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Estimation of deterministic component of monthly rainfall time series : A case study for Pantnagar

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सार - जलविज्ञान और पर्यावरणीय प्रबंधन क्षेत्रों के विभिन्न अनुप्रयोगों में समय- श्रृंखला विशलेषण और पुर्वानुमान प्रमुख माध्यम बने। समय - श्रृंखला में आँकड़ों के अनुक्रम पर समय - श्रृंखला परिवर्तिता के रूप मे विचार किया जा सकता है। इस समय समय - श्रृंखला परिवर्तिता को दो घटको में विभाजित किया जा सकता है निर्धारित घटक और यादृच्छिक घटक। निर्धारित घटक आवधिक या गैर आवधिक हो सकते है। गैर आवधिक घटक की स्थिति में ट्रेंण्ड और जंप श्रृंखला को विशेष बनाते है और आवधिक निर्धारित घटक की स्थिति में चक्रीय पैटर्न श्रृंखला को विशेष बनाते है। समय श्रृंखला के यादृच्छिक घटक का आकलन करने के लिए निर्धारित घटक को मुख्य समय श्रृंखला परिवर्तिता से घटाया जाता है इस शोध पत्र में भारत के उत्तराखंड राज्य में स्थित पंतनगर क्षेत्र के 1981 से 2010 तक के मासिक वर्षा के आँकडों का उपयोग निर्धारित घटकों का आकलन करने के लिए किया गया है। ऐतिहासिक आंकड़ों की श्रृंखला में स्थायित्व लाने के लिए वर्गमूल ट्रांसफॉर्मेशन का उपयोग किया गया है। आवधिक घटक के निर्धारण के लिए फुरिए विश्लेषण किया गया। आधारभूत समय और महत्वपूर्ण हार्मोनिक की संख्या ऑटोकोरेलोग्राम विश्लेषण और क्रमशः भिन्न परीक्षण के विश्लेषण द्वारा निर्धारित किया गया। सांख्यिकीय परीक्षण से यह पता चला है कि समय श्रृंखला में कोई ट्रेंण्ड घटक नहीं है। इस क्षेत्र के लिए निर्धारित घटक के समीकरण भी प्रस्तावित किए गए।

ABSTRACT. Time series analysis and forecasting has become a major tool in various applications in hydrology and environmental management fields. Data sequence in a time series can be considered as a time series variable. This time series variable can be divided into two components *viz*. deterministic component and stochastic component. Deterministic component may be periodic or non-periodic. Trend and jump characterize the series in case of non-periodic deterministic component and in case of periodic deterministic component, cyclic pattern characterizes the series. To estimate the stochastic component of a time series deterministic component is subtracted from the main time series variable. In this study, the monthly rainfall data of 1981 to 2010 of Pantnagar region situated in Uttarakhand state, India, has been used to estimate deterministic component. To bring stationarity in the historical data series, square root transformation has been used. Fourier analysis was performed for determination of periodic component. The base period and number of significant harmonics was determined by auto correlogram analysis and analysis of variance test respectively. Statistical tests revealed that there was no trend component in the time series. An equation for the deterministic component has also been proposed for this region.

Key words – Turning point test, Mann Kendall's rank correlation test, Correlogram analysis, Harmonic analysis, Trend component, Periodic component.

1. Introduction

Time series can be defined as a set of observations arranged chronologically, *i.e.*, a sequence of observations usually ordered in time but may be ordered to some other dimensions. The main aim of a time series analysis is to describe the history of movements in time of a variable at a particular site and to generate data having properties of the observed historical record. To compute properties of a historical record, the historical record of time series is broken into separate components and analyzed individually to understand the underlying mechanism of different components. Once the properties of components are understood, these can be generated with similar properties and combined together to give a generated future series. Analysis of a continuously recorded time series is performed by transforming the continuous series into a discrete time series of finite time interval. Matlas (1967) measured degree of linear dependence between hydrologic events k time units apart by serial correlation coefficients. Quimpo (1968); Chau and Wu (2010); Sang *et al*. (2012) used autocorrelation and spectral analysis techniques to model time series and showed that the spectral techniques with harmonic analysis can be efficiently used to investigate the structure of time series. Adamowaski and Smith (1972) used time series analysis in estimating a generation process and its parameter for synthesizing daily rainfall. Raudkivi and Lawgun (1972) generated a long series of serial correlated non-normal distributed rainfall duration events. The generated data results were found good in the serial correlogram, the first three moments and the distribution of historical data were reproduced. Parthasarthy and Dhar (1976) studied trend and periodicities in the seasonal and annual rainfall of India from 1901 to 1960. Shahane *et al*. (1976) analyzed mean monthly time series of precipitation, runoff, atmospheric water vapor transposed, terrestrial's storage and evapotranspiration of watersheds distributed over the entire continental United States. Time series were analyzed by harmonic analysis to compute Fourier coefficients with first order harmonics and explained variance. Linear cross-correlation coefficients for 10 combinations of the five time series were also computed. The spatial variation of these parameters was useful in the formulation of statistical models for simulation and production applications. Similar studies were also conducted by Piazza *et al*. (2011); Simmons *et al*. (2010); Vose *et al*. (2014).

