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A study of cloud spirals of tropical cyclones

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ABSTRACT. From examination of a number of tropical cyclones in the vicinity of Miami, Sen and Hiser (1957) found that the cloud spirals of tropical cyclones, in general, fit with equiangular spirals. We have also found the same characteristic to be valid in case of tropical cyclones of Indian seas. Moreover, the average trajectories and streamlines drawn by various workers have also been found to closely approximate with equiangular spirals.

In this paper, the general axially symmetric wind field with streamlines as equiangular spirals has been obtained and it has been shown that the ratio of divergence to vorticity equals to the cotangent of the angle of spirals in such cases. Using this result, it has been shown that the rate of generation of latent heat within a tropical cyclone is proportional to the cotangent of the angle of spirals, and certain well known satellite and radar characteristics have been established.

1. Introduction

Analyses of cyclonic storms over tropical oceans suffer from lack of availability of data. An analysis over Indian seas is extremely difficult and subjective because of scanty or complete absence of observations from the Arabian Sea and Bay of Bengal. As the satellite and radar cloud imageries represent, to some extent, the tangible manifestations of many of the dynamical and physical processes taking place within a tropical cyclone, the pictures have become a potential tool for analysis.

The satellite and radar imageries have been widely used for estimating location and intensity of tropical cyclones. Sikka (1971) studied the location and intensity of tropical cyclones obtained from satellite pictures and found that the location and intensity determined from satellite picture agrees well with those derived from conventional synoptic analysis. From examination of a number of hurricanes in the vicinity of Miami, Sen and Hiser verified that the characteristic bands of precipitation are, in general, equiangular spiral in shape. They found that 300-400 miles ahead of the system of spiral bands, there usually exist narrow well defined lines of echoes which move in the direction of the hurricane. Based on their work, the method of determination of location of the cyclonic storm centres by fitting equiangular spirals to the spiral bands of the radar imageries of tropical cyclones have widely been used. The accuracy of this method is also comparable to that of conventional synoptic analysis.

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Various empirical studies have been made to obtain estimates of intensity, maximum wind etc and to evaluate their forecasts on the basis of satellite pictures. Fett (1966) and Fritz *et al.* (1966) categorized the tropical cyclones according to the degree of organisation of cloud systems and their horizontal extent. The maximum wind speed was estimated by fitting regression against the diameter of the cloud mass. Dvorak (1973, 1975) approached the problem in a more systematic manner and categorised various tropical disturbances potentially capable of intensifying into a hurricane stage on the basis of two-way classification based on two characteristics what he described as central features and outer banding features. The estimates of current intensity and future developments were again obtained (by drawing) out statistical correlations.

The main difficulty encountered in the empirical studies is that these do not provide any information on the dynamical parameters of the wind field. Moreover, the maximum wind etc obtained from these studies are true for 'average storms' of a particular locality and differ significantly from observations in individual cases. Smith (1972) derived estimates of divergence, vorticity and radial and tangential winds based on determination of the wind field from calculations of cloud motions at different cloud levels. No direct method exists for estimating these parameters from the cloud pictures.

2. Basic assumptions

In this paper an attempt has been made to establish the basic observed characteristics of cloud spiral. No effort has been made to evaluate the deterministic potential and practical applicability of the model. We have assumed a simplified model based on following assumptions:

(a) The cyclone is axially symmetric and stationary.

(b) The treamlines are equiangular spirals.

The first of these assumptions needs no special consideration as it was made for simplifying the mathematical formulations, though some observations have indicated marked asymmetry in tropical storms.

The cloud imageries do not exhibit spirals inside the Radius of Maximum Winds (RMW) and it is not possible to obtain the streamline patterns from cloud pictures. As the RMW range from 20 to 75 km, not much of attention need be devoted to the streamline pattern inside RMW for synoptic purposes. Outside the radius of maximum wind, clouds are well arranged along streamlines. We have verified with the help of radar and satellite pictures of following cyclones that the cloud spirals fit with equiangular spirals: (*i*) Coxbazar cyclone (3 May 1971), (*ii*) Arabian Sea cyclone (Nov 1971), (*iii*) Bangladesh cyclone (1972), (*iv*) Kakinada cyclone (1969) and Bay cyclone (1974). Moreover, with the help of average trajectory drawn by Hughes (1952), we found that the trajectory is equiangular spirals with angle $\cot^{-1}(0.05)$. The departure from the equiangular spirals may be attributed to the errors of averaging different storms with different intensity, subjective analysis and the inherent asymmetry of the tropical storms.

3. Divergence and vorticity

Under the assumption of axial symmetry, the horizontal wind vector may be represented as vector function of radius r and vertical coordinate z only.

$$\mathbf{V} = \mathbf{V}(r, z) \tag{1}$$

Therefore, resolving the vector $\mathbf{V}(r, z)$ along and perpendicular to $\mathbf{r}$

$$\mathbf{V} = \left[\mu(r, z) \frac{\mathbf{r}}{r} + \lambda(r, z) \frac{\mathbf{k} \times \mathbf{r}}{r} \right] \tag{2}$$

where λ and μ are scalar functions of r and z, and $\mathbf{k}$ is unit vertical vector. The corresponding equation for the eastward and northward components of velocity are :

$$u\mathbf{i} + v\mathbf{j} = \left[\mathbf{i} \left(\frac{x\mu(r, z)}{r} - \frac{y\lambda(r, z)}{r} \right) + \mathbf{j} \left(\frac{y\mu(r, z)}{r} + \frac{x\lambda(r, z)}{r} \right) \right] \tag{3}$$

where u, v are the eastward and northward velocity components, $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the eastward and northward directions and x, y are the coordinates measured along latitude and longitude circles. The differential equation for the family of steamlines of this wind field is, therefore,

$$\frac{dy}{dx} = \frac{v}{u} = \frac{y\mu(r, z) + x\lambda(r, z)}{x\mu(r, z) - y\lambda(r, z)} \tag{4}$$

