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On the calculations of vertical motion

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ABSTRACT. An expression has been obtained for calculating the altitude dependence of vertical motion on the basis of the equation of mass continuity and the thermodynamic equation. The amount of precipitation is calculated using the proposed method and two other methods and is then compared with the actual amount of precipitations.

1. Introduction

The ordered vertical motions are one of the main factors affecting the variations of the vertical profiles of air temperature and humidity, the evolution of the *As*, *Ns* cloud system, and the precipitations. Such motions take place over a significant atmospheric depths and vast territorics, which are comparable with the sizes of cyclones and anticyclones. In the lower troposphere the velocities of this motion do not exceed 7 cm/sec (Yrolova 1979).

Various researchers have computed vertical motion field using different techniques such as adiabatic and Bellamy method. Busbhy (1952) computed vertical motion field using Sutcliff's development theory. In general, these methods differ from each other in physical approximations used and numerical solutions. Therefore, some form of comparison between the methods should be found.

In the present work, consideration is given to another method for calculating the vertical velocity and, after that, the proposed method is compared with two other known methods.

2. The essence of the method (proposed)

The equation of mass continuity may be written as :

$$\frac{d\rho}{dt} + \rho \operatorname{div.} \mathbf{V} = 0 \tag{1}$$

The thermodynamic equation is of the form :

$$\frac{dT}{dt} - \frac{AR}{c_p}\,\frac{T}{P}\,\frac{dP}{dt} = \frac{Q}{c_p} \tag{2}$$

where, ρ is the density of air; T is the temperature of air; R is universal gas constant; c_p is the specific heat at a constant pressure; P is the atmospheric pressure; Q is the diabatic factors.

It can readily be obtained from the equation of state that :

$$\frac{d\rho}{dt} = \rho\left(\frac{1}{P}\,\frac{dP}{dt} - \frac{1}{T}\,\frac{dT}{dt}\right) \tag{3}$$

Eqns. (1)-(3) give the following expression of the total variation in pressure (τ) :

$$\tau = P\,\frac{c_p}{c_v}\left(\frac{Q}{c_p T} - \operatorname{div.} V\right) \tag{4}$$

To high degree of accuracy correlation between the individual variations in pressure (τ) and the vertical velocity (ω) can be written as :

$$\tau = -\rho\, g\, \omega \tag{5}$$

The total divergence can be expressed as :

$$\operatorname{div.} \mathbf{V} = D + \frac{\partial \omega}{\partial P}$$

where, D is the horizontal divergence.

After substituting (5) in (4) the vertical velocity at a given pressure P will be expressed as :

$$w_p = -\frac{1}{e^k}\left[\int_{P_o}^{P} A\,e^k\,dP - w_o\right] \tag{6}$$

$$\text{where, } k = \int_{P_o}^{P} x\,dP$$

$$x = -\frac{g}{RT}\frac{c_v}{c_p};\ A = D - \frac{Q}{c_p T}$$ (in case of adiabatic flow, A stands for horizontal divergence); ω_0 is the vertical velocity on the surface.

In case of humid air, the term x is expressed as :

$$x = \frac{-g\propto}{RT}\frac{c_v}{c_p},$$

$$A = D - \frac{Q\alpha}{c_p T}$$

where, $\alpha = (1+.605\frac{m}{1+m})^{-1}$ (Here, m is the water vapour mixing ratio).

At $T_d < T$ and $q < q^*$ $\quad m = \frac{dq}{dt} = 0$

and at

$T_d = T$ and $q = q^*$ $\quad m = -\frac{dq}{dt}$

Here, $q = .622\frac{e}{P-e}$, where, q is the specific humidity; e is the pressure of water vapour, q^* specific humidity (saturation case).

The non-adiabatic effects are estimated using the formulae :

$$Q_1 = \frac{c_p}{\rho}\left(\frac{\partial}{\partial z}K\frac{\partial T}{\partial z}\right) \qquad (7)$$

$$Q_2 = -.622\,L\frac{\rho}{P}\left(\frac{de^*}{dT}\frac{dT}{dt} - \frac{e^*}{P}\frac{dP}{dt}\right) \qquad (8)$$

where, Q_1 and Q_2 are the turbulent and phase transition respectively, and K is the turbulent coefficient in cm²/sec. e^* is the pressure of water vapour (Saturation case).

3. The scheme of comparison

The proposed method will be compared with two well-known methods for calculating the vertical velocity.

