

# Estimates of effective long wave radiation from the Bay of Bengal

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**ABSTRACT.** A number of empirical expressions have been used to estimate the effective outgoing long wave radiation from a sea surface. We have used the observations recorded during the Indo-USSR Monsoon Experiment of 1977 to test the efficiency of these expressions. We find that an expression derived by Girduk *et al.* (1973) tends to provide a better fit with the actinometric observations than the other empirical expressions currently in use.

## 1. Introduction

The effective outgoing long wave radiation ( $E$ ) is made up of two components—(i) the net long wave radiation directed upwards from the sea surface ( $F^\uparrow$ ) and (ii) the downward counter radiation ( $F^\downarrow$ ) from the cloud base. Usually,  $F^\uparrow$  exceeds  $F^\downarrow$  but they are of the same order of magnitude; consequently,  $E$  is the small difference of two terms of opposite sign.

The outgoing radiation is a function of the sea surface temperature, and the emissivity of the sea surface, while the counter radiation depends on the air temperature, the liquid water content of the atmosphere and its cloud cover.

The purpose of this paper is to compare the computations of  $E$ , using semi-empirical expressions, with actual observations during the Indo-USSR expedition of 1977. We believe this could lead to improvements in numerical models of the monsoon.

## 2. Symbols

The following symbols, which are not defined in the text, have been used :

Temperature of the sea surface  $T_w$  (°K)

Temperature of the overlying air	$T_a$ (°K)
Liquid water content	$w$
Cloud cover in tenths	$N$
Sea surface emissivity	$\delta$ (0.91)
Stefan-Boltzman constant	$\sigma$ ( $567 \times 10^{-7}$ watts. $m^{-2} \text{ } ^\circ K^{-4}$ )
Specific humidity of the air	$q$ (g/kg)
Water vapour pressure at 10 m above the sea surface	$e$ (mb)
Relative humidity of the overlying air	$r$ (%)

## 3. Empirical expressions for effective long wave radiation

A number of empirical expressions have been derived from time to time for the effective long wave radiation ( $E$ ). Two of the more recent ones are due to Hastenrath and Lamb (1979) and Girduk *et al.* (1973). Hastenrath and Lamb express  $E$  by,

$$E = \delta \sigma T_w^4 (0.39 - 0.05 q^{\frac{1}{2}}) \times 1.053 N^2 + 4.0 \delta \sigma T_w^3 (T_w - T_a) \quad (3.1)$$

The first term in (3.1) is the outgoing radiation modified by the emissivity of the sea surface, the moisture content of the atmosphere and the cloud cover. The second term is a correction for the temperature of the air overlying the sea surface.

TABLE 1

Mean value of the effective longwave radiation ( $\text{ly. min}^{-1}$ )  
(Values in parenthesis are only for clear skies)

Cloud amount (okta)	No. of observations	Empirical formulae for calculation of effective longwave radiation							Observed ( $E$ )
		Hasten-rath and Lamb (3.1) (1979)	Girduk <i>et al.</i> (3.2) (1973)	Swinbank (3.5) (1963)	Geiger (3.6) (1961)	Angstrom (3.7) (1916)	Brunt (3.8) (1932)	McDoland (3.9) (1957)	
8	187	0.0540	0.0312	0.0223 (0.0995)	0.0409 (0.1156)	0.0409 (0.1157)	-0.0032 (0.0773)	0.0398 (0.0887)	0.0428
7	113	0.0631	0.0455	0.0360 (0.0950)	0.0555 (0.1125)	0.0555 (0.1125)	0.0131 (0.0745)	0.0309 (0.0904)	0.0461
6	74	0.0739	0.0604	0.0511 (0.0944)	0.0702 (0.1121)	0.0703 (0.1121)	0.0295 (0.0745)	0.0474 (0.0912)	0.0515
5	47	0.0848	0.0774	0.0656 (0.0957)	0.0839 (0.1130)	0.0839 (0.1130)	0.0449 (0.0761)	0.0609 (0.0912)	0.0579
4	36	0.0914	0.0841	0.0751 (0.0943)	0.0935 (0.1120)	0.0935 (0.1121)	0.0552 (0.0752)	0.0723 (0.0917)	0.0587
3	51	0.0958	0.0917	0.0827 (0.0935)	0.1011 (0.1115)	0.1011 (0.1116)	0.0624 (0.0737)	0.0802 (0.0911)	0.0604
2	40	0.1003	0.0977	0.890 (0.0938)	0.1069 (0.1115)	0.1069 (0.1116)	0.0689 (0.0739)	0.0862 (0.0910)	0.0595
1	8	0.1010	0.1036	0.0915 (0.0927)	0.1099 (0.1111)	0.1010 (0.1111)	0.0709 (0.0721)	0.0894 (0.0906)	0.0613
Total	556	0.0718	0.0571	0.0480 (0.0961)	0.0668 (0.1133)	0.0668 (0.1133)	0.0254 (0.0754)	0.0414 (0.0902)	0.0500

