Universal spectrum for short period (days) variability in atmospheric total ozone

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सार — सामयिक अस्थिरता के शक्ति स्पैक्ट्म के प्रतिलोम शक्ति नियम के साथ सहवर्ती स्थानिक प्रतिमान के स्वत: समरूप फ्रक्टल ज्यामिति के रूप में निर्घारित किए गए दूरी और समय से संबंधित दीर्घ अवधि सहसंबंध वास्तविक विश्व की गतिकीय पढ़ति में बहुव्यापी है और हाल ही में यह पता चला है कि वे स्वयं संगठित क्रांतिक के लक्षण हैं। वायुमंडलीय प्रवाह में स्वयं संगठित क्रांतिक भूमंडलीय मेघ आच्छादन प्रतिमान की फ्रक्टल ज्यामिति के रूप में होते हैं और वायुमंडलीय भंवर ऊर्जा मानावली में प्रतिलोम शक्ति नियम के रूप में होते हैं। इस शोध पत्र में वायुमंडलीय प्रवाह हेतु हाल ही में विकसित कोष्ठक गतिकीय पढ़ति मॉडल का सारांश दिया है। समताप मंडल और उससे ऊपर के स्तरों तक के वायुमंडलीय प्रवाहों के संचालित करने की क्वान्टम जैसी यांत्रिकी के स्वाभाविक परिणाम के रूप में मॉडल के उपयोग से वायुमंडलीय भंवर ऊर्जा स्पैक्ट्म हेतु सांख्यिकीय सामान्य वितरण के प्रतिलोम शक्ति नियम की प्रयुक्ति तैयार की गई। मॉडल से तैयार की गई प्रायुक्ति, वायुमंडल के कुल ओजोन के निरन्तर आवर्तित वक्र विश्लेषण से मेल खाती है। वायुमंडल की कुल ओजोन परिवर्तिता (दिनों में) से स्वयं संगठित क्रांतिक के स्थानिक लक्षण—अर्थात् शक्ति मानावली हेतु प्रतिलोम शक्ति नियम का पता चलता है। इसके अतिरिक्त जो दीर्घ-अवधि स्थानिक सहसंबंध, स्वयं

ABSTRACT. Long-range spatio-temporal correlations manifested as the self-similar fractal geometry to the spatial pattern concomitant with inverse power law form for the power spectrum of temporal fluctuations are ubiquitous to real world dynamical systems and are recently identified as signatures of self-organized criticality. Self-organised criticality in atmospheric flows is exhibited as the fractal geometry to the global cloud cover pattern and the inverse power law form for the atmospheric eddy energy spectrum. In this paper, a recently developed cell dynamical system model for atmospheric flows is summarized. The model predicts inverse power law form of the statistical normal distribution for atmospheric eddy energy spectrum as a natural consequence of quantum-like mechanics governing atmospheric flows extending upto stratospheric levels and above. Model predictions are in agreement with continuous periodogram analyses of atmospheric total ozone. Atmospheric total ozone variability (in days) exhibits the temporal signature of self-organized criticality, namely, inverse power law form for the power spectrum. Further, the long-range temporal correlations implicit to self-organized criticality can be quantified in terms of the universal characteristics of the normal distribution. Therefore, the total pattern of fluctuations of total ozone over a period of time is predictable.

Key words — Power spectra. Eddy. Perturbation. Turbulent. Periodogram. Fluctuation. Periodicities. Autocorrelation.

1. Introduction

Atmospheric total columnar ozone exhibits nonlinear variability on all time scales from days to years (WMO 1985, Gao and Stanford 1990, Prata 1990). The quantification of the nonlinear variability, in particular, long-term trends in atmospheric total ozone is an area of intensive research since the identification in recent years of the major spring-time Antarctic ozone hole and the general decreasing trend in stratospheric ozone throughout the high latitudes (Bojkov *et al.* 1990, Callis *et al.* 1991). In this paper, a recently developed cell dynamical system model for atmospheric flows (Selvam 1990, Selvam *et al.* 1992, Selvam 1993, Selvam and Radhamani 1994, Selvam and Joshi 1994) is summarized. The model predicts quantum-like mechanics for atmospheric flows extending upto the stratosphere and above (Sikka *et al.* 1988). The model predictions are in agreement with continuous periodogram analyses of sets of twenty to twenty five daily or up to 6-days' means of atmospheric columnar total ozone content at five different locations.

