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A Markov chain model for daily rainfall occurrences at east Thanjavur district

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सार — वर्षा होने और न होने के बारे में द्विस्थितीय एक मार्कोव चेन की सहायता से बताया जा सकता है। वर्षा होने की स्थिति को 0 स्थिति द्वारा तथा वर्षा न होने की स्थिति को 1 स्थिति द्वारा दर्शाया गया है। ज्ञात प्राचल के साथ प्वासों प्रक्रिया का अनुसरण करने वाला नमूना हमने अपनाया है। इस प्वासों नमूने का उपयोग करते हुए मार्कोव चेन के सिद्धांतों और भूतकालिक अप्रेक्षित मानों अथवा अब तक अनप्रेक्षित भावी मानों से संबंधित स्थिति का आकलन करने हेतु सांख्यिकी निष्कर्षों को प्रभावित करने के लिए हमने एक नया तरीका अपनाया है। इस शोध का उद्देश्य नए दृष्टिकोण के साथ आंकड़ों की पूर्व उपयुक्तता का तुलनात्मक अध्ययन करना है।

ABSTRACT. The occurrences and non-occurrences of the rainfall can be described by a two-state Markov chain. A dry date is denoted by state 0 and wet date is denoted by state 1. We have taken the sample which follows a Poisson process with known parameter. Using this Poisson sample we have given a new approach to affect statistical inference for the law of the Markov chain and state estimation concerning unobserved past values or not yet observed future values. The paper aims at comparing the earlier fit of the data with the new approach.

Key words — Markov chain, Probability, Dry and wet days, Matrix, Monsoon.

1. Introduction

Karr (1986) has given the following procedure to problems on inference for Markov chains based on point process samples :

Given a process $X = [X(t)]$ on R with unknown probability law, but usually of fairly narrowly specified structure, and a Poisson process N on R, often but not invariably preassumed homogeneous, such that X and N are independent, the goal is to affect statistical inference for the law of X and state estimation concerning unobserved past values or not yet observed (and possibly never to be observed) future values, given as of single realizations of the Poisson samples $X(T_n)$, that is, the values of X at the points of N, together possibly with observations of N itself. Observable aspects are in the marked point process N, marked by the current value of X.

We follow the assumptions made by Karr (1986) in applying Markov chain model for studying the patterns of occurrences of dry and wet days. The days were recorded under two categories, dry and wet. A day is called dry if the rainfall during the 24 hours commencing from 6 a.m. on that day is either nil or trace, otherwise it is termed wet. One of the two periods is pre-monsoon from 1 January to 30 June and the other period is monsoon season from 1 July to 30 December. Two sets of pre-monsoon season are considered: Set I covering a total $2 \times 180 = 360$ days in the two years — 1987 and 1988. The data are taken from records maintained by the revenue officials of Government of Tamil Nadu at the concerned centres.

In redrafting the information in a table suitable to Karr's (1986) discussion, data are chosen to be a Poisson process with parameter 2 and to decide the observation to fall on specific dates in every month.

2. Method and analysis

To describe the occurrences of dry and wet days at Thanjavur district we use a two-state Markov chain model as considered by Gabriel and Neumann (1983). We assume that the probability of dry (or wet) days depends on the conditions of previous day. Such a probability model is referred to as a Markov chain, whose parameters are the two conditional probabilities,

$$P_0 = \text{Prob (Wet day/previous day dry)}$$

$$P_1 = \text{Prob (Wet day/previous day wet)}$$

We denote the dry day by state 0 and wet day as state 1, the occurrence of dry and wet day can, then,

TABLE 1

Centre : Thiruthuraipoondi — Years : 1987-1988

Day	Jan		Feb		Mar		Apr		May		Jun		Jul		Aug		Sep		Oct		Nov		Dec	
	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988
4	D	D	D	D	D	D	D	D	D	W	D	D	D	D	D	D	D	D	W	D	W	W	W	W
5	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	W	D	D	W	W	D	W	W	D
10	D	D	D	D	D	D	D	D	D	D	D	D	D	W	D	D	D	W	W	D	D	W	D	D
11	W	D	D	D	D	D	D	D	D	D	D	D	D	D	D	W	D	W	D	D	D	W	D	D
18	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	W	W	W	D	D	D	D	D
25	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	W	W	D	W	D	W	D
29	D	D	—	D	D	D	D	D	D	D	D	D	D	D	D	W	W	D	W	D	D	D	D	D
30	D	D	—	—	D	D	D	D	D	D	D	D	D	D	D	W	W	D	W	D	D	D	D	D

