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Variability in rainfall dispersion in Madhya Pradesh

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सार — गामा वितरण संभाविता निदर्श के प्रयोग द्वारा मध्य प्रदेश में वर्षा परिवर्तनशीलता में वितरण और विक्षेपण का परीक्षण करने का प्रयास किया गया है। गामा वितरण के आकार और मापक्रम प्राचलों के आकाशीय और क्षेत्रीय वितरण का परीक्षण किया गया। जल आवश्यकताओं के साथ संभावित वर्षा की तुलना द्वारा जल अधिशेषों और कमियों की अवधियों का पता लगाया गया। माध्य वर्षा से संभावित वर्षा का पता लगाने के लिए समाश्रयण समीकरण का विकास किया गया। फसल योजना के लिए जल स्रोतों के सही इस्तेमाल के लिए सस्य विज्ञान प्रयोगों का मूल्यांकन किया गया।

ABSTRACT. An attempt has been made to examine distribution and dispersion in rainfali variability in Madhya Pradesh by applying Gamma distribution probability model. The spatial and regional distribution of shape and scale parameters of the Gamma distribution have been examined. Periods of water surpluses and deficiencies have been identified by comparing the probability rainfall with the water requirement. Regression equations have been developed to find probabilitistic rainfall from the mean rainfall. Agronomic practices have been evaluated for efficient utilization of water resources for crop planning.

Key Words — Homogeneous, Physiological, Skewness, Stress, Maturity, Agroclimatic, Flowering.

1. Introduction

Rainfall is the most important natural input for agricultural production in India. However, it varies the most, both temporarily and spatially, particularly in the State of Madhya Pradesh which is nearly bifurcated by the tropic of cancer. With limited irrigation facilities, rain water shortages lead often to moisture stress causing substantial reduction in crop yields. Agricultural planning in this region as such calls for a scientific and rational analysis of the rainfall for increasing crop output.

In the publications of India Meteorological Department rainfall is available mostly as monthly averages. These averages neither indicate agroclimatic homogeneous zones nor gives information on rainfall variability. In rainfed agriculture, one is interested in knowing how much rainfall could be expected in as high a probability level, as possible say in 4 out of 5 years or 3 out of 4 years. Rainfall data in climatological summaries are rarely given in forms of probabilities.

The present study attempts to examine distribution and dispersion in rainfall variability over Madhya Pradesh by determining shape and scale parameters. Agronomic practices believed to be helpful in soil water conservation have also been evaluated.

The importance of weekly rainfall in governing and selecting crop varieties and calendar of agricultural operations has already been emphasised by Sarker and Biswas (1980). Da Mota (1978) had also suggested that analysis for agriculture should be based on rainfall of shorter periods of the crop growth. Here week has been chosen as a unit of time.

2. Data

The study utilised daily rainfall data from 1901-1970 in respect of nearly 100 stations (Fig. 1) in the State. The crops are normally sown during the middle of June and by the end of September or early October the physiological growth and reproduction phases are completed. The period chosen in the study spans from 23rd standard week (3—10 June) to 43rd standard week (1—7 October). Estimates of statistical parameters according to Mooley (1973), would be stable, if the data is more than 50 years. For nearly all stations, this condition was fulfilled.

Das (1987), on the basis of moisture availability, soil types and crop characteristics, divided the State into four major agroclimatic zones (Fig. 1). In the present study mean picture of the variability is obtained for each of these regions from the stations falling within them and discussed.

3. Methodology

Monthly, seasonal or annual rainfall over most parts of India, perhaps, cannot be represented by the normal distribution (Sankaranarayanan 1933, Pramanik and Jagannathan 1953, Mooley and Crutcher 1968, etc). Rainfall series transformed through square root, cube root, logarithmic transformation, etc. for normalisation have been found to be of limited success, (Mooley 1973). On the other hand, many studies (Sarker and Biswas 1980, Das 1987 etc) have confirmed that Gamma distribution function (GDF) fits well to the weekly rainfall. According to Lewis and Ndolo (1987) also, the GDF is one of the most appropriate models for interpreting rainfall data for different stages of crop growth.