TABLE 2

Standard deviation of the effective longwave radiation ( $\text{ly. min}^{-1}$ )  
(Values in parenthesis are only for clear skies)

Cloud amount (okta)	Hasten-rath and Lamb (1979)	Empirical formulae for calculation of effective longwave radiation						Observed ( $E$ )
		Girduk <i>et al.</i> (1973)	Swinbank (1963)	Geiger (1961)	Angstrom (1916)	Brunt (1932)	McDoland (1957)	
8	0.0100	0.0077	0.0102 (0.0106)	0.0060 (0.0069)	0.0060 (0.0069)	0.0104 (0.108)	0.0046 (0.0029)	0.0138
7	0.0077	0.0054	0.0072 (0.0074)	0.0043 (0.0048)	0.0044 (0.0048)	0.0084 (0.0085)	0.0029 (0.0024)	0.0149
6	0.0050	0.0031	0.0041 (0.0042)	0.0026 (0.0028)	0.0026 (0.0028)	0.0053 (0.0053)	0.0018 (0.0017)	0.0152
5	0.0061	0.0041	0.0053 (0.0054)	0.0034 (0.0036)	0.0034 (0.0036)	0.0060 (0.0060)	0.0021 (0.0020)	0.0154
4	0.0036	0.0021	0.0026 (0.0027)	0.0018 (0.0018)	0.0018 (0.0018)	0.0037 (0.0037)	0.0022 (0.0021)	0.0168
3	0.0040	0.0022	0.0028 (0.0028)	0.0019 (0.0020)	0.0019 (0.0020)	0.0040 (0.0040)	0.0021 (0.0020)	0.0082
2	0.0052	0.0034	0.0043 (0.0043)	0.0029 (0.0029)	0.0029 (0.0029)	0.0049 (0.0049)	0.0016 (0.0015)	0.0121
1	0.0027	0.0009	0.0013 (0.0013)	0.0008 (0.0008)	0.0008 (0.0008)	0.0027 (0.0027)	0.0018 (0.0018)	0.0084
Total	0.0252	0.0188	0.0257 (0.0078)	0.0248 (0.0051)	0.0248 (0.0052)	0.0279 (0.0081)	0.0284 (0.0026)	0.0155

The details have been discussed by Budyko (1958) and Wyrski (1966). Hastenrath and Lamb used this expression to compute  $E$  over the Indian Ocean.

Girduk *et al.* (1973), on the other hand, used a different formulation for the impact of clouds. Their expression is

$$E = \delta\sigma T_w^4 - \delta [1.63 (T_a^4)^{\frac{1}{2}} - 0.775] \times (1 + KN^2) \quad (3.2)$$

In (3.2),  $K$  is the ratio of (i) the difference between the long wave counter radiation for an overcast and clear sky to (ii) the value corresponding to a clear sky. We have

$$K = (F_o \downarrow - F_c \downarrow) \div F_c \downarrow \quad (3.3)$$

where,  $F_o \downarrow$ ,  $F_c \downarrow$  stand for the downward radiation for an overcast and clear sky respectively.