Tao and Delleur (1976) showed that the logarithmic transformation of the runoff data has the merit of reducing the periodicities in the seasonal standard deviation. The cyclic standardization was found to be effective in removing the periodicities in the seasonal mean and variance, but it was not able to remove seasonal correlations of data series. Sen (1979) investigated autocorrelation structure of a seasonal hydrologic series. Sen (1979) proposed a method of adaptive Fourier analysis of periodic stochastic hydrological sequences using monthly river flow data of Columbia (U.S.A.). Richardson (1981) observed that the deterministic mathematical model, which was used to evaluate the long-term effects of proposed hydrologic changes, necessitates daily weather data as input. Narayana (1982) suggested a transformation to transform a non-normal monthly hydrological data into a normal one. Yevjevich and Dyer (1983) observed that the daily rainfall series exhibited intermittency, periodicity and stochasticity. They found that coefficient of variation, skewness and kurtosis had less periodicity than other parameters for all the series of the model. Gordin *et al.* (1987) proposed a random pulse generator for simulated rainfall using statistical data from 420 rainfall periods in Minsk, Russia. Chiew *et al*. (1993) compared simple polynomial equation, simple process equation, simple time series equation, complex time series model and simple conceptual model for simulation of daily, monthly and annual flows in eight unregulated catchments. Bo *et al.* (1994); Kossieris *et al*. (2016); Kim *et al*. (2013) proposed a statistical approach based on modified Bartlett-Lewis rectangular pulse model to disaggregate rainfall statistics form daily data. Benalaya *et al.* (1998) studied the temporal rainfall trends by time series of rainfall data collected from six weather stations in Northern Tunisia. Gyasi and Agyei (1999) disaggregated daily rainfall using the binary nonrandomized Barlett-Lewis rectangular pulse model and long normal autoregressive model. They used second harmonic Fourier series to represent the seasonal variation of the parameter governing the storm life time. Sanso and Guenni (1999) modeled the serial structure present in rainfall by imposing a serial structure to a normal variate. They used a dynamic linear model on normal variate, using a Fourier representation to allow for the seasonality of the data. Thompson (1999) used complex demodulation of monthly precipitation data to determine temporal changes in the decadal averages of the annual precipitation cycle.

Time series analysis and forecasting has become a major tool in various applications in hydrology and environmental management fields. It is used for building mathematical models to generate synthetic hydrologic records, to forecast hydrologic events, to detect trends and shifts in hydrologic records and to fill in missing data and extend records. Most of the hydrologic systems have both deterministic and stochastic components. Time series analysis is of immense practical use in dealing with forecasting of various hydrological parameters. These data sequences, particularly monthly times series, are widely used in water resources planning and operation studies to assess the reliability of alternative system designs and policies and to understand the variability of future system performances. Objective of present study is to analyze monthly rainfall data of Pantnagar and to identify the deterministic component of the monthly rainfall time series.

2. Materials and method

2.1. *Description of study area*

The daily rainfall data were collected from meteorological observatory, G. B. Pant University of Agriculture and Technology, Pantnagar (Fig. 1) situated in Udham Singh Nagar district of Uttarakhand state. Pantnagar falls in sub-humid and subtropical climatic zone and situated in Tarai belt of Shivalik range of foot hills of Himalayas. Geographically it is located at 79.29° E longitude and 29° N latitude. Generally, monsoon starts in the third week of June and continues upto September. The

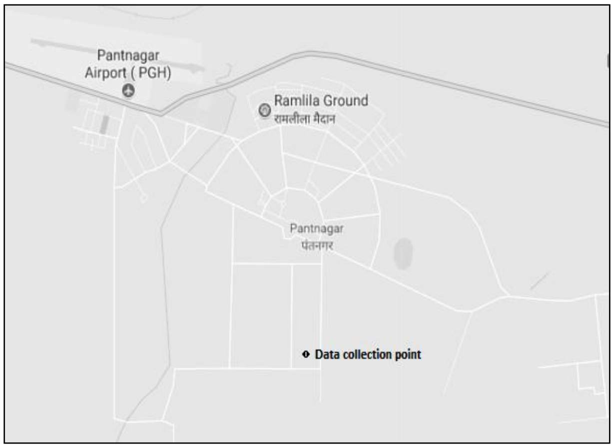

Fig.1. Data collection point at Pantnagar

mean annual rainfall is 1364 mm of which 80-90 per cent occurs during June to September. The daily data were then converted to monthly data by adding the daily rainfall values for the respective months. Monthly rainfall data for the period 1981 to 2010 were considered for estimation of deterministic component.

2.2. *Statistical analysis of time series*

A time series can be divided into the deterministic components which consist of non-periodic and periodic behavior and the stochastic components that consist of chance and chance dependent effects. Mathematically the i^{th} observation on a time series is expressed as:

$$X_i = \Delta_i + E_i \qquad (1)$$

where, Δ_i is deterministic components and E_i is stochastic components. The deterministic components consist of a non-periodic behaviour *i.e.*, trend and a periodic (or cyclical) behaviour. The trend and periodicity could be formulated in a manner in which exact prediction of its value is made. Mathematically, it is expressed as:

$$\Delta_i = T_i + P_i \qquad (2)$$

where, T_i is trend component and P_i is periodic component.