TABLE 1

Values of variance test ratio

Weeks	Region I		Region II		Region III		Region IV	
	Datia	Gird	Basoda	Now-gong	Panna	Rewa	Raigarh	Jabalpur
26	47.3	37.0	49.5	62.31	70.33	69.1	61.98	39.17
30	59.8	45.3	50.0	59.8	58.63	52.5	56.66	59.91
34	57.3	58.0	64.3	70.7	51.54	45.9	42.83	52.17
38	39.0	30.8	44.3	49.0	40.02	67.9	57.25	42.16

In tropics, even in the rainy season, in some of the weeks, weather remains dry. Thus, the weekly rainfall distribution includes both zero and non-zero values, which can be represented by GDF as suggested by Thom (1951). Accordingly,

$$F(x) = q + p\,G(x)$$

where $F(x)$ is the mixed GDF, q is the probability of zero rainfall and p is $(1-q)$

$G(x)$, the GDF is given by

$$G(x) = \int_0^x \frac{x^{\gamma-1}\, e^{-x/\beta}}{\beta^{\gamma}\,\overline{|\gamma}}\, dx \quad \text{for } x>0$$

$$=0 \qquad\qquad \text{for } x<0$$

Here β is the scale parameter and indicates range or dispersion of the rainfall x, larger values indicating a greater tendency of x to deviate from the mean γ, the shape parameter, is inversely related to skewness. Larger values of γ cause the probability function of x to approach normality. Both β and γ assume value > 0.

In order to fit the mixed GDF, it is essential to estimate β and γ from observed data series. Thom (1951) suggested maximum likelihood estimate (MLF) to these parameters as follows :

$$\hat{\gamma} = \frac{1 + (1 + 4A/3)^{\frac{1}{2}}}{4A}$$

$$\hat{\beta} = \bar{x}/\hat{\gamma}$$

$$A = \ln \bar{x} - \frac{1}{n} \sum_{i=1}^{n} \ln x_i$$

The variables $x_1, x_2, \ldots\ldots\ldots\ldots, x_n$ are the non-zero values of rainfall whose mean is $\bar{x}$, ln being the natural logarithm.

TABLE 2

Weekly distribution of shape and scale parameters

Week No.	Region I		Region II		Region III		Region IV	
	γ	β	γ	β	γ	β	γ	β
23	1.26	15.5	0.85	21.2	0.88	22.5	0.89	36.7
24	1.06	26.1	0.79	37.7	0.91	34.3	1.05	53.3
25	0.97	32.7	0.94	39.9	0.87	50.4	1.11	53.7
26	0.98	42.7	0.91	68.3	0.89	70.5	1.53	47.5
27	1.16	42.6	1.28	57.9	1.31	61.5	1.74	41.1
28	1.09	59.5	1.19	69.3	1.49	56.7	1.46	58.8
29	1.29	54.8	1.48	57.0	1.41	68.1	1.60	44.4
30	1.15	66.9	1.33	68.1	1.61	58.9	2.05	46.0
31	1.38	48.7	1.30	60.7	1.32	72.9	1.44	58.8
32	1.19	63.5	1.23	68.3	1.50	70.1	1.43	47.7
33	1.17	58.7	1.17	69.9	1.27	68.1	1.51	50.4
34	1.30	47.3	1.25	60.4	1.28	65.5	1.45	51.3
35	1.15	55.7	0.91	69.9	1.28	55.4	1.24	58.6
36	1.16	57.7	1.07	68.5	1.28	55.4	1.67	40.3
37	1.04	56.3	0.82	68.0	1.10	54.9	1.09	55.3
38	0.91	52.7	1.07	42.5	0.94	51.9	1.05	50.2
39	0.98	47.0	0.72	54.4	0.89	40.2	0.94	45.7
40	0.89	39.0	0.85	41.8	1.01	43.1	0.92	51.9
41	0.69	83.5	0.77	49.6	0.79	43.3	0.99	33.3
42	0.83	56.2	0.72	37.0	0.85	23.6	0.94	28.6
43	0.50	61.1	0.60	33.1	0.90	14.2	0.94	25.6

