

## Some characteristics of dynamical initialization of mass and velocity fields in the lower latitudes\*

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सार — क्षेत्रीय दाब घनत्व पूर्ण समीकरण (पी० ई०) प्रतिदर्श के ढांचे के अन्तर्गत निम्न अक्षांशों में भूविभव एवं पवन क्षेत्रों के गतिकीय प्रारंभीकरण के अनेक आंकिक प्रयोग किये गये हैं। किवगनोव और मोहन्ती (1979) ने अपने पहले लेखों में निम्न अक्षांशों में पी० ई० मॉडल के प्रारंभिक मान और परिसीमा मान की समस्या को ध्यान में रखते हुए गतिकीय प्रारंभीकरण की ईष्टतम प्रक्रिया का वर्णन किया था। इस लेख में भी प्रारंभीकरण की प्रक्रिया का उपयोग कर तथा वेग क्षेत्र को निवेश आंकड़े मानकर निम्न अक्षांशों में गतिकीय प्रारंभीकरण के कुछ खास खास गुणों तथा सतुलित आंकड़ों का उपयोग करके पूर्ण समीकरण प्रतिदर्श के समाकलन के प्रभाव को दर्शाया है।

ABSTRACT. A number of numerical experiments on dynamical initialization of geopotential and wind fields in the lower latitudes are carried out within the framework of a regional barotropic primitive equation (P.E.) model. In the earlier works of Kivganov and Mohanty (1979) an optimal procedure of dynamical initialization with consideration of an initial-value and boundary-value problem of a P.E. model in the lower latitudes is stated. In this paper with use of this initialization procedure and velocity field as input data some of the important properties of the dynamical initialization in the lower latitudes and the effectiveness of the integration of a P.E. model with the use of balanced data are demonstrated.

### 1. Introduction

Primitive equation (P. E.) numerical weather prediction model is very much sensitive to the initial data and boundary conditions. Hence, a P. E. model requires an improvement of the quality of the data, their mutual balance at initial time and mathematically corrected boundary conditions. Any imbalance between the mass and velocity fields leads to the generation of high-frequency fictitious gravity-inertia waves, if time integrations are attempted with a P. E. model. The mathematical and physical approaches taken in a general balanced initialization of geopotential and wind fields within the framework of dia-

gnostic filtered equations (quasi-geostrophic, quasi-solenoidal or balance equations) give inexact conditions of balance when applied to the primitive equations. On the other hand, properly balanced and free from high-frequency meteorological noise initial data can increase to a great extent computational stability of the model with increase of forecasting period and thus improve the quality of forecasting.

The sources of imbalance are not only the quality of data but also the boundary conditions and finite-difference scheme used in a regional model for its integration and the truncation error

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consistency between the initialization equations and forecast equations. These factors lead to the necessity of an initialization procedure where the primitive forecast equations are used directly with its all physical and mathematical limitations for the purpose of attaining mutual adjustments between mass and velocity fields. Such an initialization procedure is known as "Dynamical initialization" which was proposed by Miyakoda and Moyer (1968) and improved by Nitta and Hovermale (1969).

As one of the important problems in numerical weather prediction is the initialization of the dependent variables used in a P. E. model as input, this procedure got wide use and modifications (Temperation 1973, Haltiner and Mc Collough 1975, Fedorova and Fuks-Ravinovich 1972, Spektorman and Fuksh-Ravinovich 1978). However, most of these works were conducted for middle and higher latitudes where geopotential and geostrophic wind were used as input data. But in the lower latitudes use of wind field as initial data and derivation of geopotential from wind field in the P. E. model was proposed by Houghton and Washington 1969 [The use of wind field as input in an operational P. E. model involves various difficulties (objective analyses, derivation of geopotential field from wind field, initialization etc)]. In this work a series of numerical experiments with wind field as initial data were carried out to find out some of the important characteristics of the dynamical initialization of mass and velocity fields in the lower latitudes and the role of such balanced dependent variables in a simple forecasting model.

## 2. Description of the model

All the experiments were carried out within the framework of a regional barotropic P. E. model. The rectangular domain of integration was extending from 3 deg. N to 40 deg. N and from 60 deg. E to 120 deg. E with grid length equal to 210 km.

The basic equations of the model in the matrix form are:

$$\frac{\partial Z}{\partial t} = -\left(A \frac{\partial Z}{\partial x} + B \frac{\partial Z}{\partial y} - C\right) = F \quad (1)$$

where,

$$Z = \begin{pmatrix} u \\ v \\ \phi \end{pmatrix}; A = \begin{pmatrix} u & 0 & l \\ 0 & u & 0 \\ \phi & 0 & u \end{pmatrix}; B = \begin{pmatrix} v & 0 & 0 \\ 0 & v & l \\ 0 & \phi & v \end{pmatrix};$$

$$C = \begin{pmatrix} lv \\ -lu \\ 0 \end{pmatrix}$$

$u, v$  — the zonal and meridional components of wind in cartesian coordinates  $(x, y)$ , where  $x$  and  $y$  increased to the east and north respectively;  
 $\phi$  — geopotential;  
 $l$  — Coriolis parameter.