Trend is the steady and regular movement in a time series through which the values are, on an average, either increasing or decreasing. The trend component is approximated either by exponential, polynomial or simple type of models. To check the presence of trend, the tests for randomness were performed on the time series. The following tests were performed to check the randomness of the time series, X_t.

2.3. *Turning point test*

In an observed sequence Xt for t = 1, 2, 3 upto N, a turning point occurs at time $t = i$, if Xt is either greater than X_{i-1} and X_{i+1}, or less than the two adjacent values. The number of turning points p, in a series is expressed as a standard normal variate Z, in the form:

$$Z = \frac{p - \bar{p}}{\sqrt{\mathrm{var}(\bar{p})}} \qquad (3)$$

where, p is the expected number of turning points in a random series. The expected number of turning points p and its variance, *var (p)* are calculated from the following equations:

$$\bar{p} = \frac{2(N-2)}{3} \qquad (4)$$

$$\mathrm{var}\left(\bar{p}\right) = \frac{16N - 29}{90} \qquad (5)$$

where, N is the number of observations. The calculated value of Z was compared with the table value at 5% level of significance to test the presence of trend. If the calculated value of Z is within the limits of ±1.96, the hypothesis of no trend, is accepted.

2.4. *Stationarity*

A time series is called as stationary, if its statistical parameters such as mean, standard deviation etc do not change from one segment to another, or otherwise it is called as non-stationary. Available statistical methods are designed to analyze stationary time series. Therefore, a non-stationary time series must be transformed into a stationary one before these methods are applied. To bring stationarity in the recorded rainfall series Z_t, the square root transformation as suggested by Richardson (1981) was used as:

$$X_t = \sqrt{Z_t} \qquad (6)$$

where, X_t is transformed monthly rainfall series at time t and Z_t is observed monthly rainfall series at time *t.*

2.5. *Mann Kendall's rank correlation test*

This test measures the disarray in the hydrologic data and is based on the proportional number of subsequent observations which exceed a particular value. To conduct this test, the number of times the values of X_j is greater than X_i is determined in all pairs of observation

(X_i, X_j, $j>i$) of the sequence of $X1$, $X2$ upto X_N. The ordered subsets (i, j) can be expressed as (i = 1, j = 2, 3, 4 upto N), (i = 2, j = 3, 4, 5 upto N), ….. (i = N-1, j = N). Number of times X_j is greater than X_i is counted to compute the test statistic. The test statistic was then calculated by the following equation:

$$\tau = \frac{4P}{N(N-1)} - 1 \tag{7}$$

where, P is number of times X_j is greater than X_i. The test statistic has following parameters:

$$E(\tau) = 0 \tag{8}$$

$$\mathrm{var}(\tau) = \frac{2(2N+5)}{9N(N-1)} \tag{9}$$

$$\text{Standard test statistic } Z = \frac{\tau - \bar{\tau}}{\sqrt{\mathrm{var}(\bar{\tau})}} \tag{10}$$

Since the statistic converges rapidly to standard normal distribution as N increases, the hypothesis of no trend in the series was tested by comparing the calculated value of standard test statistic with the tabulated value of standard normal variate at 5% level of significance. If the calculated value of Z is within the region of acceptance, the hypothesis of no trend is accepted. After determination of trend component T_t, it was separated from the series X_t, as expressing in the mathematical form:

$$Y_t = X_t - T_t \tag{11}$$

2.6. *Periodic component*

The periodic effects are short-term fluctuations which occur in a series due to imposition of a cyclic phenomenon and are repetitive over a fixed period of time. The periodic component in a time series can be represented through a system of sine functions after trend component, if it exists, has been estimated and removed. The periodic component in the series Y_t was determined by the method of harmonic analysis. Investigation of periodic component by harmonic analysis requires the identification of base period and computation of significant number of harmonics. The identification of the base period was done by correlogram analysis.

2.7. *Correlogram analysis*

Serial correlogram is a graphical relationship of auto-correlation function r_k with lag k. It measures the degree of linear dependence between the events k time units apart within the time series. The auto-correlation function r_k of the series Y_t were computed using the following relationship given by Kottegoda and Horder (1980):

$$r_k = \frac{[\sum_{t=1}^{N-k}\{(Y_t - \bar{Y})(Y_{t+k} - \bar{Y})\}]}{[\sum_{t=1}^{N}(Y_t - \bar{Y})^2]} \tag{12}$$

where, r_k is auto-correlation function of time series Y_t at lag k, Y_t is rainfall series after removing trend component, Y is mean of time series Y_t, k is lag of time unit and N is total number of discrete values of time series Y_t. The base period in the series was obtained by visual inspection of peaks and troughs in the auto-correlogram.