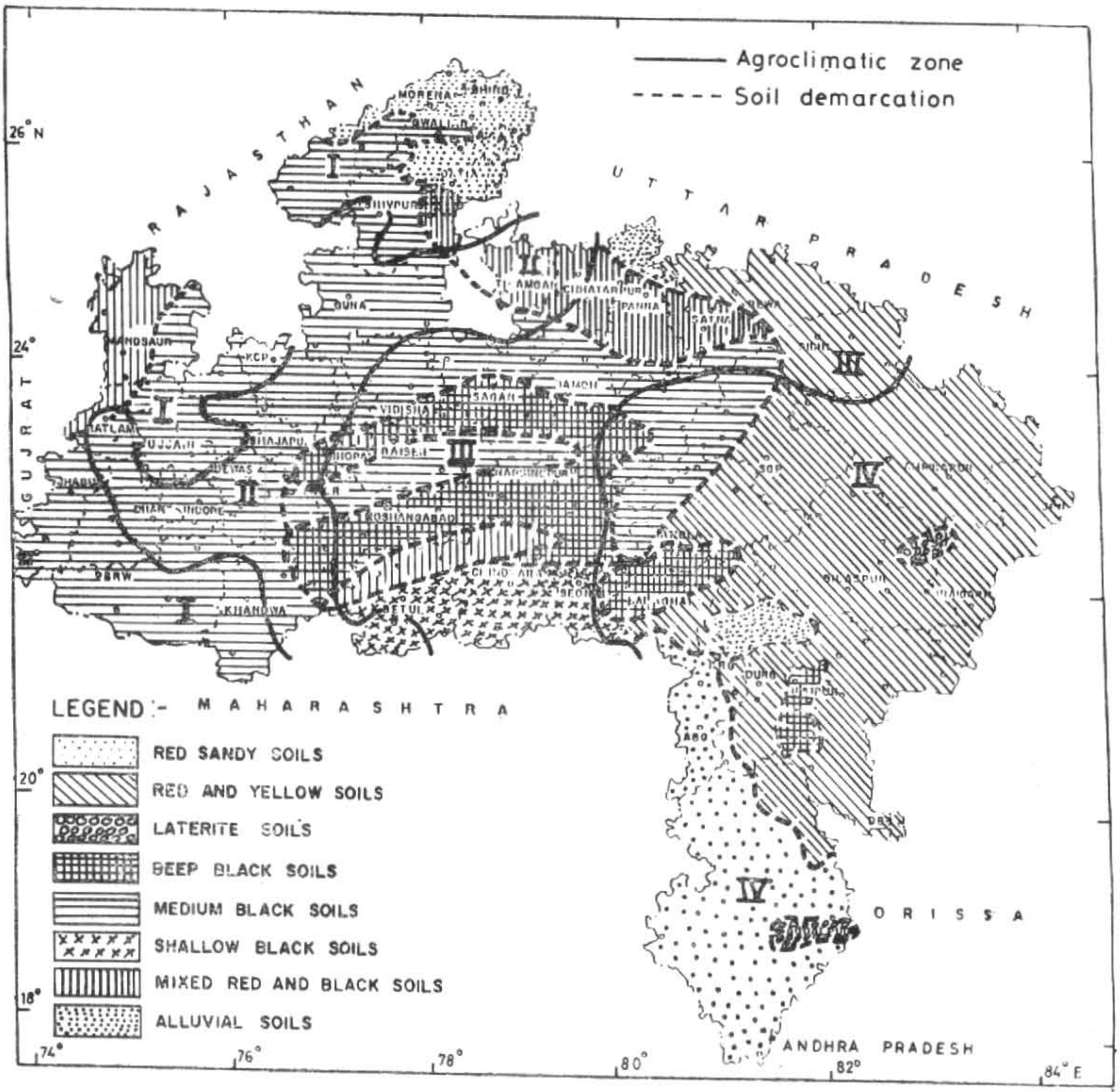

Fig. 1. Agroclimatic classfication by moisture availability index and soil types : (as per Das, 1987)

In the present analysis the probability level chosen was 75%. The minimum rainfall Px' at this level is obtained by putting $p=0.75$, $q=0.25$ in the equation

$$P_{x}' = \left(1+q+p\right) \int_{0}^{x'} \frac{x^{\hat{\gamma}-1}\, e^{-x/\hat{\beta}}}{\hat{\beta}^{\hat{\gamma}}\, \Gamma(\hat{\gamma})}\, dx$$

4. Results and discussion

4.1. *Test for goodness of fit*

Variance ratio test suggested by Cochran (1954) was used to test the goodness of fit of the Gamma distribution function. This test can be applied to any distribution for which the theoretical variance is computed independently from parameters estimated by a method other than the method of moments. The test statistic is

$$\chi^2_{n-1} = \sum_{i=1}^{n} (x_i-\bar{x}) / \hat{\beta}^2 \hat{\gamma}$$

where, χ^2_{n-1} is the Chi-square statistic with $(n-1)$ degrees of freedom.