Finite — difference approximations of the right hand side terms of the system (1) were carried out with Shuman's semi-momentum form (Shuman and Stackpole 1969).

## 3. Initial data

For this model we used vertically averaged wind fields at 850, 700, 500, 300 and 200 mb. The geopotential field was derived from the wind field by calculating stream function and later by reverse balance equation. It was found in an earlier work of Kivganov and Mohanty (1979 a) that the maximum difference between derived geopotential and observed averaged geopotential at 850, 700, 500, 300 and 200 mb remained within 3 gpm in the centre of tropical cyclones and mean difference always less than 1 gpm.

## 4. Boundary conditions

In an earlier work, Kivganov and Mohanty (see Ref.) studied the influence of various boundary conditions on the dependent meteorological variables during the initialization procedure and it was found out that for such a rectangular domain suitable boundary conditions are:

At northern and southern walls (half-way between inner and outer grid points):

$$\bar{u}_t^y + (\bar{u}^{xy} \bar{u}_x^y + \phi_x^{-y})^x = 0 \quad (2)$$

$$\bar{v}^y = 0 \quad (3)$$

$$l\bar{u}^y + \phi_y = 0 \quad (4)$$

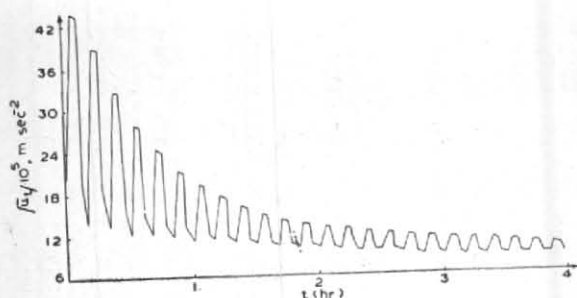
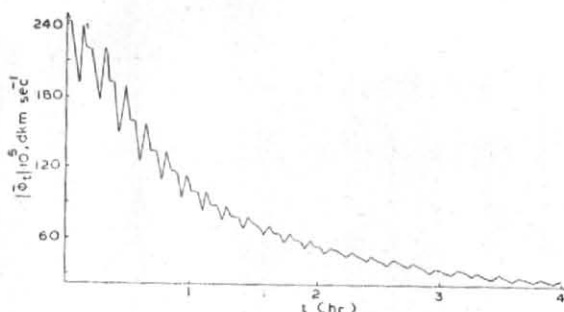
At the eastern and western walls:

$$Z_{i,1} = Z_{i,M-1}; Z_{i,M} = Z_{i,2}; i=1, 2, \dots, N \quad (5)$$

Where  $M$  and  $N$  - numbers of grid points along  $x$  and  $y$  axes respectively. Boundary condition (3) ensures no flow through the northern and southern walls at any point and condition (4) ensures the geostrophic flow along the walls. Condition (5) stands for periodically cyclic conditions at eastern and western boundaries.

## 5. Method of initialization

It was experimentally found out (Kivganov and Mohanty 1979 b) that in the lower latitudes with wind field as initial input the effective method of

Fig. 1. Change of  $|\bar{u}_t|$  in every step of 'Pseudo forecasting'Fig. 2. Change of  $|\bar{\phi}_t|$  in every step of 'Pseudo forecasting'

initialization is a four step scheme of Matsuno. This method consists of a forward forecasting, followed by a backward forecasting which is called as "Pseudo forecasting". For both forward and backward forecasting of the system (Eqn. 1) used the well-known two step 'Euler-backward' method which has the properties to stimulate the computational viscous effect of damping high-frequency waves (Nitta and Hovermale 1969).

The four steps of "Pseudo-forecasting" are:

Forward forecast:

$$\begin{aligned} Z_{n+1}^* &= Z_n^v + \Delta t \cdot F(Z_n^v) \\ Z_{n+1}^v &= Z_n^v + \Delta t \cdot F(Z_{n+1}^*) \end{aligned} \quad (6)$$

Backward forecast:

$$\begin{aligned} Z_n^* &= Z_{n+1}^v - \Delta t \cdot F(Z_{n+1}^v) \\ Z_n^v &= Z_{n+1}^v - \Delta t \cdot F(Z_n^*) \end{aligned}$$

where  $Z_n^*$  and  $Z_{n+1}^*$  — intermediate values of  $Z$  at the time steps  $n$  &  $n+1$  respectively;

$\Delta t$  — time interval;  
 $v$  — number of iterations (number of cycles of Pseudo-forecasting).