Comparing this with the differential equation of the family of equiangular spirals r=const. exp $(-a\theta)$,

$$\frac{dy}{dx} = \frac{ay - x}{ax + y} \tag{5}$$

We find $$\mu(r, z) = -a\lambda(r, z) \tag{6}$$

Therefore, we can rewrite Eqn. (2) as :

$$\mathbf{V}(r, z) = \lambda(r, z)\left[\mathbf{k} \times \left(\frac{\mathbf{r}}{r}\right) - a\left(\frac{\mathbf{r}}{r}\right)\right] \tag{7}$$

$$V_r(r, z) = -a V_\theta(r, z) \tag{8}$$

where V_r and V_θ denote the radial and tangential components of $\mathbf{V}(r, z)$. Substituting this in the expression for divergence and vorticity in cylindrical coordinates :

$$D(r, z) = \frac{1}{r}\frac{\partial\, r V_r}{\partial r} = -\frac{a}{r}\frac{\partial}{\partial r}\frac{r V_\theta}{r}$$

$$= -a\,\zeta(r, z) \tag{9}$$

where $\zeta(r, z)$ is the vorticity and $D(r, z)$ the divergence. This equation shows that the divergence and vorticity are proportional to each other.

The observations indicate that the cloud spirals tend to be more and more circular with the growth of the cyclone. Therefore, the value of the constant a decreases with the growth of the storm. Eqns. (8) and (9) thus embody the well-known fact that the divergence and radial velocity decrease with the growth of the storm.

4. Latent heat considerations

To understand the physical significance of the parameter a of organisation of cloud spirals with reference to growth of the storm, we consider heating by cumulus convection inside the storm. Following Charney and Eliassen (1964) we shall parameterize the turbulent transport process of the cumulus convection field by means of a single parameter m expressing the mean degree saturation of air in the region of active convection.

$$\bar{q}(r, z) = m(z)\,\bar{q}_s(r, z) \tag{10}$$

where $\bar{q}$ and $\bar{q}_s$ are the mean specific humidity and saturation specific humidity respectively. Then the rate of convergence of moisture within a vertical column extending through the atmosphere based on a unit horizontal cross-section may be written as (Charney & Eliassen 1964) :

$$I = -\int_0^\infty \frac{\bar{m}}{r}\frac{\partial}{\partial r}\left(\bar{\rho} r\, \bar{V}_r\, \bar{q}_s\right) dz + \left(\bar{m}\,\bar{\rho}\,\bar{w}\,\bar{q}_s\right)_0 \tag{11}$$

where $\bar{\rho}$ is the mean density of air, $\bar{m}$ is the average value of m, $\bar{V}_r$ and $\bar{w}$ are the mean radial and vertical velocities, and the subscript "0" denotes a value at $z = 0$, the top of the friction layer. Integrating the equation of continuity for air in steady state :

$$\frac{\partial\,\bar{\rho}\,\bar{w}}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\bar{\rho}\, r\, \bar{V}_r(r, z) = 0 \tag{12}$$

We may write the second term of (11) as :

$$(\bar{m}\,\bar{q}_s)_0\,(\bar{\rho}\bar{w})_0 = (\bar{m}\bar{q}_s)_0 \int_0^\infty \frac{1}{r}\frac{\partial}{\partial r}(r\bar{\rho}\,\bar{V}_r)\, dz \tag{13}$$

Substituting (13) into (11) we have :

$$I = \int_0^\infty \frac{1}{r}\frac{\partial}{\partial r}\left[\left(\bar{m}\,\bar{\rho}\,\bar{q}_s\right)_0 - \bar{m}\,\bar{\rho}\,\bar{q}_s\right] r\,\bar{V}_r\, dz \tag{14}$$

Integrating this equation over the radius of satellite clouding R and changing the order of integration and using (8) we have the total release of latent heat as :

$$H = \int_0^R L\, I\, 2\pi r\, dr = 2\pi\, a\, R\, L \int_0^\infty \left[\left(\bar{m}\,\bar{\rho}\,\bar{q}_s\right)_0 - \bar{m}\,\bar{\rho}\,\bar{q}_s\right]_{r=R} \bar{V}_\theta(R, z)\, dz \tag{15}$$

where, L is the latent heat of evaporation.

Eqn. (15) shows that the rate of generation of latent heat within the storm is proportional to the constant of the spiral a. Therefore, in mature stage, where H is very small, a approaches zero and the spiral tend to be circular.

References

Charney, J.G. and Eliassen, A., 1964, *J. atmos. Sci.*, **21**, pp. 68-75.

Dvorak, V.F., 1973, NOAA Tech. Memo. NESS, 45, US Dep. of Commerce, NESS, Wash., D.C.

Dvorak, V.F., 1975, *U.S. Month. Weath. Rev.*, **103**, 5, pp. 420-430.

Fett, R., 1966, *U.S. Month. Weath. Rev.*, **94**, 1, pp. 9-18.

Firtz, S., Hubert, L. and Timchalk, P., 1966, *U.S. Month. Weath. Rev.*, **94**, 4, pp. 231-236.

Hughes, L.A., 1952,, *J. Met.* **9**, p. 422.

Senn, H.V., and Hiser, H.W., 1957, *Proc. Sixth Weath Radar Conf.*, Boston, Am. met. Soc., pp. 165-170.

Sikka, D.R., 1971, *Indian J. Met. Geophys.*, **22**, pp. 305-312.

Smith, C.L., 1972, NOAA Tech. Memo. ERL, NHRL-100