2.8. *Harmonic analysis*

The Fourier series representation of a trend free time series having a set of variables Y_1, Y_2, Y_3 upto Y_ω, equi-spaced at time interval t, is expressed as:

$$Y_t = \mu + \sum_{j=1}^{m} A_j \,\mathrm{Sin}\left(\frac{2\pi\tau}{\omega} + \theta_j\right) + E_i \tag{13}$$

where, μ is population mean, j/ω is frequency of occurrence of harmonics, j = 1, 2, 3 upto 6, Ei is stochastic component, A is amplitude and θ_j is phase. The periodic component in the mean from the set of observations X_1, X_2, X_3 upto X_n with reference to equation (13) was estimated by the following equation:

$$\mu_t = m_y + \sum_{j=1}^{m}\left(A_j Cos\frac{2\pi j\tau}{\omega} + B_j Sin\frac{2\pi j\tau}{\omega}\right) \tag{14}$$

where, μ_t is periodic component in mean monthly value, m_y is mean of series Y_t, t is 1, 2 upto N with N number of discrete values of rainfall A_j and B_j are Fourier co-efficient of mean series,τ is 1, 2, 3 upto ω with ω as the base period of the series, j is 1, 2, 3 upto m with m number of significant harmonics, ω is base period (= 12 for monthly rainfall data). The parameters A_j and B_j were computed by the following equations:

$$A_j = \frac{2}{\omega}\sum_{\tau=1}^{\omega} m_\tau \,\mathrm{Cos}\left(\frac{2\pi j\tau}{\omega}\right) \tag{15}$$

$$B_j = \frac{2}{\omega}\sum_{\tau=1}^{\omega} m_\tau \,\mathrm{Sin}\left(\frac{2\pi j\tau}{\omega}\right) \tag{16}$$

where, m_τ is individual monthly mean values.

$$m_\tau = \frac{1}{n}\sum_{p=1}^{n} X_{p,\tau} \tag{17}$$

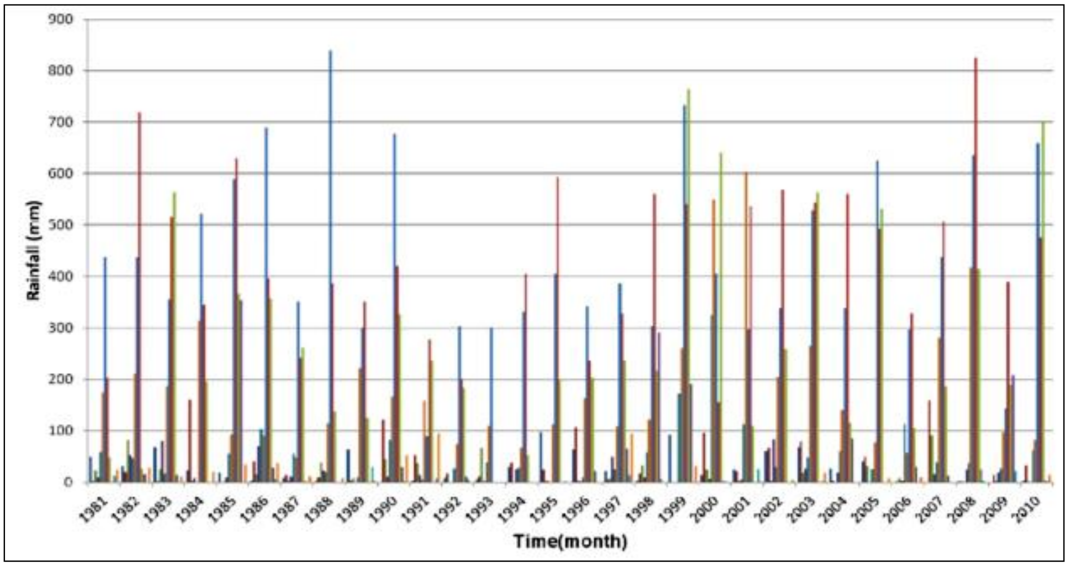

Fig. 2. Measured monthly rainfall at Pantnagar

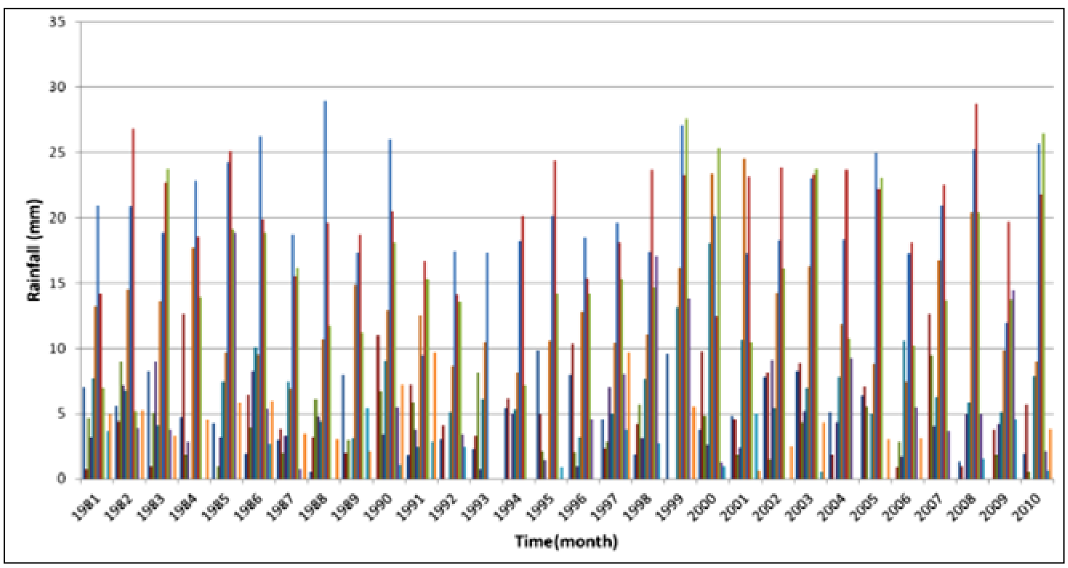

Fig. 3. Transformed monthly rainfall at Pantnagar

The periodic component in the standard deviation was calculated by using the following relationship:

$$\sigma_\tau = S_y + \sum_{j=1}^{m}\left(A'_j Cos\frac{2\pi j\tau}{\omega} + B'_j Sin\frac{2\pi j\tau}{\omega}\right) \quad (18)$$

where, σ_τ is evaluated standard deviation of monthly value, S_y is standard deviation of series Y_t, t is 1, 2, 3 upto N with N number of discrete values of rainfall, A'_j & B'_j are Fourier co-efficient of standard deviation series, τ is 1, 2, 3 upto ω with ω as base period of the series, j is 1, 2, 3 upto m with m number of significant harmonics and ω is period (= 12 for monthly rainfall data). The parameters $A'j$ and $B'j$ were estimated by following equation:

$$A'_j = \frac{2}{\omega}\sum_{\tau=1}^{\omega} S_\tau \operatorname{Cos}\left(\frac{2\pi j\tau}{\omega}\right) \quad (19)$$

$$B'_j = \frac{2}{\omega}\sum_{\tau=1}^{\omega} S_\tau \operatorname{Sin}\left(\frac{2\pi j\tau}{\omega}\right) \quad (20)$$

$$S_\tau = \left\{\frac{1}{n}\sum_{\omega=1}^{n}(Y_{p,\tau} - m_\tau)^2\right\}^{\frac{1}{2}} \quad (21)$$

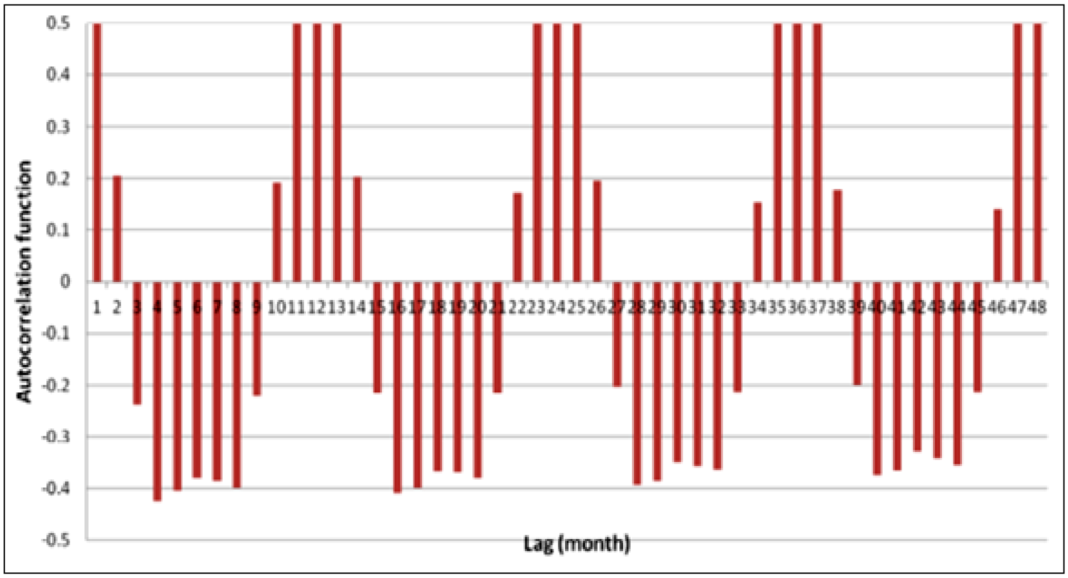

Fig. 4. Autocorrelogram for series

where, S_τ is standard deviation of individual rainfall values and n = numbers of years of rainfall records. The number of significant harmonics was required to compute periodic component in the mean and standard deviation. A limited number of harmonics are significant to explain themajor part of variance of a periodic parameter, it is not necessary to estimate all *ω/2* harmonics for even and odd value of *ω*.

In general, it is found that first six harmonics of a periodic parameter for time series of any interval less than or equal to 30 days are significant and should be tested for significance (Mutreja, 1986). The number of significant harmonics to be fitted in monthly rainfall series after removal of trend was found through the analysis of variance (ANOVA) test.

The null hypothesis is that the variance explained by the harmonic j, $\frac{N}{2}\left(A_j^2 + B_j^2\right)$ is equal to zero. Therefore, A_j and B_j value for j = 6, 5, 4, 3, 2, 1 (in order) were tested to obtain the *F-ratio*. The computed value of *F* was compared with the tabulated value of *F* to test the null hypothesis.