This scheme was first used by Nitta and Hovermale (1969) for dynamical initialization which obtained wide use in later works.

In this procedure after each complete cycle of iteration:

$$Z^{v+1} = R Z^v \quad (7)$$

where,  $R$  — amplification rate.

It is easy to find out that

$$R = 1 - \omega^2 \Delta t^2 + \omega^4 \Delta t^4$$

where,  $\omega$  — angular frequency.

Thus for a given value of  $\Delta t$  which satisfies the computational stability condition, waves of  $\omega \Delta t < 1$  are reduced in amplitude.

It was found (Kivganov and Mohanty — see Ref.) that the process of mutual adjustment of observed velocity and derived geopotential fields in a "Pseudo-forecasting" (Eqn. 6) filtered out fictitious high-frequency gravity waves faster than the process of forced adjustment of one field (wind or geopotential) to the other.

#### 6. Some characteristics of dynamical initialization

In order to study the role of the simulated computational viscosity generated during "Pseudo-forecasting" by "Euler-backward" method, we have calculated:

$$\left| \bar{Z}_t \right| = \frac{1}{(M-2)(N-2)} \sum_{i=2}^{N-1} \sum_{j=2}^{M-1} \frac{|Z_{n+1} - Z_n|}{\Delta t} \quad (8)$$

at the end of each step of the iteration cycles.

The values  $|\bar{u}_t|$  and  $|\bar{\phi}_t|$  are demonstrated in the Figs. 1 and 2. These figures are showing the change of  $|\bar{u}_t|$  and  $|\bar{\phi}_t|$  in the process of initialization. As it is evident from Figs. 1 and 2, during the first 2 to 3 hours of initialization damping of the amplitude of the fictitious high-frequency waves in the atmosphere is more intensive and after damping oscillation is negligible. This phenomenon is comparable with the characteristic period of adaptation of mass and velocity fields in the atmosphere. It is also found out that the mass and velocity field suffered from damping oscillation with a phase 90 deg., which explained the process of mutual adjustment of both the fields in the process of initialization.

Dynamical initialization has the properties of selective filtration of only fictitious high-frequency gravity waves without altering the mode

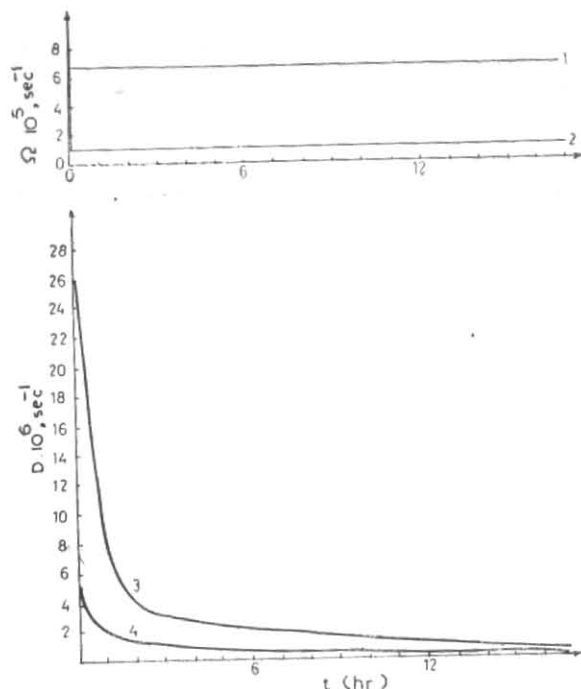


Fig. 3. Change of  $\Omega$  and  $D$  in the process of dynamical initialization.

(1)  $|\Omega|_m$ , (2)  $|\bar{\Omega}|$ , (3)  $|D|_m$  and (4)  $|\bar{D}|$

of the atmospheric motion responsible for synoptic and large-scale phenomenon. To illustrate this characteristics two experiments were carried out.

First experiment consisted of calculation of absolute maximum value of divergence  $|D|$ , absolute mean divergence  $|\bar{D}|$ , absolute maximum vorticity  $|\Omega|_m$  and absolute mean vorticity  $|\bar{\Omega}|$  of the velocity field at the end of each cycle of "Pseudo-forecasting". The results are demonstrated in the Fig. 3. In the process of initialization  $|\bar{D}|$  and  $|D|_m$  suffered a damping in a way similar to  $|\bar{u}|$  and  $|\bar{\phi}_t|$ . The damping is very intensive during the first 2-3 hours initialization and after 8-10 hours period the values of  $|D|_m$  and  $|\bar{D}|$  are almost constant. But the values of  $|\Omega|_m$  and  $|\bar{\Omega}|$  remained almost constant during entire period of initialization. These facts supported the basic concepts of dynamical initialization that the divergence component of the velocity which is responsible for generation of gravity waves damped out substantially during the process of initialization, whereas the velocity field which characterised the synoptical and large-scale features of the atmosphere remain unchanged.