The Chi-square values of the variance ratio test for a few selected stations from each of the four agroclimatic zones for selected weeks is given in Table 1. At all the stations the Chi-square values indicated non-significance at 5% level. In fact in none of the stations/weeks considered in the study, these values were significant. It was, therefore, concluded that the weekly rainfall in Madhya Pradesh follows the Gamma distribution.

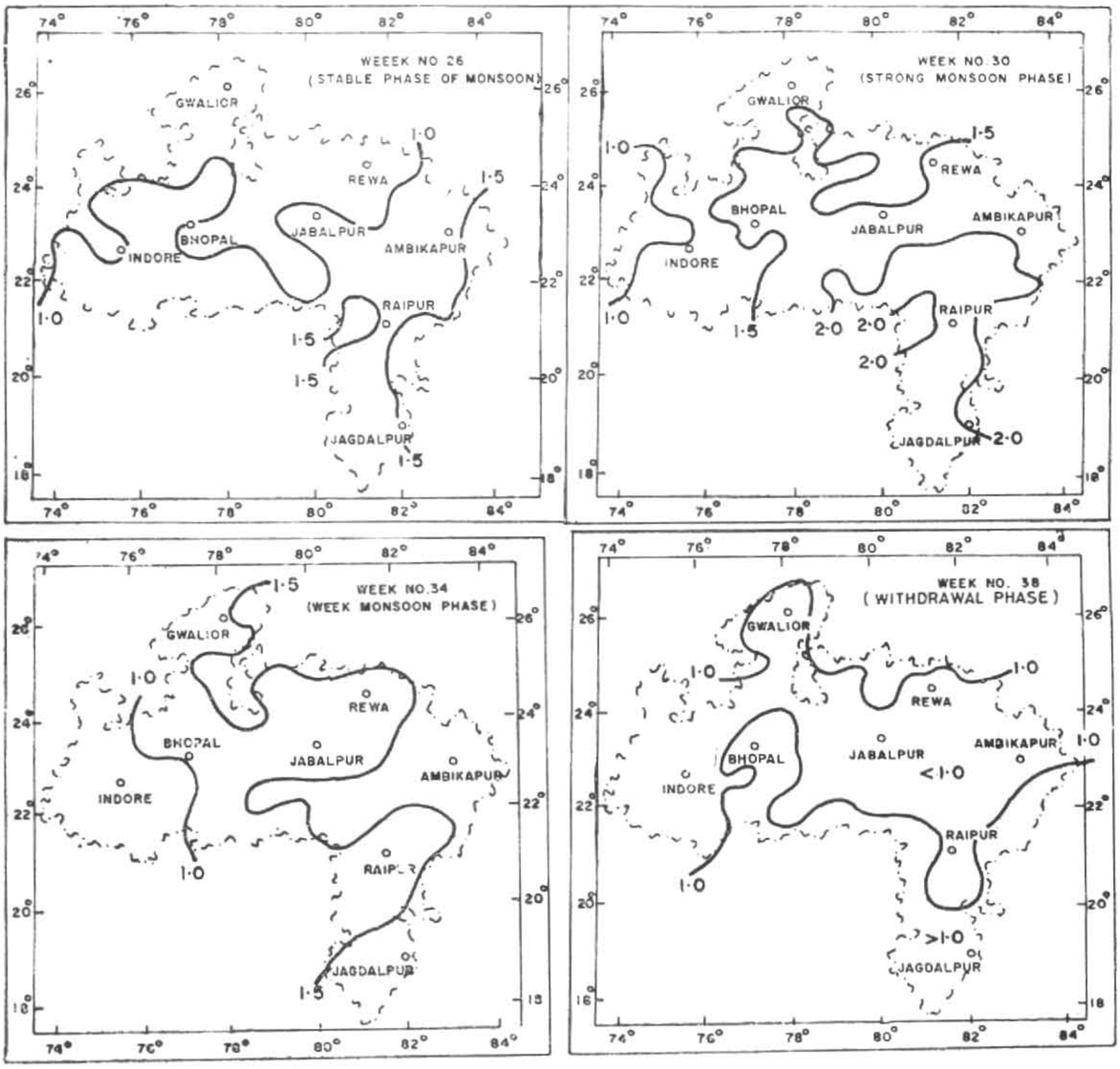

Fig. 2. Spatial distribution of shape parameter (γ)

4.2. *Regional variations of shape and scale parameters*

The estimated values of γ and β for the crop season is given for each week for the 4 agroclimatic zones of the State in Table 2. It is seen that the Regions I and III, γ values are quite low after 38th standard week, meaning thereby that positive skewness, in general, exists. A smaller γ also indicates that a few larger values cause positive skewness and that the mean departs further from the median values. In Region II also, this trend is observed but only in the 37th week, while in Region IV it is seen in the 39th week. Except Regions II and III where skewness is observed towards the beginning of the crop season, in the rest of the cases, the γ values are quite large, *i.e.*, the weekly rainfall is 'more' normally distributed.