The power spectra of atmospheric columnar total ozone follows the inverse power law form of the statistical normal distribution. Inverse power law form for the power spectra of temporal fluctuations is ubiquitous to real world dynamical systems and is a temporal signature of self-organized criticality (Bak *et al.* 1988) or deterministic chaos (Selvam 1990) and implies long-range temporal correlations. Universal quantification for self-organized criticality in the
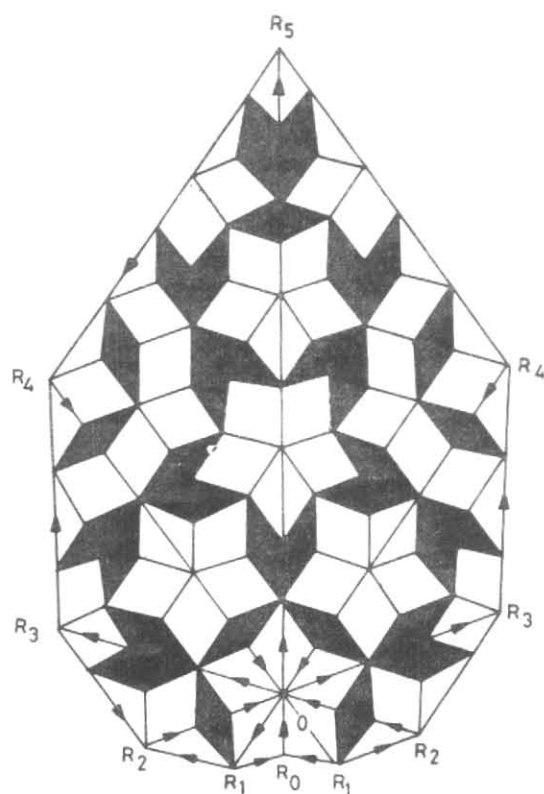
Fig. 1. The quasi-periodic Penrose tiling pattern which forms
the internal structure of large eddy circulations

temporal fluctuations of atmospheric columnar
total ozone content implies predictability of the
total pattern of fluctuations. Further. trends in
atmospheric total ozone may also be predictable.

2. Cell dynamical system model

In summary (Selvam 1990. Selvam *et al.* 1992.
Selvam 1993. Selvam and Radhamani 1994.
Selvam and Joshi 1994). the mean flow at the
planetary atmospheric boundary layer (ABL)
possesses an inherent upward momentum flux of
surface frictional origin. This upward momentum
flux is progressively amplified by the exponential
decrease of atmospheric density with height
coupled with latent heat released during micro-
scale fractional condensation by deliquescence on
hygroscopic nuclei even in an unsaturated
environment. This mean upward momentum flux
generates helical vortex roll (or large eddy) cir-
culations in the ABL seen as cloud rows/streets
mesoscale cloud clusters (MCC) in the global
cloud cover pattern. Townsend (1956) has shown
that large eddy circulations form as the spatial
integration of enclosed turbulent eddies intrinsic to
any turbulent shear flow. The relationship between
the root mean square (rms) circulation speeds W

and w_* of large and turbulent eddies of respective
radii R and r is then obtained as,

$$W^2 = \frac{2r}{\pi R} w_*^2 \qquad (1)$$

A continuum of progressively larger eddies
grows from the turbulence scale at the planetary
surface with two-way ordered energy feedback bet-
ween the larger and smaller scales as given in Eqn.
(1). Large eddy is visualized as the envelope of
enclosed turbulent eddies and large eddy growth
occurs in unit length step increments equal to the
turbulent eddy fluctuation length r. Such a concept
is analogous to the non-deterministic cellular
automata computational technique where cell
dynamical system growth occurs in unit length step
increments during unit intervals of time (Oona and
Puri 1988). Also. the concept of large eddy growth in
length step increments equal to r, the turbulence
length scale. *i.e.*, length scale doubling is identified
as the universal period doubling route to chaos
eddy growth process. The large eddy of radius R_n at
the nth stage of growth goes to form the internal cir-
culation for the next stage. *i.e.*, $(n+1)$th stage of large
eddy growth. Such a concept. leads as a natural con-
sequence. to the result that the successive values of
the radii R and the rms eddy circulation speeds W
follow the Fibonacci mathematical number series,
i.e.. the ratio of the successive values of R (or W) is
equal to τ. the golden mean $\{\tau = (1 + \sqrt{5})/2 = 1.618\}$.

Incidentally. the golden mean is associated with
self-similar fractal structures found in nature [Jean
1992 (a & b). Stewart 1992]. The self-similar fractal
geometry of atmospheric flow structure has been
documented and discussed by Tessier *et al.* (1993)
The name 'fractal' means broken (fractured) Eucli-
dean structures such as those commonly found in
nature. Self-similarity implies that sub-units of a
system resemble the whole in shape.