TABLE 2

Centre : Vedaranyam — Years : 1987-1988

Day	Jan		Feb		Mar		Apr		May		Jun		Jul		Aug		Sep		Oct		Nov		Dec	
	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988
4	D	D	D	D	D	D	D	D	W	D	D	D	D	D	D	D	D	D	W	D	D	W	W	W
5	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	W	W	W	D
10	D	D	D	D	D	D	D	D	D	D	D	D	D	W	D	D	D	D	W	W	W	W	D	W
11	D	W	D	D	D	D	D	D	D	D	D	D	D	D	W	W	D	D	W	D	D	W	W	D
18	D	D	D	D	D	D	D	W	D	D	D	D	D	D	W	D	D	D	W	D	W	D	D	D
25	D	D	D	D	D	D	D	W	D	W	D	D	D	D	D	W	D	D	W	D	D	D	W	D
29	D	D	—	D	D	D	D	D	D	D	D	D	D	D	D	W	W	D	W	D	D	D	D	D
30	D	D	—	—	D	D	D	D	D	D	D	D	D	D	D	W	W	D	W	D	D	D	D	W

be denoted by a Markov chain with two ergodic states 0 and 1 and transition probability matrix given by.

$$A = \begin{matrix} & 0 & 1 \\ 0 & 1-p_0 & p_0 \\ 1 & p_1 & 1-p_1 \end{matrix}$$

The n-step transition probabilities are given by the elements of A^n, where A^n is given by,

$$A^n = \frac{1}{p_0+p_1}\begin{pmatrix} p_1 & p_0 \\ p_1 & p_0 \end{pmatrix} + \frac{(1-p_0-p_1)^n}{p_0+p_1}\begin{pmatrix} p_0 & -p_0 \\ -p_1 & p_1 \end{pmatrix}$$

Further $\lim_{n\to\infty} A^n = \begin{pmatrix} v_0 & v_1 \\ v_0 & v_1 \end{pmatrix}$

where $v_0 = \frac{p_1}{p_0+p_1}$, $v_1 = \frac{p_0}{p_0+p_1}$

A wet spell W of length k is defined as a sequence of k wet days preceded and followed by dry days. Dry spells D are defined correspondingly. Weather cycles are defined as combinations of a wet spell and an adjacent dry spell. It follows that the distribution of W and D are geometric with parameters p_1 and p_0 respectively, that is Probability $(W=r) = p_1\ (1-p_1)^{r-1}$, $r = 1, 2, \ldots.$ and Probability $(D=r) = p_0\ (1-p_0)^{r-1}$ so that the mean $E(W) = \frac{1}{p_1}$ and $E(D) = \frac{1}{p_0}$

The central limit theorem for dependent variables asserts that the distribution of the number of wet days, Y_n in a sequence of n transitions is asymptotically normal with,

$$E(Y_n) = \frac{np_0}{p_0+p_1},\ \mathrm{VAR}(Y_n) = np_0p_1\frac{(2-p_0-p_1)}{(p_0+p_1)^3}$$

From Table 1, we observe that

		Actual day 0	Actual day 1	Total
Preceding day	0	62	6	68
	1	7	7	14
	Total	69	13	82

$$A = \begin{pmatrix} 0.91176 & 0.08824 \\ 0.5 & 0.5 \end{pmatrix}$$

$$A^{20} = \begin{pmatrix} 0.85003 & 0.15002 \\ 0.85002 & 0.15002 \end{pmatrix}$$

$$\underset{n\to\infty}{\mathrm{Lt}} A^n = (0.85,\ 0.15)$$

		Actual day 0	Actual day 1	Total
Preceding day	0	62	6	68
	1	8	7	15
	Total	70	13	83

$$A = \begin{pmatrix} 0.91176 & 0.08824 \\ 0.53333 & 0.46667 \end{pmatrix}$$

$$A^{20} = \begin{pmatrix} 0.85804 & 0.14196 \\ 0.85804 & 0.14196 \end{pmatrix}$$

$$\underset{n\to\infty}{\mathrm{Lt}} A^n = (0.858,\ 0.142)$$

From Table 2, we observe that

		Actual day 0	Actual day 1	Total
Preceding day	0	58	7	65
	1	8	9	17
	Total	66	16	82