The periodic mean and standard deviation were calculated after determination of significant number of harmonics. The periodicity of the time series was removed from the trend free time series by the process of standardization with the value of m_τ and S_τ by using the following equation :

$$Y'_{p,\tau} = \frac{Y_{p,\tau} - \mu_\tau}{\sigma_\tau} \quad (22)$$

Since in equation (22) the variable $Y'_{p,\tau}$ is only an approximately standardized variable, its mean Y' and standard deviation S'_y were determined to compute the standardized stochastic component. The standardized stochastic components were computed by the following equation (Mutreja, 1986) :

$$Y"_{p,\tau} = \frac{Y'_{p,\tau} - \overline{Y'}}{S'_y} = E_i \quad (23)$$

where, E_i is the stochastic component of the series. This standardization makes the expectation of the mean and variance equal to zero and one respectively at any time *t*, thus approximating stationary condition. The series E_i was subjected to checks for stationary behaviour and model identification.

3. Results and discussion

The measured monthly rainfall data were plotted against time as shown in Fig. 2. In the measured rainfall data series, the rainfall is zero in some months: therefore to make the series continuous, the zero data were substituted by a value of 0.001 mm. An addition of such a low value is not likely to affect the analysis significantly, but makes the series continuous. The historical data series of monthly rainfall was found to be non-stationary. To bring stationarity in the

TABLE 1

Fourier coefficients of periodic mean series

Harmonic No.	Value of A_j	Value of B_j	Total
1	-1.93	-3.33	-5.26
2	-0.11	0.45	0.34
3	-0.86	0.87	0.01
4	1.34	0.98	2.32
5	0.07	-0.032	0.038
6	0.78	-0.0012	0.7788
Total	- 0.71	- 1.0632	-1.7732

TABLE 2

Fourier coefficients of periodic standard deviation series

Harmonic No.	Value of A'_j	Value of B'_j	Total
1	-0.221	-1.13	-1.351
2	-0.0013	-0.26	-0.2613
3	-0.123	0.178	-0.055
4	0.135	-0.023	0.112
5	0.198	-0.056	0.142
6	0.35	-0.0012	0.3498
Total	0.3377	-1.2655	-0.9278

TABLE 3

Analysis of variance test for fitting of actual number of harmonics

S. No.	Variance	Sum of squares	Degree of freedom	F- ratio
1	A_j, B_j $j = 5,6$	$\sum_{j=5}^{6} \frac{N}{2}(A_j^2 + B_j^2) = 0.527$	2	00.0459
2	A_j, B_j $J = 1,2,3,4$	$\sum_{j=1}^{4} \frac{N}{2}(A_j^2 + B_j^2) = 3782.5$	5	46.8166
3	Residuals	2018.0	352	-
4	Total	5801.027	359	-

TABLE 4

Estimated values of periodic monthly mean and standard deviation

Month, τ	Mean, μ_τ	Standard deviation, σ_τ
1	4.304578	3.072223
2	5.071245	3.69657
3	3.429174	2.771299
4	3.529251	2.666728
5	6.590717	3.63827
6	12.90787	4.423713
7	20.44223	4.379653
8	19.8996	5.417976
9	15.36668	6.503724
10	4.202289	5.446996
11	1.289795	1.734964
12	2.93304	2.934446

recorded monthly rainfall series, the series was transformed by square root transformation using Equation (1). The transformed monthly rainfall data were plotted against the time as shown in Fig. 3.

3.1. *Modeling of deterministic components*

3.1.1. *Trend component*

The plot of transformed monthly rainfall values shown in Fig. 2 reveals that there is no trend present in the series X_t. However, the presence of no trend was confirmed by the following tests:

3.1.2. *Turning point test*

The number of turning points p observed in the annual data series was found to be 17. The expected number of turning points p and the variance are 18.66 and 5.011 mm^2, respectively. The value of test statistic Z computed by equation (2.4) was found to be -0.33127 and the table value of Z at 5% level of significance is ± 1.96. Since the calculated value is within the limit of ± 1.96, the null hypothesis of no trend was accepted at 5% level of significance.

3.2. *Mann Kendall's rank correlation test*

The value of Mann Kendall's rank correlation test statistic Z was computed by equation (2.7) and its value

TABLE 5

Mean and standard deviation of stochastic series

Month, τ	Mean, μ_τ	Standard deviation, σ_τ
1	-1.034	0.789
2	-0.838	0.950
3	-1.259	0.712
4	-1.234	0.685
5	-0.447	0.935
6	1.176	1.137
7	3.114	1.125
8	2.974	1.392
9	1.808	1.671
10	-1.061	1.400
11	-1.809	0.446
12	-1.387	0.754

were found to be 1.08. The calculated value was found to be within the limits of ±1.96, therefore the null hypothesis of no trend in series was accepted at the 5% level of significance. The tests revealed the absence of trend component; therefore, the long-term value of trend component was taken to be zero.

3.2.1. *Periodic component*

Since, there was no trend component present in the new series X_t, the series X_t was expressed as series Y_t. The periodic component in the series was investigated by the method of harmonic analysis. Base period and significant number of harmonics are two essential tools which are required to be estimated to determine the periodic component for the present analysis. Base period in the data series Y_t was identified through correlogram analysis.