The overall envelope of the large eddy traces a
logarithmic spiral with the quasi-periodic Penrose
tiling pattern for the internal structure as shown in
Fig. 1. Atmospheric flow structure consists of
overall logarithmic spiral trajectory $R_0 R_1 R_2 R_3 R_4$
R_5 with dominant internal eddy circulations
OR_0R_1. OR_1R_2. OR_2R_3 etc. such that OR_0. OR_1.
OR_2. .... follow the Fibonacci mathematical num-
ber series. *i.e.*, $OR_1/OR_0 = OR_2/OR_1 = .. = \tau$.
where τ is the golden mean equal to $\frac{1 + \sqrt{5}}{2}$
$= 1.618$. Atmospheric circulation structure, there-
fore. consists of a nested continuum of vortex roll

circulations (vortices within vortices) with a two-way ordered energy flow between the larger and smaller scales. Such a concept is in agreement with the observed long-range spatio-temporal correlations in atmospheric flow patterns.

The cell dynamical system model also predicts the following logarithmic wind profile relationship in the ABL.

$$W = (w_*/k) \ln Z \qquad (2)$$

where. the Von Karman's constant k is identified as the universal constant for deterministic chaos and represents the steady state fractional volume dilution of large eddy by turbulent eddy fluctuations. The value of k is shown to be equal to $1/\tau^2$ ($=0.382$) where τ is the golden mean. The model predicted value of k is in agreement with observed values. Since the successive values of the eddy radii follow the Fibonacci mathematical number series the length scale ratio Z for the nth step of eddy growth is equal to $Z_n = R_n/r = \tau^n$. Further. W represents the standard deviation of eddy fluctuations. since W is computed as the instantaneous rms eddy perturbation amplitude with reference to the earlier step of eddy growth. For two successive stages of eddy growth starting from primary perturbation w_*. the ratio of the standard deviations W_{n+1} and W_n is given from Eqn. (2) as $(n+1)/n$.

Denoting by σ. the standard deviation of eddy fluctuations at the reference level ($n=1$). the standard deviations of eddy fluctuations for successive stages of eddy growth are given as integer multiples of σ. i.e., σ, 2σ. 3σ. etc.

The concept of large eddy formation as the spatial integration of enclosed turbulent eddies leads as a natural consequence to the result that the atmospheric eddy energy spectrum follows normal distribution characteristics. i.e., the square of eddy amplitude represents the eddy probability density. Incidentally. the above result. namely. that the additive amplitudes of eddies when squared represent the eddy probability density, is inherent to the observed sub-atomic dynamics of quantum systems and is accepted as an-ad hoc assumption in quantum mechanics (Maddox 1988).

Atmospheric flow structure, therefore. follows quantum-like mechanical laws where the eddy energy spectrum represents the eddy probability density and the apparent wave-particle duality is physically consistent in the context of atmospheric flows since the bimodal (formation and dissipation)

form for energy manifestation in the bi-directional energy flow intrinsic to eddy circulations results in the formation of clouds in updrafts and dissipation of clouds in downdrafts.

The conventional power spectrum (Section 3) plotted as the variance *versus* the frequency in log-log scale will now represent the eddy probability density on logarithmic scale *versus* the standard deviation of the eddy fluctuations on linear scale since the logarithm of the eddy wavelength represents the standard deviation. i.e., the rms value of eddy fluctuations (Eqn. 2).

The rms value of the eddy fluctuations can be represented in terms of statistical normal distribution as follows. A normalized standard deviation. $t=0$. corresponds to cumulative percentage probability density equal to 50 for the mean value of the distribution. Since the logarithm of the wavelength represents the rms value of eddy fluctuations the normalized standard deviation t is defined for the eddy energy distribution as $t = (\log \lambda/\log T_{50}) - 1$ where λ is the period in years and T_{50} is the period up to which the cumulative percentage contribution to total variance is equal to 50 and $t=0$. Log T_{50} also represents the mean value for the rms eddy fluctuations and is consistent with the concept of the mean level represented by rms eddy fluctuations. Spectra of daily or several days' average values should follow the model predicted universal spectrum. It will be possible to identify model predicted longer periodicities in spectra of several days' average values.

The universal spectrum predicted for total ozone temporal (days) variability is independent of the exact details of ozone generation mechanisms. Fluctuations of all scales in atmospheric flows self-organize into a unified whole eddy continuum and contribute to the observed ozone variability.

