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Persistency in sequences of wet and dry pentads over Bombay

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ABSTRACT. Seventy years pentad rainfall data for the southwest monsoon of India (June-September) of Bombay have been examined for their precipitation patterns using statistical techniques. It is found that the character of one pentad has an influence on the character of the following pantad. Further, there is slight tendency for wet pentads to be followed by further wet pentads. There is no tendency for dry period to be further followed by further dry period.

1. Introduction

A country like India where agriculture is of paramount importance, study of precipitation patterns is basic for agricultural operations. Much work has been done in this direction. Gold (1929) discussed whether, run of meteorological events has a real significance or is due to mere chance. Beer *et al.* (1946) have shown that sequence of wet and dry months observed at Kew and many places in the British Isles followed the mathematical law of probability. Srinivasan (1954) analysed monthly rainfall data of Rayalaseema and concluded that consecutive rainfall amounts are independent and perhaps shorter period may show persistency. Mooley and Appa Rao (1968, 1970) have studied the statistical distribution of pentad rainfall. Ananthakrishnan and Pathan (1971) have studied the pentad rainfall pattern over India and adjacent sea area. Mooley and Appa Rao (1972) have studied the dependence and independence of pentad rainfall during monsoon season.

Thus lot of work has been done on pentad rainfall. Workers in this direction have emphasised that the independence of the consecutive rainfall figures is due to the length of the period examined and that a statistical aftereffect will come into play for sufficiently short periods. It is worthwhile to see independence or otherwise of pentad rainfall. With this in view, as a pilot study, the present paper aims at finding the precipitation in successive pentads and in sequences of pentads over Bombay.

2. Data and method

As a pilot study, rainfall data of one station, *viz.*, Bombay was utilized. From the daily rainfall records of Bombay, five days (non-overlapping) rainfall totals were computed for the pentads covering the southwest monsoon of India for each year 1901 to 1970. The pentads used are the standard pentads as used by Mooley (1970). The frequency distribution of the observed pentad rainfall is shown in Fig. 1. Normals of each of the above pentads were computed. The same are shown in Table 1. These normal values formed the threshold values for considering a pentad to be wet or dry. A wet pentad designated as W was defined to be a pentad whose observed value was greater than or equal to its threshold value. Similarly a dry pentad D was defined for a pentad rainfall amount less than the threshold value. Precipitation ranges above and below the corresponding threshold values were obtained. A frequency distribution of the above precipitation ranges were formed. The same is shown in Fig. 2. The distribution shows that there are 652 wet pentads and 1588 dry pentads. The position of the mode shows that the precipitation deviation for any pentad is most probably in the range from —1.00 to 0.00 inches.

3. Precipitation in successive pentads

A χ^2— test for independence was applied to the sequence of wet and dry pentads to understand whether one occurrence has any effect on the character of the following occurrence. A 2×2 contingency table was used to arrange the data. There were four classifications of the pentads

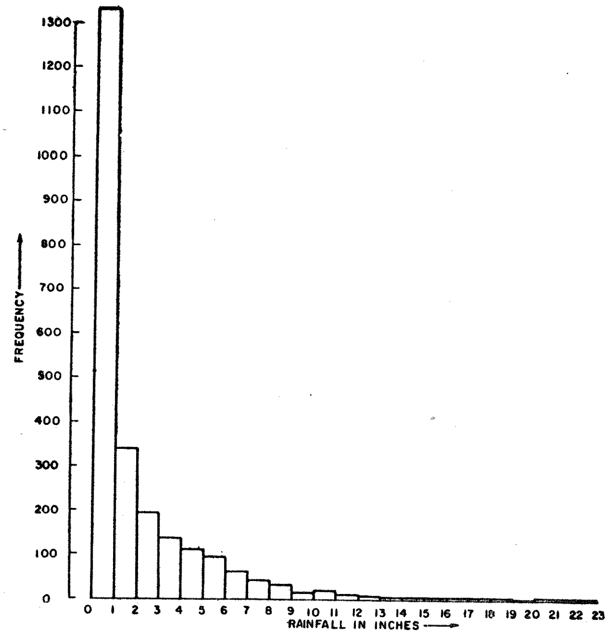

Fig. 1. Frequency distribution of pentad rainfall at Bombay (1901-1970)