The β values, by and large, are found to increase till 35/36 th week in the four zones. A larger value indicates greater tendency of rainfall to deviate from the mean. In other words, it means that substantial dispersion exists till first week of September or so. The dispersion is about 3 times that observed in the beginning of the season.

4.3. *Geographical distribution of γ and β*

Geographical distribution of γ and β has been studied by plotting these parameters for each week and analysing them. The illustrations are given only for 4 weeks in general representing onset, active, weak and withdrawal phases of the monsoon (Figs. 2 & 3).

During the commencement of crop growth stages, γ is less than one in NW Madhya Pradesh. Thus, weak skewness in rainfall exist only in the northwest; over rest of the regions hardly any skewness is seen at emergence and early development phases. As the monsoon season advances, rainfall distribution becomes less and less skew, *i.e.*, it approaches normality. The 30th week clearly brings out this feature. However, rainfall pattern again becomes skew in the western parts of the State in 34th week and in central parts in 38th week.

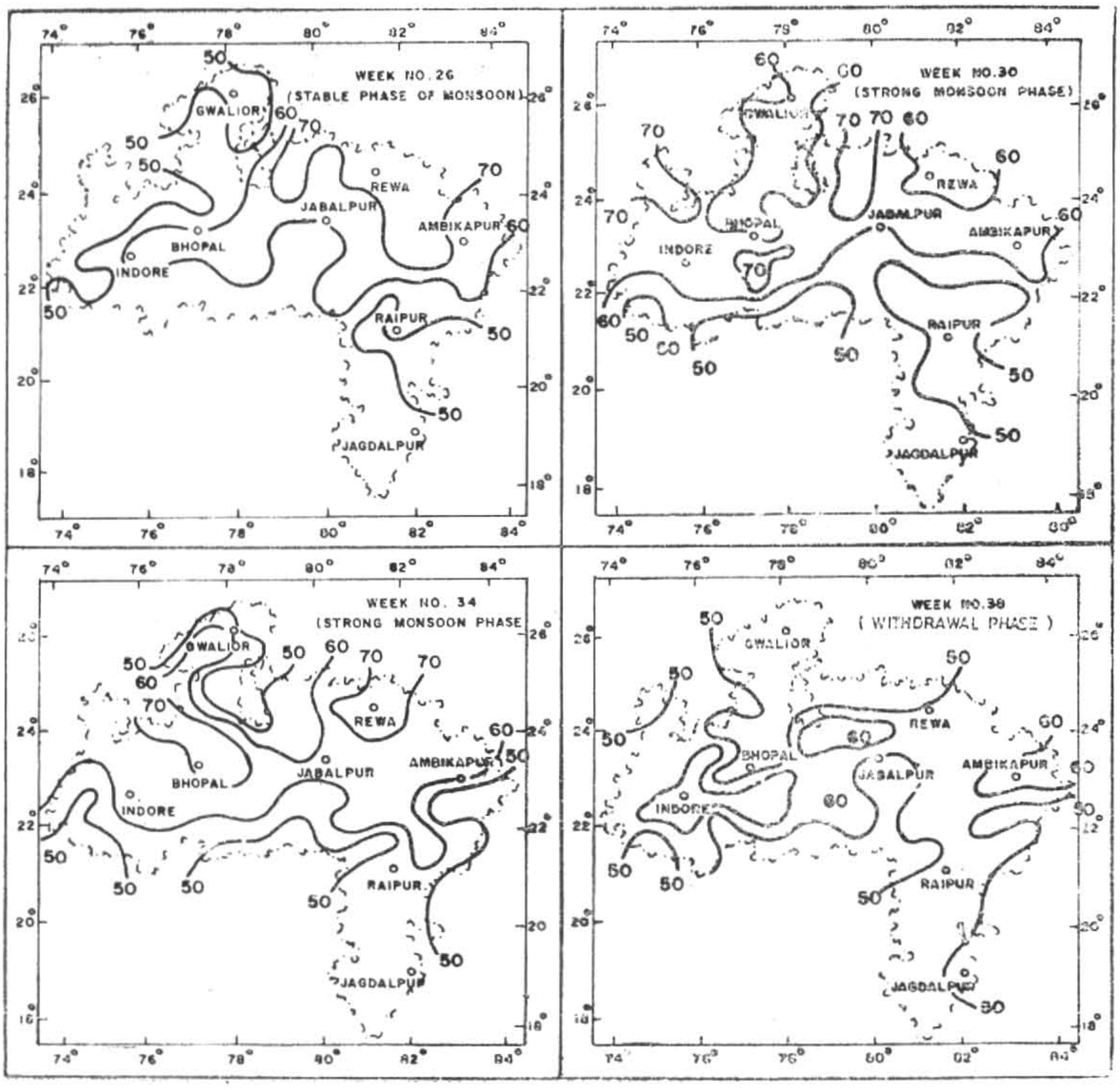

Fig. 3. Spatial distribution of scale parameter (β)

This skewness only not persist but increase in magnitude, till monsoon finally withdraws.

Analysis of the scale parameter reveal some interesting features during advancing stage of the monsoon over the State. Distribution of β shows large departures from mean in NE Madhya Pradesh and least over to SE and NW Madhya Pradesh. By 30th week β exceeds a value 60 north of tropic of cancer and persist till 34th week. It is only towards the end of the monsoon that β values start decreasing. Nevertheless values less than 50 are seen only in few districts, *i.e.*, Mandla, Balaghat, Durg, Chindwara, Betul etc. and the extreme east.