Second experiment consisted of the calculation of the complex Fourier series (Cooley and Tukey 1965) of the  $u$  and  $v$  components of the velocity field. From the amplitudes of Fourier series spectral distribution of kinetic energy (K.E.) was calculated for the initial wind field and the wind field after 18 hours (108 cycles) of initialization.

It was found that percentage change of K.E. during initialization in the zonal component of atmospheric spectrum did not exceed 1 per cent; it was less than 10 per cent in large-scale waves and less than 25 per cent in synoptic scale waves whereas in the short wave length spectrum was more than 50 per cent. This result once again confirmed the fact that during initialization process short-wave length mode of atmospheric motion suffered damping whereas large scale motion practically remained unchanged.

Finally, in order to study the effect of initialized fields compared to the initial fields we have carried out forecasting with use of the system (1) for a period of 24 hours. First, time step of integration was that of "Euler-backward" and the subsequent time steps by Adam's method:

$$Z_{n+1} = Z_n + \left[ \frac{3}{2} \left( \frac{\partial Z}{\partial t} \right)_n - \frac{1}{2} \left( \frac{\partial Z}{\partial t} \right)_{n-1} \right] \Delta t + D (\Delta t^3) \quad (9)$$

At each time step we have calculated  $|\bar{z}_t|$ . In the Fig. 4 values of  $|\bar{u}_t|$  are illustrated. Three experiments were conducted with (i) initial fields (without initialization), (ii) dependent variables with 6 hours (36 cycles) initialization and (iii) input data with 18 hours (108 cycles) initialization. As the P.E. model is a barotropic model which is isolated from external sources of energy the values  $|\bar{z}_t|$  should be more or less stable and constant



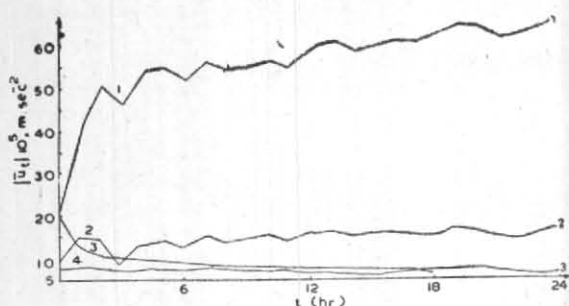


Fig. 4. Change of  $|\bar{u}_t|^2$  in the process of :

- (1) Forecasting without initialization;
- (2) Forecasting with 6 hrs "Pseudo-Forecasting";
- (3) Forecasting with 18 hrs "Pseudo-forecasting"
- (4) Dynamical initialization.

during a 24 hours forecasting period. But as it is evident from Fig. 4 during a first 6 hours period the forecast with initial fields showed a clear sign of unstable rapid increase of  $|\bar{u}_t|^2$  which is a sign of computational instability. The fields with 6 hours initialization also showed some unusual changes of  $|\bar{u}_t|^2$  where as fields with 18 hours initialization remained almost constant during entire 24 hours forecasting period.

These results explained the facts that during the first few hours of forecasting, due to the existence of imbalance between dependent variables fictitious high-frequency gravity-inertia waves were generated and after the period of adaptation the value of  $|\bar{u}_t|^2$  remained more or less stable. In case of a properly balanced input data (18 hours initialization fields) such type of phenomenon did not occur. The quantitative evaluation of the forecast meteorological fields also indicated similar type of unrealistic values in case of initial data without dynamical initialization.

## 7. Summary and conclusions

On the basis of the above stated numerical experiments, we can make the following concluding remarks:

- (1) Dynamical initialization has the property of selective filtration of only fictitious high-frequency gravity waves without

affecting the large scale atmospheric phenomenon.

- (2) It was found out that maximum change of geopotential field after initialization from the observed values did not exceed 1 to 2 gpm. Initialization process made the mass and velocity fields somehow smoother compared to the observed fields as in the case of geopotential field, there were increase of geopotential values by 1 to 2 gpm in the region of cyclones and decrease of geopotential by almost same value in the region of anti-cyclones.
- (3) For integration of a P. E. model with wind field as input data, dynamical initialization process is highly essential as without dynamical initialization it is quite impracticable to integrate a P. E. model due to generation of fictitious high-frequency gravity waves. Moreover, the wind field is not homogeneous as the geopotential and each of the terms  $\partial u/\partial x$  and  $\partial v/\partial y$  are of the order of  $10^{-5}$  where as divergence ( $D = \partial u/\partial x + \partial v/\partial y$ ) is of the order of  $10^{-6}$ , so calculation of divergence field from the observed wind field for a P. E. model contains substantial errors. Therefore, before start of the forecasting, it is required to damp out the fictitious amplitude of divergence and to bring it to a reasonable low level.

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