3.2.2. *Correlogram analysis*

For the development of autocorrelogram, the autocorrelation function r_1 of the series Y_t for lag 1 to 180 were computed by Equation (12). Box and Jenkins (1976) suggested that the maximum of the total number of observations. The autocorrelogram of series Y_t was developed by plotting autocorrelation function r_1 with lag up to 48 and are shown in Fig. 4. The autocorrelogram of the series Y_t showed that autocorrelation functions are significantly different from zero, which meant that all the monthly values are mutually dependent.

It was observed from the autocorrelogram shown in Fig. 4 that the narrow peak and broad troughs combining together makes one cycle which completes in a period of 12 months. Kottegoda and Horder (1980) stated that the autocorrelogram of any time series with a cyclic component will itself be periodic. Therefore, the series Y_t is characterized as periodic with a periodicity of 12 months. The similar results were also reported by Asfawa *et al*. (2018) and Dabral *et al*. (2016).

3.3. *Determination of number of harmonics*

The periodic mean μ_τ and the periodic standard deviation σ_τ were determined by Equation (14) and Equation (18), respectively. The coefficients of Fourier series of periodic mean and periodic standard deviation A_j, B_j, A'_j and B'_j were computed by using Equations 15 to 20. The computed values of coefficients A_j, B_j, A'_j and B'_j upto six harmonics are given in Table 1 and Table 2, respectively. The actual number of harmonics to be fitted in the series Y_t were determined through an ANOVA test which is given in Table 3. It is observed from the table of F-distribution, that F (5, 359) is equal to 1.988 at 5% level of significance. It implies that four harmonics are considered significant for determination of periodic component while other harmonics are not significant and therefore, can be ignored for further analysis.

After determination of significant harmonics, month wise periodic mean and periodic standard deviations for twelve months were computed by Equations 14 and 18, respectively, as given in Table 4. The periodicity of the series X_t was then removed by Equation 22. After removal of the periodicity, the components thus obtained were standardized with the help of Equation 23 as suggested by Srikanthan and McMohan (1983). The series produced after standardization was termed as a stochastic component series S_t. The mean and standard deviation of the series S_t were computed month wise to check the stationary. The computed values of mean and standard deviation do not change with time, *i.e*., the series properties are independent of the time. The mean and standard deviation of stochastic series are given in Table 5, which indicates that the mean values are around zero (0.00024) and the standard deviation values are around one (0.9998). Therefore, the series S_t was considered as stationary series.

Finally, the mathematical model of the time series was obtained by substituting the values of trend and periodic components in Equation 2 which is expressed as:

$$Z_t = (\mu_\tau - 0.3312\, \sigma_\tau + 1.08\, \sigma_\tau E_i)^2 \tag{24}$$

After applying the value if stochastic component of the time series, the above mathematic model can be used for generation of synthetic data and short term forecasting.

4. Conclusions

The objective of this study was to analyze deterministic component of the monthly rainfall time series of Pantnagar. The deterministic component consisted of trend and periodic components. No trend in rainfall data has been found for Pantnagar. Also, using Mann Kendall's rank correlation test statistic, no trend component has been observed. Thus, from above both the tests, the trend component for the observed annual data series was found to be non-significant. The autocorrelogram of the series showed that autocorrelation functions are significantly different from zero, which meant that all the monthly values are mutually dependent and the narrow peak and broad troughs combining together makes one cycle which completes in a period of 12 months. Therefore, the series Y_t is characterized as periodic with a periodicity of 12 months. The mathematical model for the time series in terms of deterministic components of monthly rainfall for Pantnagar station was found as: $Z_t = (\mu_\tau - 0.3312\ \sigma_\tau + 1.08\ \sigma_\tau\ E_i)^2$. The deterministic component of monthly rainfall values can help the planners and farmers to schedule irrigation ahead of sowing. The deterministic component also facilitate the advanced plan of water demand in urban areas. Moreover, the quantity of drainage water can also be properly managed through the aforesaid component of rainfall series.

References

Adamowaski, K. and Smith, A. F., 1972, " Stochastic generation of rainfall", *Journal of Hydraulic Division, Proceedings ASCE*, **98**, 11, 1935-1945.

Asfawa, A., Simane, B., Hassen, A. and Bantider, A., 2018, "Variability and time series trend analysis of rainfall and temperature in northcentral Ethiopia: A case study in Woleka sub-basin", *Weather and Climate Extremes*, **19**, 1, 29-41

Benalaya, A., Torchde, F., Sebei, H., Wheather, H. and Kirby, C., 1998, "Characterization and stochastic modelling of monthly time series", *Hydrology in changing environment*, volume I, Proceedings of the British Hydrological Society International Conference, Exeter, U. K., July 15-18, 1988, 53-61.

Bo, Z., Islam, S. and Eltahir, E. A. B., 1994, " Aggregation disaggregation properties of stochastic rainfall model", *Water Resources Research*, **30**, 12, 3423-3435.

Box, G. E. and Jenkins, G. M., 1976, "Time series analysis : forecasting and control", revised ed. Holden-Day.

Chau, K. W. and Wu, C. L., 2010, "A hybrid model coupled with singular spectrum analysis for daily rainfall prediction", *Journal of Hydro Informatics*, **12**, 4, 458-473.