In the following section it is shown that continuous periodogram analyses of atmospheric total columnar ozone exhibit the signatures of quantum-like mechanics. namely. the cumulative percentage contribution to total variance. computed starting from the high frequency end of the spectrum, follows the cumulative normal distribution.

3. Data and analysis

Daily values of atmospheric total ozone for 5 different stations were obtained from Ozone Data for the World (Dept. of Environment). 1988-91. Continuous periodogram analysis (Jenkinson 1977) was
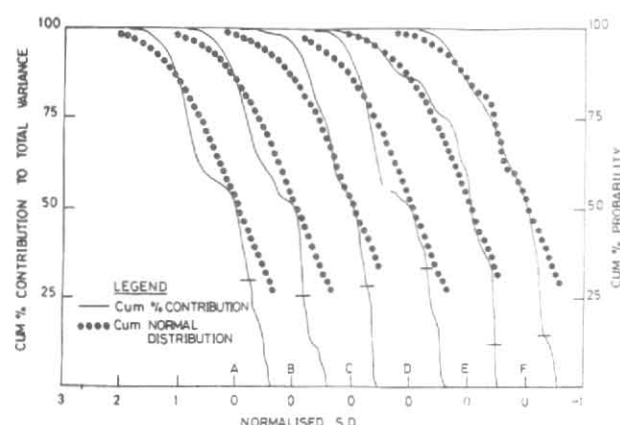
Fig. 2. Power spectra of total ozone for the six (A to F) time series in Table 1

done for 6 sets of twenty to twenty five daily or up to 6 days, averages of atmospheric total ozone content. The broad band power spectrum of total ozone time series can be computed accurately by an elementary but powerful method of analysis developed by Jenkinson (1977) which provides a quasi-continuous form of classical periodogram allowing systematic allocation of the total variance and degrees of freedom of the data series to logarithmically-spaced elements of the frequency range (0.5, 0).

A brief summary of traditional spectral analysis techniques and their drawbacks for analyses of broadband spectra are given in the following lines. Spectral analysis of discrete meteorological and hydrological data sequences plays an important role in data analysis, interpretation, and search for periodicities. Various methods for estimating spectral density function of physical data sequences are now well-known and widely used (Oladipo 1988). Briefly, two distinct classes of spectral estimation techniques based on fourier transform operations have evolved. The power spectral density estimate based on the indirect approach via auto-correlation estimation was popularized by Blackman and Tukey (1959). The other power spectral density estimate based on the direct approach, via the Fast Fourier Transform (FFT) operation on the data, is the one typically referred to as the periodogram (Jones 1965). In the periodogram method the squared magnitude of the Discrete Fourier Transform (DFT) of the data is computed by FFT. The Blackman and Tukey technique first estimates a finite number of discrete auto-correlation function values, or lags, for a finite data sequence. The DFT of the estimated auto-correlation lag estimates is, then, computed to obtain a smoothed power-spectrum estimate. The two classical methods are

statistically equivalent and produce similar results and differ only in their computational approach.

None-the-less the periodogram is more popular today because of the high computational efficiency of FFT used for the calculation of DFT, and it has found extensive application as a standard spectral technique in different fields of research, specially in geophysics and engineering. The computation of the DFT for climatological/meteorological data series creates several difficulties due to the constraint that the DFT must operate on sampled natural waveforms over relatively short, finite intervals. Further, since the spacing of the DFT values is related solely to the reciprocal of the record length or period, many of the true peaks in the spectra are never sampled and the analysis is "forced" because of the discrete and fixed record length. This consideration creates a serious resolution problem in the various regions of the climatological/meteorological spectra particularly in the wavelength of 5-25 years/days (Schikedanz and Bowen 1977). Meteorological time series data show fluctuations on all time scales within the time period under study: the spectrum of fluctuations is broad band with embedded dominant periodicities. Conventional power spectral analysis discussed earlier will not be able to resolve the broad band characteristics of the observed fluctuations. A more appropriate technique for spectral analysis of meteorological fluctuations is the continuous periodogram spectral analysis developed by Jenkinson (1977) which is discussed below.