TABLE 1

Isobaric level	Coefficient a_1	a_2	a_3
850	.014	.048	.87
700	.025	.032	.69
500	.047	.029	.55
300	.086	.026	.23
200	.14	.024	.14
100	.19	.028	.07

* means the saturation case.

Method 1 — This method is based on solving the quasigeostrophic omega Eqn. (3) :

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial P^2}\right)w = \frac{f_0}{\sigma}\frac{\partial}{\partial P}\left[V_g.\nabla\left(\frac{1}{f_0}\nabla^2\phi\right) + f + \frac{1}{\sigma}\nabla^2\left(V_g.\nabla\left(-\frac{\partial\phi}{\partial P}\right)\right)\right] \qquad (9)$$

where, σ is the static stability determined from the relation, the coriolis parameter f is determined as $f=f_0+\beta y$. where, β is the Rossby parameter $= \partial f/\partial y$.

The choice of the horizontal grids is limited in order to apply this methods at one point (Hellwan). Thus the used model is 10-level model and the horizontal domain with dimensions (6×6) with grid distance 390 km.

The necessary input data is the geopotential heights at the different isobaric levels.

The values of ω are calculated from Eqn. (9) up to a 100 mb level using the Liebmann relaxation method. The relaxation coefficient for different isobaric surfaces were chosen as in (Abdel Wahab 1977).

Method 2 — This method is based on the values of a geostrophic wind components to be substituted in the expression :

$$w = -\int_0^z\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right)dz$$

where,

$$u' = -\frac{g}{f^2}\left[\frac{\partial^2 H}{\partial x\partial t} + \frac{g}{f}\left(H, \frac{\partial H}{\partial x}\right)\right]$$

$$v' = -\frac{g}{f^2}\left[\frac{\partial^2 H}{\partial y\partial t} + \frac{g}{f}\left(H, \frac{\partial H}{\partial y}\right)\right]$$

By the aid of this expression, Yrolova (1958) derived that the vertical velocity may be expressed as :

$$\tau = a_1\triangle(T,H) + a_2[(T,\triangle H) + (H,\triangle T)] - a_3\triangle P_0$$

where,

H is the height of isobaric surface.

T is the relative thickness between 1000/500 M.b.

a_1, a_2 and a_3 are coefficients depending on the height of isobaric surface.

$a_1 = 3.7\times10^{-2}\ (H,P_0)$, $a_2 = 3.7\times10^{2}\ (H,T)$,

$a_3 = 0.589(H_p - H_{p0})/P_0$

$$\triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

and P_0 is the surface pressure.

The values of the coefficients used in Eqn. (10) for a case 13 December 1962, are given in Table 1.