Chiew, F. H. S., Stewardson, M. J. and McMohan, T. A., 1993, "Comparison of six rainfall-runoff modelling approaches", *Journal of Hydrology*, **147**, 1, 36.

Dabral, P. P., Saring, T. and Jhajharia, D., 2016, "Time series models of monthly rainfall and temperature to detect climate change for Jorhat (Assam), India", *Global Nest Journal*, **18**, 3, 494-507.

Gordin, I. V., Nechaev, A. P. and Vinitskaya, A. A., 1987, "Stochastic models of rainfall for planning construction to regulate surface runoff", *Soviet Meteorology and Hydrology*, **8**, 70-75.

Gyasi, Y. and Agyei, Y., 1999, "Identification of regional parameter of a stochastic model for disaggregation", *Journal of Hydrology*, **223**, 3-4, 148-163.

Kim, D., Olivera, F., Cho, H. and Socolofsky, S. A., 2013, "Regionalization of the Modified Bartlett-Lewis Rectangular Pulse Stochastic Rainfall Model", *Terrestrial, Atmospheric & Oceanic Sciences*, **24**, p3.

Kossieris, P., Makropoulos, C., Onof, C. and Koutsoyiannis, D., 2018, "A rainfall disaggregation scheme for sub-hourly time scales: Coupling a Bartlett-Lewis based model with adjusting procedures", *Journal of Hydrology*, **556**, 980-992

Kottegoda, N. T. and Horder, M. A., 1980, "Daily flow model based on rainfall occurrences using pulses and transfer function", *Journal of Hydrology*, **47**, 215-234.

Matlas, N. C. 1967, "Time Series analysis", *Water Resources Research*, **3**, 817-829.

Mutreja, K. N., 1986, "Applied Hydrology", Tata McGraw-Hill, New Delhi

Narayana, I. R., 1982, "Stochastic modelling of monthly rainfall", *Journal of Hydrology*, **57,** 357-387.

Parthasarthy, B. and Dhar, O. N., 1976, "A study of trends and periodicities in the seasonal and annual rainfalls in India", *Indian Journal of Met. Hydrological Geophysics*, **27**, 1, 23-28.

Piazza, Di. A., Conti, F. L., Noto, L. V., Viola, F. and La Loggia, G., 2011, "Comparative analysis of different techniques for spatial interpolation of rainfall data to create a serially complete monthly time series of precipitation for Sicily, Italy", *International Journal of Applied Earth Observation and Geoinformation*, **13**, 3, 396-408.

Quimpo, R. G., 1968, "Autocorrelation and spectral analysis in hydrology", *Journal of Hydraulic Division Proceedings ASCE*, **94**, 2, 363-373.

Raudkivi, A. J. and Lawgun, N., 1972, "Generation of serially correlated non-normally distributed rainfall durations", *Water Resource Research*, **8,** 2, 398-409.

Richardson, C. W., 1981, "Stochastic simulation of daily precipitation, temperature and solar radiation", *Water Resources Research*, **17**, 1, 182-190.

Sang, Y. F., Wang, Z. and Liu, C., 2012, "Period identification in hydrologic time series using empirical mode decomposition and maximum entropy spectral analysis", *Journal of Hydrology*, **424**, 154-164.

Sanso, B. and Guenni, L., 1999, "Stochastic model for tropical rainfall at a single location", *Journal of Hydrology*, **214**, 1, 4, 64-73.

Sen, Z., 1979, "Effect of periodic parameters on the autocorrelation structure of hydraulic series", *Water Resources Research*, **15**, 6, 1639-1642.

Simmons, A. J., Willett, K. M., Jones, P. D., Thorne, P. W. and Dee, D. P., 2010, "Low-frequency variations in surface atmospheric humidity, temperature and precipitation: Inferences from re-analyses and monthly gridded observational data sets", *Journal of Geophysical Research: Atmospheres*, **115**, D1.

Shahane, F. N., Thomas, D. and Bock, P., 1976, "Correlation analysis of hydrometeorological data", *Journal of Hydraulic Division, Proceeding ASCE*, **102**, 10, 1561-1572.

Srikanthan, R. and McMohan, T. A., 1983, " Stochastic simulation of daily rainfall for Australian Stations", *Transactions of ASAE*, **26**, 3, 754-759.

Tao, P. C. and Delleur, J. W., 1976, "Multistation, multilinear synthesis of hydrologic time series of disaggregation", *Water Resources Research*, **12**, 6, 1303-1312.

Thompson, R., 1999, "A time series analysis of the changing seasonality of precipitation in the British Isles and neighbouring areas", *Journal of Hydrology*, **224**, 169-183.

Vose, R. S., Applequist, S., Squires, M., Durre, I., Menne, M. J., Williams Jr, C. N. and Arndt, D., 2014, "Improved historical temperature and precipitation time series for US climate divisions", *Journal of Applied Meteorology and Climatology*, **53**, 5, 1232-1251.

Yevjevich, V. and Dyer, T. G. J., 1983, "Basic structure of daily precipitations series", *Journal of Hydrology,* **64,** 49-67.