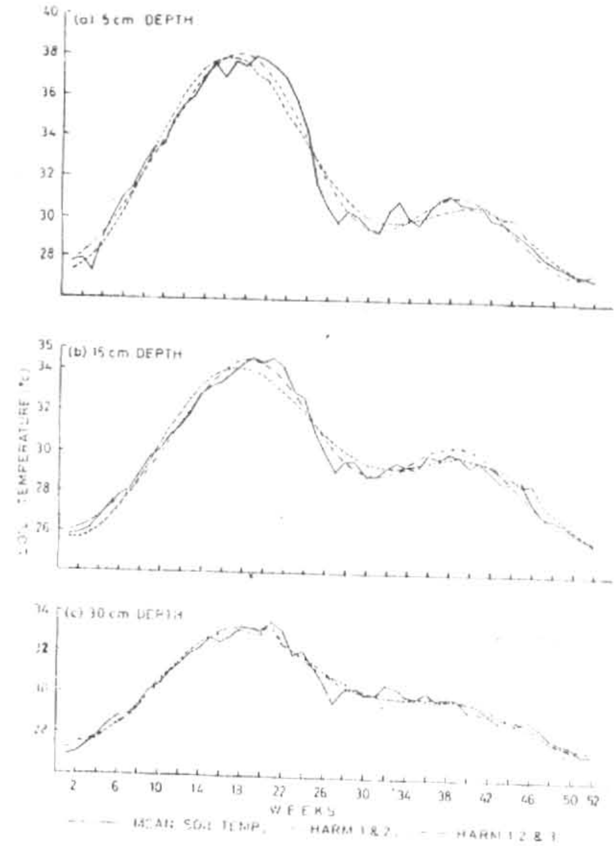

Fig. 2. Weekly mean observed soil temperature at Rajahmundry at (a) 5 cm, (b) 15 cm & (c) 30 cm depths and those estimated by first two and first three harmonics

The experimental site is always kept free of vegetation. The soil temperatures were recorded at 0700 and 1400 LMT. These hours of observations roughly correspond to the minimum and maximum temperature epochs of air respectively. Normal weekly means (1951-1980) are used in this investigation. Weekly means of soil temperatures at various depths are computed for the whole year divided into 52 standard weeks beginning from 1 January. The 52nd week (24-31 December) and 9th week during leap years (26 February-4 March) are assumed to contain 8 days.

3. Method of analysis

Any periodic function can be represented by a sine and cosine series or in other words, the periodical variation can be expressed as a superposition of several sinusoidal variations. In this case, the weekly mean soil temperatures at various depths are the periodic functions to be studied. The general form of Fourier series representing the weekly mean soil temperature is as follows :

$$T = \overline{T} + \sum_{n=1}^{N/2} \left\{ A_n \cos\left(\frac{360}{P} nx\right) + B_n \sin\left(\frac{360}{P} nx\right) \right\} \quad (1)$$

where, $\overline{T}$ is the mean of the 52 weekly averages for the year, N=No. of equally spaced data points which is 52 in this case, A_n and B_n are the amplitudes (half range) of the various terms; P is the period of the fundamental cycle, x is time factor varying from 0-51. Eqn. (1) can also be expressed as follows :

$$T = \overline{T} + \sum_{n=1}^{N/2} C_n \left\{ \sin\left(\frac{360}{P} nx\right) + \phi_n \right\} \quad (2)$$

TABLE 1

Amplitude and phase angle of soil temperatures in respect of first three harmonics and observed mean annual temp. (°C) at different depths

Stations	1st harmonic		2nd harmonic		3rd harmonic		Mean observed temp. (T) (°C)
	Amplitude	Phase angle	Amplitude	Phase angle	Amplitude	Phase angle	
			5 cm depth				
Solapur	3.99	341.13	3.05	244.31	0.74	59.27	30.73
Bhubaneshwar	5.33	308.49	2.73	249.04	0.91	43.61	30.31
Rajahmundry	3.93	328.02	2.49	248.55	0.56	44.72	31.72
Pantnagar	9.31	272.45	3.08	235.99	0.54	44.66	26.65
Pattambi	3.38	29.50	2.16	269.29	0.52	116.62	31.56
Hisar	10.67	269.10	3.62	234.41	0.80	354.06	28.33
Pune	3.52	341.36	3.20	245.72	0.54	14.30	28.82
			15 cm depth				
Solapur	3.16	320.48	2.43	238.82	0.45	47.33	28.17
Bhubaneshwar	4.88	300.01	2.59	249.05	0.73	20.26	29.61
Rajahmundry	3.05	306.55	1.96	237.53	0.57	39.20	29.79
Pantnagar	9.02	269.10	2.91	234.51	0.50	14.07	23.38
Pattambi	2.90	24.86	1.80	263.50	0.52	96.56	30.84
Hisar	10.36	263.92	2.67	235.21	0.51	328.52	26.22
Pune	2.80	316.28	2.72	231.44	0.32	341.25	26.63
			30 cm depth				
Solapur	2.59	316.91	1.86	230.33	0.38	31.21	28.68
Bhubaneshwar	4.71	295.01	2.36	248.48	0.60	12.72	28.75
Rajahmundry	2.25	306.85	1.13	241.39	0.16	93.14	30.03
Pantnagar	8.28	266.17	2.61	230.59	0.44	25.24	25.04
Pattambi	2.90	23.55	1.69	262.01	0.45	97.05	31.09
Hisar	9.59	14.80	2.55	230.04	0.44	332.17	26.42
Pune	2.63	314.45	2.33	231.00	0.43	353.96	27.44