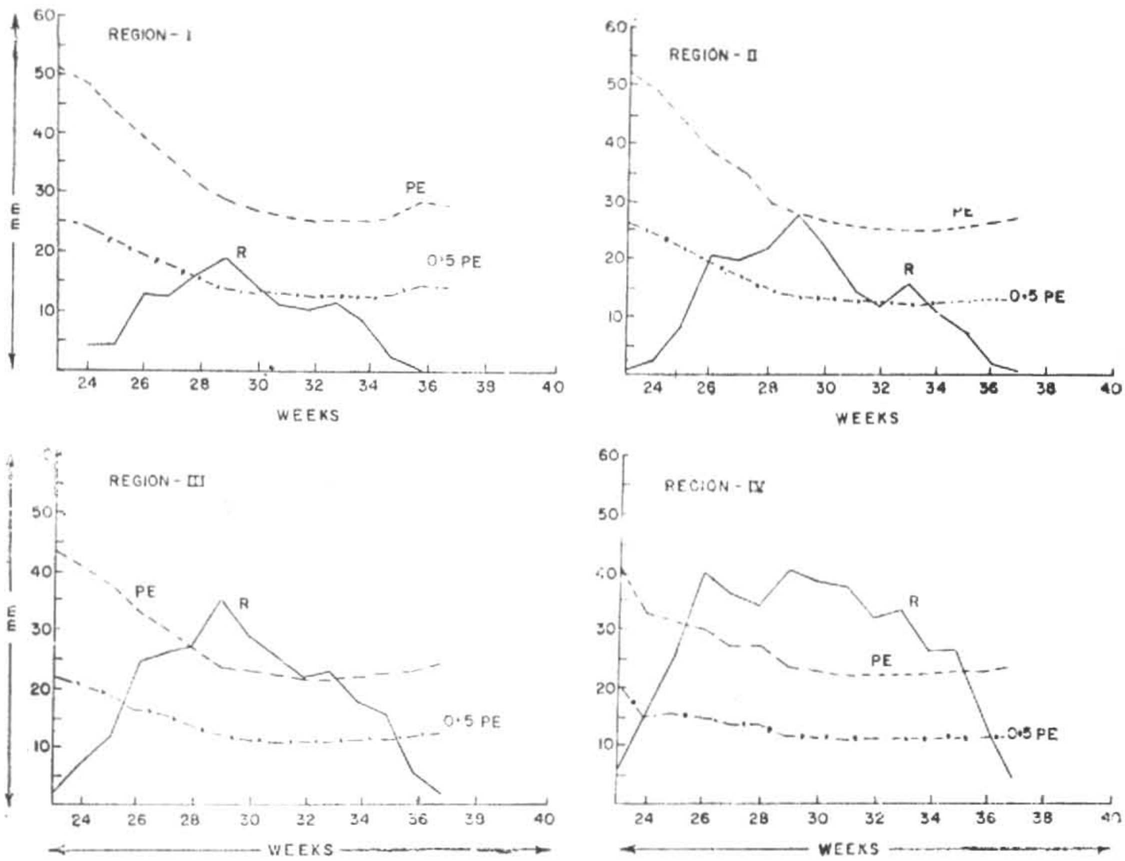

Fig. 4. March of rainfall at 75% probability level (R) and potential evapotranspiration (PE) in four regions

TABLE 3

Regression equation of assured rainfall at 75% level (y) and mean rainfall (x)

Std. week No.	Region I		Region II		Region III		Region IV	
	Equation	Correlation coefficient	Equation	Correlation coefficient	Equation	Correlation coefficient	Equation	Correlation coefficient
23	—		$y=0.06\ x-0.7$	0.27	$y=0.28\ x-4.5$	0.70**	$y=0.13\ x-0.23$	0.37
24	$y=0.08\ x-1.6$	0.60**	$y=0.21\ x-4.1$	0.55*	$y=0.42\ x-9.0$	0.78**	$y=0.48\ x-12.0$	0.68**
25	$y=0.35\ x-8.8$	0.81**	$y=0.69\ x-28.0$	0.72**	$y=0.60\ x-22.4$	0.65**	$y=0.26\ x+5.0$	0.74**
26	$y=0.42\ x-8.8$	0.90**	$y=0.28\ x+2.1$	0.56*	$y=0.34\ x-2.1$	0.47*	$y=0.45\ x-1.85$	0.96**
27	$y=0.25\ x-1.6$	0.65**	$y=0.11\ x+11.8$	0.56*	$y=0.43\ x+13.9$	0.48*	$y=0.35\ x+1.63$	0.83**
28	$y=0.33\ x-4.2$	0.66**	$y=0.14\ x+12.0$	0.48	$y=0.22\ x+9.4$	0.45*	$y=0.43\ x-4.3$	0.87**
29	$y=0.21\ x+3.4$	0.36	$y=0.23\ x+9.4$	0.50*	$y=0.24\ x+13.5$	0.48*	$y=0.43\ x+2.0$	0.92**
30	$y=0.14\ x+5.8$	0.26	$y=0.26\ x+2.1$	0.54*	$y=0.17\ x+14.1$	0.43*	$y=0.47\ x-8.7$	0.81**