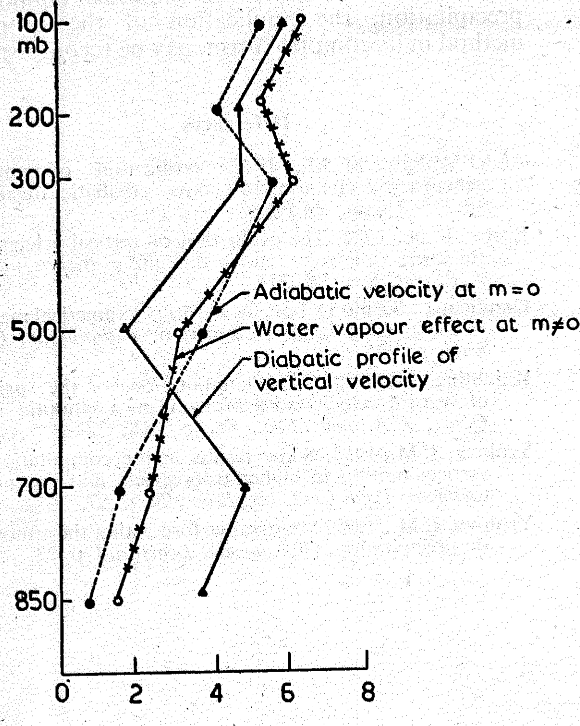

Fig. 1. Vertical velocity over Hellwan at 0000 UT on 13 December 1962

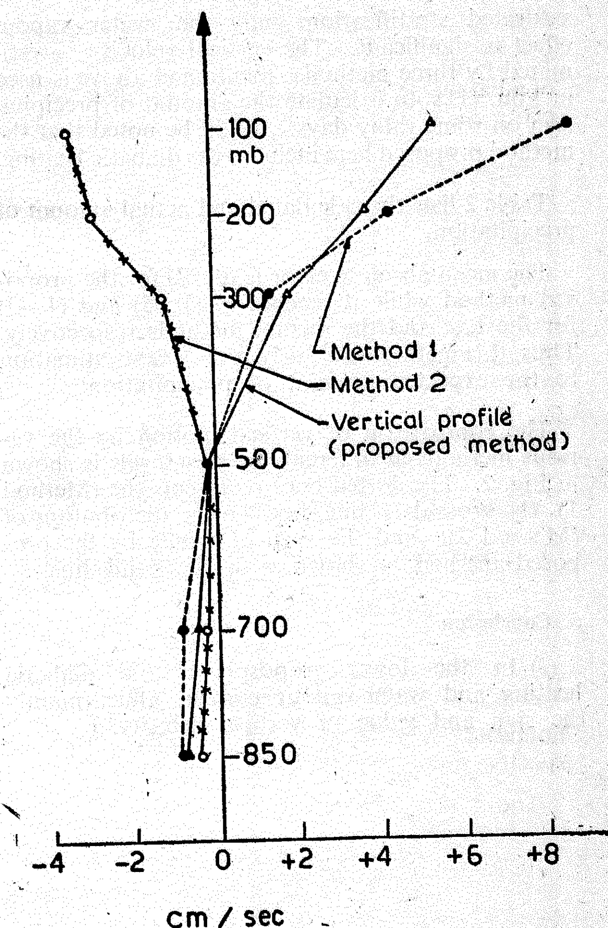

Fig. 2. Altitude dependence of vertical motion over Hellwan by different methods.

TABLE 2

Amount of precipitation over Hellwan (mm)

Date	Method			Actual amount
	1	2	Proposed	
7 Jan 1962	6.2	6.5	7.4	7.6
28 Jan 1962	12.8	13.1	14.8	14.2
13 Dec 1962	11.3	12.7	10.3	9.8
3 Jan 1964	10.5	10.0	9.8	10.7
18 Dec 1967	15.5	18.9	17.7	18.2
21 Jan 1968	9.7	10.4	13.1	12.4

Let the three methods be applied to calculating the amount of precipitation. Use will be made of the formula (6) :

$$I = -\frac{P_0}{g}\int_{\xi_1}^{\xi_2} F^*(\xi, T)\, w\, d\xi \qquad (11)$$

where, ξ_1, ξ_2 are the dimensionless coordinates of pressure $\xi_2 > \xi_1$

$$F^*(\xi,T) = \frac{-1+\frac{x-1}{x}\,T\left(\frac{1}{\propto^*}-\frac{1}{T}\right)}{\frac{L}{c_p}\left(\frac{1}{\propto^*}-\frac{1}{T}\right)+\frac{1}{q^*}}\cdot\frac{1}{\xi}$$

Here, L is latent heat of condensation $\propto^*$ is the specific air volume at saturation, q^* is specific air humidity at saturation.

4. Results and discussion

The vertical velocity is calculated on isobaric surface up to 100 mb and for various synoptic conditions. Fig. 1 shows the vertical velocity over Hellwan at 0000 UT on 13 December 1962. The dotted line represents the adiabatic velocity, *i.e.*, the term A in section 2 expresses the horizontal divergence only. The water vapour effect at $m \neq 0$ is shown with the cross-dash line. The diabatic profile of vertical velocity is shown with the solid line. It can clearly be seen that the diabatic heating affects mainly the vertical motion, especially in the lower troposphere. In case of saturated stratification, only the water vapour effect is significant. The vertical velocity ω estimated by three methods mentioned above is used in Eqn. (11) to calculate the amount of precipitation on some rainy days. It will be noted that the method proposed here includes the diabatic heating.

Table 2 lists the calculated and actual amount of precipitation.

The mean absolute error is (0.57) for the proposed method while it reaches to (1.65) and (1.43) for the first and the second methods respectively. Thus, this method can fairly gives a best estimation to the expected amount of precipitation.

The comparison of vertical motion by the various methods at different pressure levels is shown in Fig. 2. The dotted lines represents the (Method 1) the cross-dash line describe the distribution of (Method 2), and the vertical profile by the proposed method is shown with the solid line.

5. Conclusion

(*i*) In the lower troposphere, the diabatic heating and water-vapour content affect mainly the sign and value of vertical velocity.

(*ii*) At the levels above 500 mb, the method based on actual wind observations (the method proposed here) gives a better description of vertical velocity as compared with methods based on observations of geopotential and temperature.

(*iii*) When calculated the expected amount of precipitation, the application of the proposed method in its complete form may be recommended.

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