where, C_n is the amplitude of n^{th} harmonic and is given by $(A_n^2+B_n^2)^{1/2}$ and ϕ_n is the phase angle given by $\tan^{-1}(A_n/B_n)$.

The daily values of soil temperatures recorded at 0700 and 1400 LMT were averaged to obtain daily means. Weekly means were then worked out for each standard week for the data from 1951 to 1980. These means were then averaged to obtain normal soil temperature pattern for the 52 standard weeks of the year (Krishnan and Kushwaha 1972).

4. Results and discussions

Amplitudes and phase angles of soil temperatures in respect of first three harmonics are given in Table 1. From this inferences drawn are as follows :

(1) The mean observed soil temperature ($\overline{T}$) first decreases as we go down from 5 cm to 15 cm depth for all the stations but increases again as we go down further to 30 cm depth for all the stations except Bhubaneshwar where it decreases further.

(2) In case of temperatures at 5, 15 and 30 cm depths estimated by Fourier technique, the amplitudes decrease sharply with higher order of harmonics. The values have the range of 2.25° to 10.67°C for the first harmonic, 1.13° to 3.62°C for the second harmonic and 0.16° to 0.91°C for the third harmonic.

All the lowest values are recorded at Rajahmundry. Highest values for the first two harmonics are recorded at Hisar while Bhubaneshwar records the highest value of 0.91 for the third harmonic though the value 0.80 at Hisar is also quite high.

(3) In general, phase angles decrease with higher order harmonics for the same depth as well as with deeper depths for the same harmonic. I & II harmonics show

TABLE 2

Total variance (°C) of the soil temperatures at various depths and its percentage accounted for by different harmonics

Stations (Lat.,Long.)	Percentage of variance accounted for by the harmonics — 5 cm				15 cm				30 cm			
	I	II	III	Total	I	II	III	Total	I	II	III	Total
Solapur (17°40′ N, 75°54′E)	60.70	35.56	02.10	98.36	61.13	36.09	1.23	98.45	64.04	33.09	1.37	98.50
Bhubaneshwar (20°15′ N, 85°52′E)	76.62	20.09	02.21	98.92	75.99	21.33	1.69	99.01	73.38	19.68	1.27	99.33
Rajahmundry (17°00′ N, 81°46′E)	68.51	27.59	01.37	97.47	66.68	27.55	2.35	96.58	76.92	19.55	0.40	96.87
Pantnagar (29°00′ N, 79°30′E)	89.37	09.81	00.30	99.48	90.19	09.37	0.28	99.84	90.47	08.99	0.25	99.71
Pattambi (10°48′ N, 76°12′E)	69.00	28.17	01.63	98.80	69.05	26.63	2.25	97.93	72.79	24.68	1.74	97.21
Hisar (29°10′ N, 75°46′E)	88.75	10.21	00.50	99.46	93.09	6.17	0.22	99.48	92.98	06.58	0.20	99.76
Pune (18°32′ N, 73°51′E)	53.41	44.12	01.25	98.78	50.77	47.84	0.66	99.27	54.55	42.81	1.43	98.79

more consistency (only two stations Hisar and Bhubaneshwar disagree) than the III harmonic (all stations except Solapur and Bhubaneshwar disagree). Similarly 5 and 15 cm depths show more consistency (only 3 stations Hisar, Pattambi and Pune disagree) as compared to 30 cm depth (all stations except Solapur and Bhubaneshwar disagree).

(4) Amplitudes and phase angles generally decrease with higher order of harmonics as we go to lower depths.