$$A = \begin{pmatrix} 0.89231 & 0.10769 \\ 0.47059 & 0.52941 \end{pmatrix}$$

$$A^{20} = \begin{pmatrix} 0.81577 & 0.18712 \\ 0.82030 & 0.18816 \end{pmatrix}$$

$$\underset{n\to\infty}{\mathrm{Lt}} A^n = (0.816,\ 0.187)$$

TABLE 3

Centre : Nagapattinam — Years : 1987-1988

Day	Jan		Feb		Mar		Apr		May		Jun		Jul		Aug		Sep		Oct		Nov		Dec	
	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988	1987	1988
4	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	W	D	D	W	W	W
5	D	D	D	D	D	D	D	D	D	D	D	D	D	W	D	D	D	D	W	D	W	W	W	D
10	D	D	D	D	D	D	D	W	D	D	D	D	D	D	D	D	D	W	W	W	W	D	W	D
11	D	W	D	D	D	D	D	W	D	D	W	D	D	W	W	W	D	W	W	D	W	W	W	D
18	D	D	D	D	D	D	D	W	D	D	D	D	D	D	D	D	D	W	D	D	D	D	W	D
25	D	D	D	D	D	D	D	W	D	D	W	•D	D	W	D	W	D	W	W	D	W	D	W	D
29	D	D	—	D	D	D	D	D	D	D	D	D	D	D	D	W	W	D	W	D	D	D	D	D
30	D	D	—	—	D	D	D	D	D	D	D	D	D	D	D	W	D	D	W	D	D	D	D	D

TABLE 4

Limiting distribution

Centres	Thiruthuraipoondi		Vedaranyam		Nagapattinam	
Years	1987	1988	1987	1988	1987	1988
p_0	0.85	0.858	0.816	0.819	0.766	0.789
p_1	0.15	0.142	0.187	0.181	0.234	0.211

TABLE 5

Mean and variance of wet spell

Centres	Thiruthuraipoondi		Vedaranyam		Nagapattinam	
Years	1987	1988	1987	1988	1987	1988
$E(Y_n)$	4.5	4.26	5.59	5.42	7.02	6.33
$VAR(Y_n)$	9.18	8.1	11.18	7.69	13.85	9.98

		Actual day 0	Actual day 1	Total
Preceding day	0	59	9	68
	1	9	6	15
	Total	68	15	83

$$A = \begin{pmatrix} 0.86765 & 0.13235 \\ 0.6 & 0.4 \end{pmatrix}$$

$$A^{20} = \begin{pmatrix} 0.81928 & 0.18072 \\ 0.81928 & 0.18072 \end{pmatrix}$$

$$\underset{n \to \infty}{\text{Lt}} A^n = (0.819, 0.181)$$

From Table 3, we observe that

		Actual day 0	Actual day 1	Total
Preceding day	0	53	8	61
	1	9	12	21
	Total	62	20	82

$$A = \begin{pmatrix} 0.86885 & 0.13115 \\ 0.42857 & 0.57143 \end{pmatrix}$$

$$A^{20} = \begin{pmatrix} 0.76570 & 0.23430 \\ 0.76569 & 0.23431 \end{pmatrix}$$

$$\underset{n \to \infty}{\text{Lt}} A^n = (0.766, 0.234)$$

		Actual day 0	Actual day 1	Total
Preceding day	0	55	9	64
	1	10	9	19
	Total	65	18	83

$$A = \begin{pmatrix} 0.85938 & 0.14063 \\ 0.52632 & 0.47368 \end{pmatrix}$$

$$A^{20} = \begin{pmatrix} 0.78927 & 0.21089 \\ 0.78927 & 0.21089 \end{pmatrix}$$

$$\underset{n \to \infty}{\text{Lt}} A^n = (0.789, 0.211)$$

3. Conclusion

We find that the Markov Chain of order one gives an adequate fit for the observations of the pre-monsoon and the monsoon period during 1987-88. This model is based on the sample which follows a Poisson process from the observations obtained throughout the year. The limiting distribution of the transit from wet to dry and dry to wet is shown in Table 4.

The distribution of the number of wet days Y_n in a sequence of n transitions is asymptotically normal with mean and variance as shown in Table 5. We find that Markov chain model, fitted from out of the Poisson samples of the data, gives a better picture of the situation than that obtained by Thiyagarajan (1989) based on raw data. Statistical test shows that the Markov chain of order 1 gives an adequate fit for the observations of the pre-monsoon period (1987-88).

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