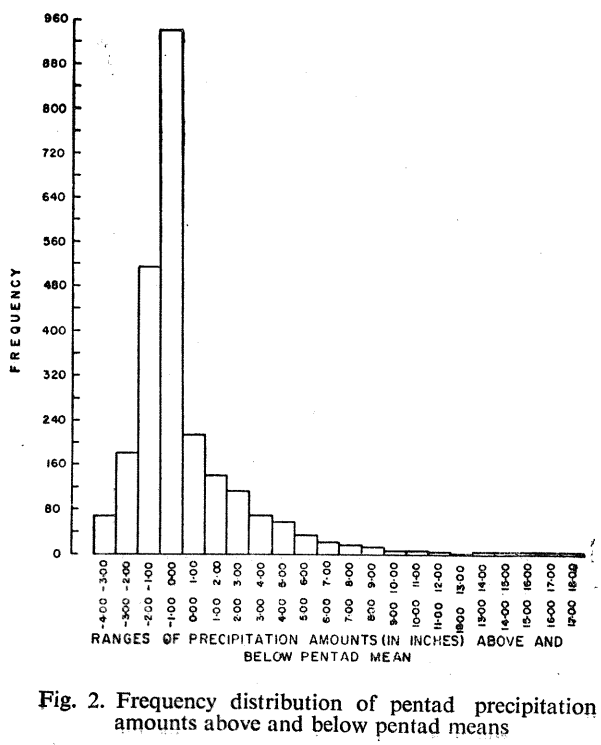

Fig. 2. Frequency distribution of pentad precipitation amounts above and below pentad means

TABLE 1

Normal pentad values (inches)

Pentad No.	Value	Pentad No.	Value
27	.05	43	3.05
28	.08	44	2.18
29	.16	45	1.90
30	.30	46	1.86
31	.63	47	2.05
32	1.36	48	2.25
33	2.51	49	1.94
34	3.19	50	1.80
35	3.39	51	1.48
36	3.93	52	1.45
37	3.66	53	1.72
38	3.96	54	0.83
39	3.07	55	1.20
40	2.91	56	0.73
41	3.22	57	0.52
42	3.18	58	0.28

TABLE 2

Current pentad	Following Pentad		Total
	W	*D*	
W	304 (187)	340 (457)	644
D	348 (465)	1248 (1131)	1596
Total	652	1588	2240

A current pentad could be *W* following *W*, *D* following *D*, *W* following *D* or *D* following *W*. The results are shown in the Table 2.

The numbers in the Table 2 without the paranthesis are the observed values '*O*' and those in the paranthesis are the expected values '*E*' on the assumption of complete independence of occurrences. The totals were used to compute the expected proportions. The χ^2 was calculated from the equation given by

$$\chi^2 = \Sigma\ (O - E)^2/E \qquad (1)$$

The χ^2 so calculated worked out to be 144.44 and this value for one d.f. was highly significant. This indicates that there is no probability of obtaining the distribution by chance and the character of one pentad has an influence on the character of the following pentad. Thus showing dependence of previous pentad.

4. Precipitation patterns in sequences of pentads

The χ^2 test showed that there is dependence between successive pantads. Further, we wish to investigate to what extent this influence exists in longer sequences of wet and dry pentads. For this the procedure followed was to compare observed integrated frequencies of runs of like pentads with the values calculated by purely theoretical application of probabilities as per appendix A.

Frequencies of wet and dry sequences of different lengths have been determined from 1901 to 1970 rainfall data. The same are given in Table 3. The table also gives the cumulative frequencies of at least *m* pentads cumulating from below (*i.e.*, cumulating from the longest to the shortest). The logarithmic values of these cumulative values are shown in Fig. 3. It is linearly related to different length (*m*) of the pantads. Theoretical justification to this relationship is obtained by approaching to the theory of probability (*see* appendix A). Applying general theory of probability, Beer *et al.* (1946) have derived theoretical relationship between *m* the length of the sequence and log *F*, *F* being the integrated frequency. Accordingly,

$$N = q\ p^m \times T \qquad (2)$$

$$n = p\ q^m \times T \qquad (3)$$

where, *N* and *n* are the integrated frequencies for wet and dry sequences and T the total number of pentads. The Table 1 gives the parameters *p*, *q* and *T* as follows :

$$p = \Sigma W/\Sigma(W+D) = 0.291$$

$$q = \Sigma D/\Sigma(W+D) = 0.709$$

$$T = 2240$$

The formulation of Eqns. (2) and (3) are given in Appendix A. The computed values of *N* and *n* and their log values are given in the Table 3 for wet and dry periods.

