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Modified numerical solution for the dispersal of air pollutants from an elevated source

RAVINDRA KUMAR

Dep. of Maths., I.I.T., New Delhi

and

R. N. GUPTA

Meteorological Office, New Delhi

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ABSTRACT. While obtaining numerical solutions for the dispersal of pollutants from an elevated point source, truncation errors due to horizontal advection invariably generate a fictitious viscosity, the magnitude of which soon exceeds vertical diffusion. This paper presents a numerical model in which the artificial viscosity appears to be very very small, *i.e.*, negligible. The results are compared with those obtained by earlier methods and the improvement in results is observed. This shows that the presence of pseudoviscosity in a numerical method earlier used to give some error, especially at the advanced stages of time.

1. Introduction

Various numerical solutions for the dispersal of pollutants from an elevated point source have been computed in the recent years. It has been shown by Gupta (1980) that the results obtained by a Gaussian plume did not provide correct estimates at larger distances downstream, because it gives concentration of pollutants for a steady state only. He also took an account of the fact that the wind and eddy diffusivity vary with height. To encounter the effect of artificial viscosity he used a method due to Mahoney and Egan (1970) which involves separating the horizontal transport and vertical diffusion in the diffusion equation. Yet, the effect of pseudoviscosity could not be completely avoided, though it was minimised.

In this paper a numerical model is presented which does not involve artificial viscosity at all.

It endeavours to find better results and to estimate the amount of incorrectness in the results due to the presence of artificial viscosity in the earlier numerical methods.

2. Numerical model

We follow Gupta (1980) but with the following modification.

The equation governing diffusion is :

$$\frac{\partial c}{\partial t} + u(Z) \frac{\partial c}{\partial x} = \frac{\partial}{\partial Z} \left[K(Z) \frac{\partial c}{\partial Z} \right] \quad (1)$$

If we leave out the advection term in (1), then the remaining equation is the familiar diffusor equation with Z and t as independent variables. As such, one analyses the equation :

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial Z} \left[K(Z) \frac{\partial c}{\partial Z} \right] \quad (2)$$

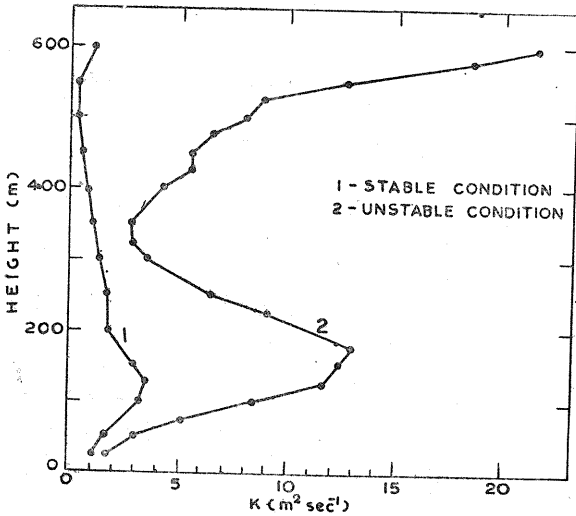


Fig. 1. Variation of eddy diffusivity \$K\$ with height on a day with (1) warm air advection and (2) cold air advection [After Lettau & Hoerber 1964]

for the purpose of stability, and uses the condition

$$K \frac{\Delta t}{(\Delta Z)^2} \leq 0.5 \quad (2.1)$$

which leaves an error of the order \$O(\Delta t) + O[(\Delta Z)^2]\$ when difference scheme is made to replace the differential expression.

With the usual notation \$t = n \Delta t\$, \$x = j \Delta x\$ and \$Z = l \Delta Z\$ if we denote, \$C(l \Delta Z, n \Delta t)\$ by \$C_l^n\$ etc, we get by Taylor's series with remainder, the following expansions :

$$C_l^{n+1} = C_l^n + \Delta t \left(\frac{\partial c}{\partial t} \right)_l + \frac{1}{2} (\Delta t)^2 \times \left(\frac{\partial^2 c}{\partial t^2} \right)_l + \frac{1}{6} (\Delta t)^3 \left(\frac{\partial^3 c}{\partial t^3} \right)_l \quad (3)$$

$$C_{l \pm 1}^n = C_l^n \pm \Delta Z \left(\frac{\partial c}{\partial Z} \right)_l + \frac{1}{2} (\Delta Z)^2 \times \left(\frac{\partial^2 c}{\partial Z^2} \right)_l \pm \frac{1}{6} (\Delta Z)^3 \left(\frac{\partial^3 c}{\partial Z^3} \right)_l + \frac{1}{24} (\Delta Z)^4 \left(\frac{\partial^4 c}{\partial Z^4} \right)_l \pm \frac{1}{120} (\Delta Z)^5 \left(\frac{\partial^5 c}{\partial Z^5} \right)_l + \frac{1}{720} (\Delta Z)^6 \left(\frac{\partial^6 c}{\partial Z^6} \right)_{l+\beta}^{l+\gamma} \quad (4)$$

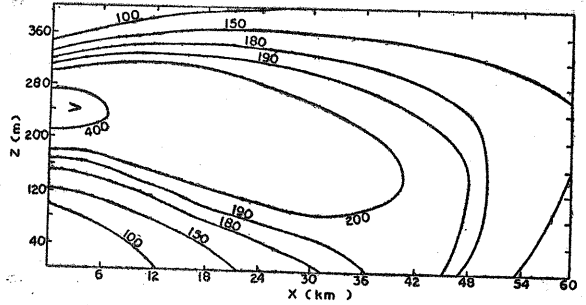


Fig. 2. Short time 1-hr concentration for \$\text{SO}_2\$ in \$X-Z\$ plane under stable condition