(5) Percentage of variance accounted for by the first three harmonics is almost same at all the depths under study and amounts to 99 per cent when taken together for each depth.

5. Soil temperature patterns

It is observed that the soil temperature curve is bimodal in case of Solapur and Rajahmundry while it is unimodal in case of Hisar—perhaps because the first two stations are in the tropics. In the first case the first mode is around 19th week for both the stations and second around 40th week in case of Solapur and around 38th week in case of Rajahmundry. The second mode is quite prominent in case of Solapur while it is not so especially at 15 cm and 30 cm depths in case of Rajahmundry. In case of Solapur the fall in temperature after first mode is more steep as compared to Rajahmundry. The single mode in case of Hisar is around 24th week and the fall of temperature is gradual, declining sharply after 40th week. While the highest temperatures recorded are almost same (about 38°C at 5 cm depth) in case of all the stations the lower limit of temperature differs much in case of Hisar as compared to the other two stations (Hisar-14° C, Solapur-24° C, Rajahmundry -27° C).

6. Variance explained by different harmonics

In order to decide how many harmonics should be computed for depicting the observed data well, one has to determine the number of harmonics required for reducing the variance of data below a selected value. Panofsky and Brier (1958) showed that the variance accounted for by a given harmonic is given by half of the square of its amplitude ($C_n^2/2$) except that of the last harmonic which is equal to the square of the amplitude (C_n^2).

The total variance of the data is the variance of data about the average (T). Hence, the ratio $C_n^2/2$ to the total variance shows the fraction of variability accounted for by each harmonic. The data in respect of the first three harmonics are presented in Table 2 (Krishnan and Kushwaha 1972).

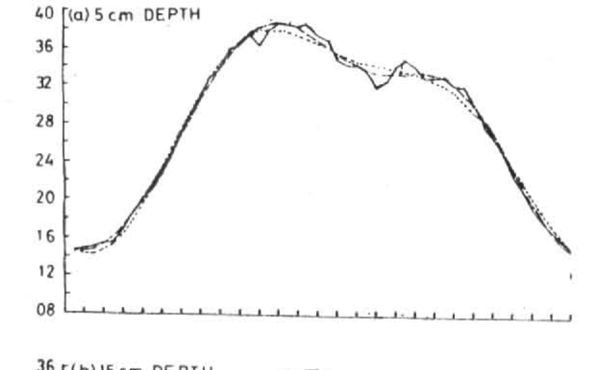

Fig. 3. Weekly mean observed soil temperature at Hisar (a) 5 cm, (b) 15 cm & (c) 30 cm depths and those estimated by first two and first three harmonics

It is interesting to note that though many workers have reported that the annual temperature cycle is well represented by the first harmonic alone, it has not been found true for the above stations. In our case, the first harmonic explains only 51-94% of the total variance at different depths, while the 2nd and 3rd harmonics represent 6-48% and 0.20-3% of the total variance respectively. This also explains the bi-modal nature of time series of soil temperatures due to occurrence of monsoon season. Types of soil temperature profiles during monsoon are different from those of other summer months due to infiltration of rainfall. The total variance explained by the first three harmonics varies from 96.6 to 99.8 per cent for all the depths.

7. Restructuring original data curve from Fourier co-efficients

From Table 2, it would be seen that with fairly good accuracy, the original data of weekly mean soil temperatures at various depths can be represented in terms of the first three harmonics.

Graphs have been drawn for actual soil temperature (observed) and those predicted by first two and first three harmonics taken together at 5 cm, 15 cm and 30 cm depths for all the seven stations under consideration. It is seen from these graphs that even the combination of the first two harmonics fit the observed values well except the fact that they fail to represent the fall of soil temperature during monsoon and rise during early post-monsoon season. These fluctuations exist at all layers under study. They are more marked in upper layers down to a depth of 30 cm than those of lower ones. The change in the pattern due to monsoon is well reflected especially in the upper layers, if the third harmonic is superimposed. Therefore, addition of third harmonic to the first two makes the fit between the observed and estimated values better during other seasons. Graphs of Solapur, Rajahmundry and Hisar are shown in Figs. 1, 2 and 3 respectively.

8. Conclusion

(*i*) Soil temperatures and their maxima/minima at different depths and at any time can be estimated for a place or a nearby place on the basis of their annual periodicity with the help of harmonics computed by Fourier technique.

(*ii*) First three harmonics taken together give good fit between the estimated and observed soil temperatures.

(*iii*) Combination of first three harmonics account for the fall of soil temperature during monsoon and its rise during early post-monsoon season.

(*iv*) Tropical stations show bimodal soil temperature pattern while extra tropical stations show a unimodal one.

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