The computed cumulative frequencies for different lengths are shown in Fig. 4. The integrated frequencies for wet pentads of various lengths form a straight line, but of slightly different slope from the theoretical line. This shows that the observed *p* and *q* have different basic values from that observed by chance. The fact that the observed values actually cross the theoretical value line is significant. This event indicates that the actual probability of getting a wet pentad of exactly one pentad is lower than to be expected by chance. On the other hand, for wet pentads of exactly two pentads or longer exact pentads, there is greater probability of this occurrence than expected by chance. This shows that a long wet pentad tends to persist by influencing the character of the subsequent pentads. On the other hand for the dry pentads, the frequency of dry sequences follow mathematical probability law. Hence one concludes that dry pentads do not influence the chance of a succeeding pentad being dry or wet.

TABLE 3

Integrated Frequency

'm'	Dry pentads					Wet pentads				
	Observed			Computed		Computed		Observed		
	f	*F*	log *F*	log *F*	*F*	log *F*	*F*	log*F*	*F*	*f*
1	103	413	6.0	6.1	462.1	6.1	462.1	5.8	345	174
2	75	310	5.7	5.8	327.7	4.9	134.5	5.1	171	92
3	62	235	5.5	5.4	232.3	3.7	39.1	4.4	79	46
4	41	173	5.1	5.1	164.7	2.4	11.4	3.5	33	20
5	30	132	4.9	4.8	116.8	1.2	3.3	2.6	13	8
6	35	102	4.6	4.4	82.8	0	1.0	1.6	5	3
7	24	67	4.2	4.1	58.7				2	0
8	13	43	3.8	3.7	41.6				2	2
9	6	30	3.4	3.4	29.5					
10	5	24	3.2	3.0	20.9					
11	8	19	2.9	2.7	14.8					
12	6	11	2.4	2.4	10.5					
13	2	5	1.6	2.0	7.5					
14	0	3	1.1	1.7	5.3					
15	0	3	1.1	1.3	3.7					
16	1	3	1.1	1.0	2.7					
17	0	2	0.7	0.6	1.9					
18	0	2	0.7	0.3	1.3					
19	0	2								
20	1	2								
21	0	1								
22	0	1								
23	0	1								
24	0	1								
25	0	1								
26	0	1								
27	1	1								