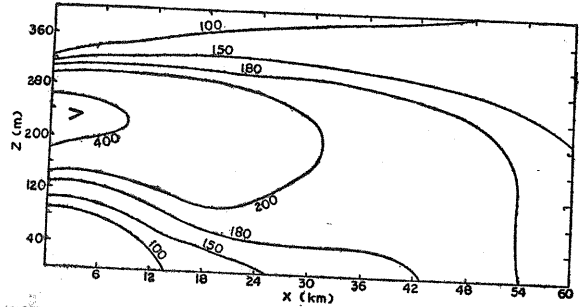


Fig. 3. Concentration after 9-hr simulation time

\$\alpha, \beta, \gamma\$ being numbers between 0 and 1. Using (3) and (4), the difference between the finite-difference expression and the corresponding differential equation is given as follows :

$$\frac{C_l^{n+1} - C_l^n}{\Delta t} - K \frac{C_{l+1}^n - 2C_l^n + C_{l-1}^n}{(\Delta Z)^2} - \left[\frac{\partial c}{\partial t} - K \frac{\partial^2 c}{\partial Z^2} \right]_l = \left\{ \left[\frac{\Delta t}{2} \frac{\partial^2 c}{\partial t^2} - K \frac{(\Delta Z)^2}{12} \frac{\partial^4 c}{\partial Z^4} \right] + \frac{\Delta t^3}{6} \left(\frac{\partial^3 c}{\partial t^3} \right)_l - \frac{K}{360} (\Delta Z)^4 \left[\left(\frac{\partial^6 c}{\partial Z^6} \right)_{l+\beta} + \left(\frac{\partial^6 c}{\partial Z^6} \right)_{l+\gamma} \right] \right\} \quad (5)$$

Now as \$C\$ satisfies the equation of heat flow, it also satisfies the equation

$$\frac{\partial^2 c}{\partial t^2} = K^2 \frac{\partial^4 c}{\partial Z^4}$$

and the R.H.S. in (5) becomes :

$$K \frac{(\Delta Z)^2}{2} \left[\frac{K \Delta t}{(\Delta Z)^2} - \frac{1}{6} \right] \frac{\partial^4 c}{\partial Z^4} + O[(\Delta t)^2] + O[(\Delta Z)^4] \quad (6)$$

Hence, if Δt and ΔZ are selected for a particular value of K such that

$$K \frac{\Delta t}{(\Delta Z)^2} = \frac{1}{6}$$

the truncation error is of the order of Δt^2 and one gets rid of the term involving $\Delta t (\partial^2 c / \partial t^2)$ which generates artificial viscosity.

The stability condition (2.1) is not violated and the error approaches zero much faster. Consequently, the solution provided by the difference equation in (5) approaches the solution of the differential equation more rapidly.

3. Data input

The following inputs were used for numerical integration :

(i) Emission rate (Q) = 1 ton hr⁻¹, (ii) Height of stack (H) = 80 m, (iii) ΔZ = 20 m, (iv) Δx = 1 km, (v) Δt = 50 sec (for Fig. 2) and 17 sec (for Fig. 3).

For this experiment we used the Pasquill (1962) and Turner (1964) classification of atmospheric stability. This classification is based on (i) wind speed, (ii) cloud cover and (iii) solar insolation, and it contains 7 categories of stability.

The variation of eddy diffusivity with stability and height was obtained from the results of Lettau and Hoerber (1964). This is shown in Fig. 1. Lettau and Hoerber provided the variation of K with Z for stable and unstable conditions.

A power law was assumed to determine the vertical profile of $u(Z)$. We have

$$u = u_1 (Z/Z_1)^p$$

where u and u_1 represent the wind speed at Z and Z_1 respectively.

The values of p were 1/9, 1/7 and 1/3 for unstable, neutral and stable conditions.

σ_y and σ_z depend on the downstream distance (x) in addition to the stability of the atmosphere. We used the expressions due to Briggs (Gifford 1976) in our work.

Following Moore (1974), the expression for plume rise $\Delta h(m)$ were :

$$(1) \text{ Unstable/neutral : } \Delta h = \frac{60 + 5H}{u} Q_H^{0.25}$$

$$(2) \text{ Stable and weak wind : } \Delta h = \frac{116}{u} Q_H^{0.25}$$

$$(3) \text{ Stable and strong wind : } \Delta h = \frac{160}{u} Q_H^{0.25}$$

Q_H stands for rate of heat emission. We used a constant value of 25.6 MW in our computations.

4. Results and discussions

Fig. 2 indicates short term 1 hour concentration for SO₂ in X - Z plane under stable condition after 9 hours simulation time, using stability criteria $K\Delta t/(\Delta Z)^2 \leq 1/2$, while Fig. 3 also depicts the concentration after 9 hours simulation time but with the stability criteria $K\Delta t/(\Delta Z)^2 = 1/6$. With the elimination of the artificial viscosity term we find that the concentration at the end points have increased and the maximum values occur at points closure to the source. Furthermore the maximum value has also increased from 183 $\mu\text{g}/\text{m}^3$ at a distance of 48 km to 192 $\mu\text{g}/\text{m}^3$ at a distance of 40 km. As such Fig. 3 gives a more realistic picture of the concentration distribution where the effect of pseudo-viscosity has been reduced to a very small quantity.

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