31	$y=0.21\ x-0.5$	0.65**	$y=0.37\ x-8.3$	0.94**	$y=0.41\ x-9.1$	0.81**	$y=0.59\ x-15.9$	0.97**
32	$y=0.21\ x-0.7$	0.48	$y=0.25\ x-2.9$	0.81**	$y=0.52\ x-16.2$	0.87**	$y=0.49\ x-12.0$	0.85**
33	$y=0.17\ x+1.9$	0.30	$y=0.29\ x-3.7$	0.72**	$y=0.28\ x+1.4$	0.73**	$y=0.38\ x+0.2$	0.77**
34	$y=0.11\ x+2.5$	0.19	$y=0.19\ x-0.9$	0.53*	$y=0.17\ x+0.6$	0.46*	$y=0.39\ x-3.9$	0.66**
35	$y=0.14\ x-4.8$	0.41	$y=0.19\ x-2.1$	0.50*	$y=0.29\ x-4.0$	0.48*	$y=0.43\ x-4.1$	0.61**
36	$y=0.33\ x-1.0$	0.35	$y=0.19\ x-6.1$	0.61**	$y=0.32\ x+4.4$	0.46*	$y=0.37\ x-6.6$	0.73**
37	—		$y=0.04\ x-0.4$	0.13	$y=0.21\ x-6.4$	0.44*	$y=0.36\ x-8.9$	0.51*

** = significant at 1%, * = sinificant at 5%

4.4. *Water availability periods*

By comparing probability rainfall R at any level with the water needs as represented by PE, it is possible to identify periods of water surplus of deficiency and their extent. As the four regions identified are considered fairly homogeneous with regard to rainfall within the region and PE, it is possible to compare R and PE. Fig. 4 illustrates the weekly pattern of estimated R at 75% *vis-a-vis* PE. Since some of the recently developed crop species can withstand moisture stress up to certain degree, curves for 1/2 PE have also been drawn.