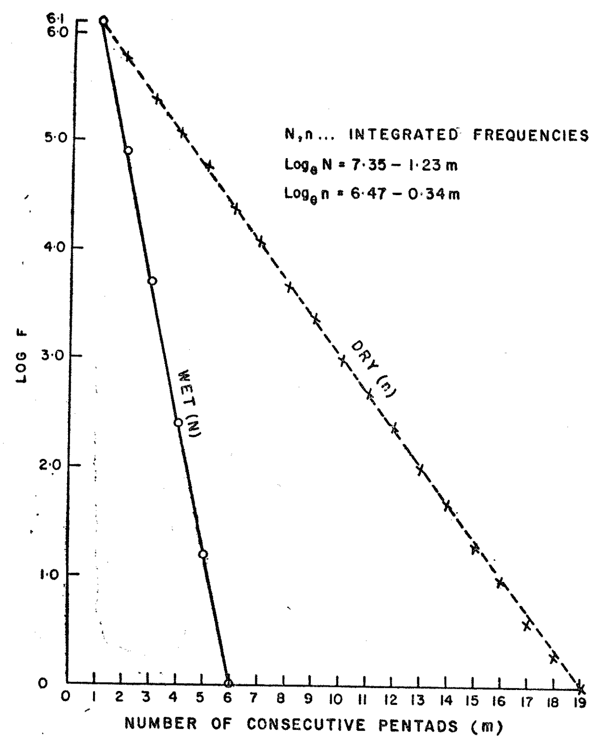

Fig. 3. Frequency of specified runs of wet and dry pentads at Bombay

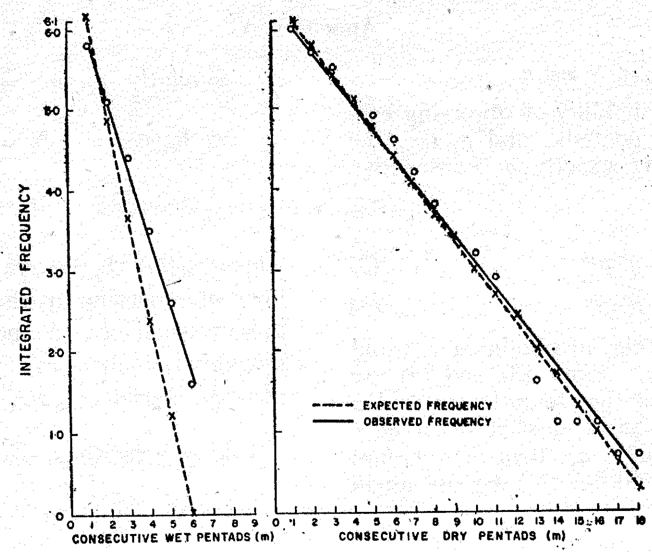

Fig. 4. Observed and integrated frequencies (*comparison*)

5. Statistical test

Eqns. (9) and (10) of Appendix A were fitted by least square method. The regression coefficients log p and log q were computed. These gave the values of p and q as 0.4504 and 0.7269 for wet and dry sequences respectively. The observed value of p, *viz.*, 0.291 was tested for significance. The observed difference between calculated and observed was three times the standard error. This shows that the two p's are significantly different for the wet period but was statistically insignificant for the dry period. This further supports that long wet pentad rainfall tends to persist by influencing the character of subsequent pentads while the dry sequence follows the general mathematical probability law.

6. Conclusions

(i) The pentads with rainfall amount above and below normal pentads in Bombay are dependent.

(*ii*) There is tendency for wet pentads to be followed by further wet pentads.

(*iii*) The dry pentads of various lengths follow the general law of probabilities.

(*iv*) The position of the mode shows that the precipitation for any pentad is most probably in the range from —1.00 to 0.00 inches below the pentad normal.

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Appendix A

Theoretical relation between m, N and n

Let $p_W(m)$ be the probability of observing exactly m consecutive wet pentads and $p_D(m)$ the probability of observing exactly m consecutive dry pentads.

Then,

$$p_W(m) = q^2 p^m \tag{3}$$

$$p_D(m) = p^2 q^m \tag{4}$$

where p, q are probabilities of obtaining wet and dry pentads respectively. Eqns. (3) and (4) are obtained due to the fact that in order to find a sequence of wet pentads there must be observed a dry pentad with probability q, then m wet ones each with probability p and finally a dry one again with probability q.

Now

$$P_W(m) = \sum_{x=m}^{\infty} p_W(x) = q^2 \sum_{m}^{\infty} p^x = q^2 p^m/(1-p)$$

$$\therefore P_W(m) = q\, p^m$$

$$[\because \Sigma\, p^x \text{ is G.P.}] \tag{5}$$

Similarly, $$P_D(m) = p\, q^m \tag{6}$$

we have $$N = q\, p^m\, T \tag{7}$$

$$n = p\, q^m\, T \tag{8}$$

where $P_W(m)$, $P_D(m)$ are the cumulative probabilities of observing at least m wet and m dry pentads respectively. N and n are the corresponding integrated frequencies. T is the total number of pentads considered, *viz.*, 2240 pentads.

Taking logarithms of both sides of (7) & (8)

$$\log_e N = m \log_e p + (\log_e q + \log_e T) \tag{9}$$

$$\log_e n = m \log_e q + (\log_e p + \log_e T) \tag{10}$$

The Eqns. (9) and (10) show logarithmic relationship first noted in a study of wet and dry sequences at Kew by Beer *et al.* (1946).