The periodogram is constituted for a fixed set of 10000 (m) periodicities which increase geometrically as $\lambda_m = 2 \exp$ (cm) where $C = .001$ and $m = 0, 1, 2, \ldots$. The Data Y_t for the N data points where used. The periodogram estimates the set of $A_m \cos (2\pi v_m t - \phi_m)$ where A_m, v_m and ϕ_m denote respectively the amplitude, frequency and phase angle for the mth periodicity. The cumulative percentage contribution to total variance was computed from high frequency side of the spectrum. The period T_{50} at which 50% contribution to total variance occurs is taken as reference and the normalized standard deviation t_m values are computed as,

$$t_m = (\log \lambda_m / \log T_{50}) - 1 \qquad (3)$$

The power spectra are plotted in Fig. 2 as cumulative percentage contribution to total variance *versus* the normalized standard deviation t. The statistical normal distribution is also plotted as crosses in Fig. 2 and represents the cumulative

TABLE 1

Periodogram estimates for total ozone

Station (Lat.-Long.)	Time series length (days averaged)	Mean M-atm-cm	S.D.	T_{50}	T_{75} (Days)	T_{90}	Periodicities (days) contributing to maximum normalised variance (H) in the wave band H > 1				
A Hobart (42.49°S-147.30°E)	25 (1) 1 Feb-25 Feb '91	276.2	12.3	5.7	26.1	41.3	2.6	3.6	4.4	5.4	—
B Melbourne* (37.50°S-144.59°E)	25 (2) 1 Oct-19 Nov '90	326.2	22.7	11.0	48.3	78.2	7.5	9.7	19.7	—	—
C Melbourne	20 (1) . 1 Feb-20 Feb '91	262.7	6.4	3.4	6.6	10.1	2.3	2.8	3.3	5.6	9.1
D Tateno (36.03°N-140.08°E)	20 (6) 1 Mar-30 Apr '90	342.6	20.5	35.8	144.0	218.7	14.9	21.4	31.8	—	—
E Reykjavik (64.00°N-21.30°W)	20 (6) 3 Mar-30 Jun '90	393.9	26.9	23.6	43.0	161.6	13.2	24.0	32.7	65.0	—
F Varanasi (25.20°N-83.00°E)	20 (4) 1 Mar-19 May '90	292.8	10.3	18.6	46.3	121.1	12.9	18.0	42.4	—	—

S.D. : Standard deviation of the time series.
* The data series is not distributed normally.

percentage probability for the normalized standard deviation t. The short horizontal lines in the lower part of the spectra indicate the lower limit above which the spectra are the same as the normal distribution as determined by the statistical Chi-square test for "goodness of fit" at 95% confidence level. It is seen from Fig. 2 that the power spectra of atmospheric total ozone are the same as the statistical normal distribution when plotted in this manner. Table 1 gives the mean and standard deviation of the data, the periodicities T_{50}, T_{75}, T_{90} up to which the cumulative percentage contribution to total variance is equal to 50, 75 and 90 respectively and the peak periodicities for dominant wave bands for which the normalized variance is equal to or more than 1.

4. Discussion and conclusion

From Fig. 2 it is seen that the spectra of temporal (days) fluctuations of atmospheric columnar total ozone content follow the universal and unique inverse power law form of the statistical normal distribution such that the square of the eddy amplitude represents the eddy probability density corresponding to the normalised standard deviation t_m equal to $(\log \lambda_m / \log T_{50}) - 1$ where T_{50} is the period upto which the cumulative percentage contribution to total variance is equal to 50. Inverse power law form for the power spectra of temporal fluctuations is a

signature of self-organized criticality in the non-linear variability of atmospheric columnar total ozone content. The unique quantification for self-organized criticality in terms of the statistical normal distribution presented in this paper implies predictability of the total pattern of fluctuations in the atmospheric total columnar ozone content over a period of time. It may, therefore, be possible to predict future trends based on a knowledge of dominant cycles in atmospheric total columnar ozone content. The applications of the above result for predictability studies will be presented in a separate paper.

The peak periodicities in the wave-bands with normalized variance equal to or more than 1 (Table 1) correspond to the time periods of the quasi-periodic Penrose tiling pattern traced by internal circulations of the large eddy (Section 2) and are respectively equal to $T(2 + \tau) = 3.6T$, $T\tau(2 + \tau) = 5.8T$, $T\tau^2(2 + \tau) = 9.5T$, $T\tau^3(2 + \tau) = 15.3T$, $T\tau^4(2 + \tau) = 24.8T$ where T, the primary perturbation time period is the diurnal cycle of solar heating. The 2.2-day wave, which is present in the data analysed, may correspond to the period $T(2/\tau + 1) = 2.2T$ of the small scale circulation internal to primary circulation according to the concept of the eddy continuum energy structure (Eqn. 1). The dominant periodicities in atmospheric columnar total ozone time series may, therefore, be

expressed as functions of the golden mean. Other studies of total ozone variability (Chandra 1986, Gao and Stanford 1990 and Prata 1990) also show quasi-periodicities close to those predicted by the model.

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