Region I appear characteristically dry with the estimated rainfall at 75% probability level never exceeds PE for even 1/2 PE except for 2 to 3 weeks. This is the dry zone where weekly variations in PE is quite large, in view of fairly high temperatures even in the monsoon season.

In Region II, variability in PE is comparatively less. Though assured rainfall in any week does not exceed PE for a large number of weeks, it does exceed 1/2 PE, implying thereby that crop prospects are somewhat better here.

There is a marked change in PE-R pattern in Region III. In this region for nearly 5 weeks (28th-33rd weeks) 75% rainfall exceed PE, whereas for a still larger period, it exceed 1/2 PE. Thus water deficiency period is restricted to only a few weeks at the beginning and end of the crop season.

An unambiguous difference emerges between period of water availability in relation to water need in the Region IV. This can be considered as a wet zone where water deficiency is minimum. Water deficit could be just for 1 to 2 weeks, most likely at sowing time or at the physiological maturity stage. Since the demand in these stages is even less than 1/2 PE (Replay 1966), it is felt, the deficiency would have no adverse effect on the crop growth or the yield during this period.

4.5. *Assured rainfall*

Areawise, the State of Madhya Pradesh is the largest State in India. The network of observatories, though adequate from climatic point of view, does not seem to be so, for, agroclimatic purposes. Recourse has, therefore, to be taken to regression analysis to get information for unrepresented stations and those locations at which data is missing for some period.

Regression equations have been developed between the mean regional rainfall as the independent variable, and the probabilistic rainfall at 75% level as the dependent parameter. These equations are given in Table 3. Correlation coefficients and the level of significance are also shown in the table. These equations establish the linear relationship between the actual (observed) regional rainfall and the theoretical values. These can be used to determine the theoretical values for any week if the corresponding mean rainfall for the region is is known.

4.6 *Agronomic evaluation of the results*

Generally warm climate, a shorter duration cold season with nearly all rainfall occuring within the summer season is the characteristic feature of all the four regions.

The southeast and central portion of the State have mostly deep soil. At 75% probability, the maximum weekly rainfall during 30-32 weeks in this area is quite high (*i.e.,* 40-60 mm). The heavy soils can retain adequate moisure throughout the entire range of reproductive and maturity stages. In this area, it is suggested bunding of water on the surface during the wet season, since most of the water is lost as surface runoff. Ponding of water would allow large quantities to move into deeper depths because it holds the water in contact with soil longer.

The northeast Madhya Pradesh experience comparatively lower amounts at the same probabilistic level. With the advance of the rainy season, one may be tempted to suspect a surplus of available moisture of 35-55 mm in a week in late vegetative growth period. However, the mixed red and black or red and yellow soil variety precludes such a possibility.

The rainfall though substantial, may not be frequently large enough to overcome the excessive transpiration losses and to percolate down due to lower rate of moisture movement in the moist soil. Moisture conservation methods may be adopted so that the crop does not suffer moisture inadequacy during flowering and post flowering stages.

The rest of the State is a low rainfall belt. The assured rainfall at 75% level in any week hardly exceed 20 mm. These are so low that they do not allow sufficient movement in deeper soils to ensure crop every year, though the soils are alluvial to medium black. In such areas, it is suggested that alternate fallowing and cropping or fallowing every fourth year must be practiced to conserve as much water from the brief wet period as possible. A system of bench terrace would also help to conserve the rain water in these areas to prevent runoff, thus helping to ensure good crop yield.

5. Conclusions

Some aspects of spatial distribution of weekly rainfall at 75% level of probability has been studied. Utility of the two statistic parameters emerging out of Gamma distribution model in producing reliable information during crop phenology, has been demonstrated. The study also helps assessing the timing mechanism for the crop usage in Madhya Pradesh.

The following broad conclusions could be drawn:

(*i*) Weekly rainfall is normally distributed between 26/27 to 37/38th week particularly over southeast Madhya Pradesh.

(*ii*) The rainfall does not disperse much from the mean in the beginning of the crop season.

(*iii*) In the northwest parts of the State, rainfall does not seem to be adequate to meet even half of the transpiration demands, in 3 out of 4 years.

(*iv*) Regression equations developed here enables to find rainfall at 75% probability level from mean rainfall values.

(*v*) Ponding of water must be practiced as a moisture conservation method to over-come loss of water.

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