$F_o \downarrow$  and  $F_c \downarrow$  are

$$F_c \downarrow = 1.63 (T_a^4)^{\frac{1}{2}} - 0.775 \quad (3.4a)$$

$$F_o \downarrow = 1.48 (T_a^4)^{\frac{1}{2}} - 0.569 \quad (3.4b)$$

The numerical constants in (3.2) and (3.4) were based on actinometric observations over a sea surface. These constants were validated against GATE data by Egorov (1976).

Swinbank (1963) had yet another expression for  $E$ , namely,

$$E = \delta\sigma T_w^4 - 9.35 \times 10^{-6} (\delta\sigma) T_a^6 \quad (3.5)$$

The first term of (3.5) is the upward radiation emitted by the sea surface, while the second term is an expression for the downward emittance from clouds. The dependence on the sixth power of air temperature ( $T_a$ ) arises from the dependence of water vapour pressure ( $e$ ) on air temperature ( $T_a$ ).

An expression in which the downward emittance was proportional to  $(T_a)^4$ , instead of  $(T_a)^6$  as in (3.5), was developed by Geiger (1961). He finds :

$$E = \delta\sigma T_w^4 - \delta\sigma T_a^4 [0.82 - 0.25 \exp(-0.216e)] \quad (3.6)$$

This is only slightly different from Angstrom's (1916) estimate of :

$$E = \delta\sigma T_w^4 - \delta\sigma T_a^4 [0.82 - 0.326 \exp(-0.216e)] \quad (3.7)$$

Finally, we have from Brunt (1932) :

$$E = \delta\sigma T_w^4 - \delta\sigma T_a^4 (0.66 + 0.039 \sqrt{e}) \quad (3.8)$$

As is well known, Brunt's equation predicts a close correlation between the intensity of counter radiation and the square root of water vapour pressure. Several workers have found wide variations in the constants used in the second term of (3.8), but in this study we used Brunt's value of the numerical constants.

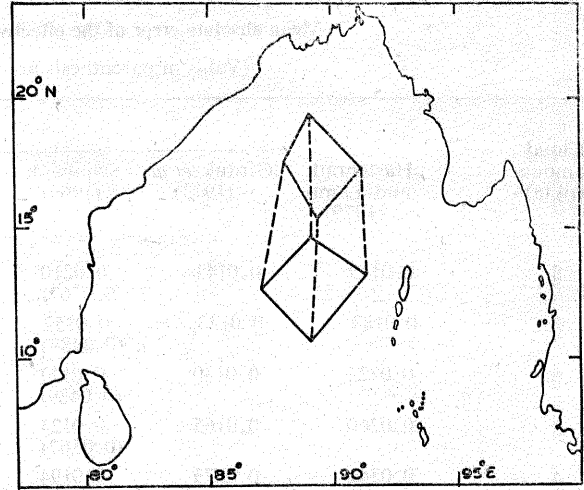


Fig. 1. Location of Russian research vessels over the Bay of Bengal during Monsoon-1977

There is yet another relation due to McDonald (1957) in which he obtained :

$$E = (0.165 - 0.769 \cdot 10^{-3} r) \quad (3.9)$$

This relation was obtained by computing the infrared flux by an Elsasser radiation chart, and the mean monthly soundings for January and July over the United States.

Eqns. (3.1) and (3.2) take account of cloud cover in the estimation of  $E$ , while the remaining equations are for clear skies.

In reality, the sky is almost always cloudy, especially, over the Indian seas during the southwest monsoon. Consequently, the influence of cloud cover on atmospheric long wave radiation is important. For a cloudy sky, the empirical relations for a clear sky should be modified to include the effect of cloudiness. A suitable modification was suggested by Bolz and is referred to by Geiger (1965) and Morgan *et al.* (1971). We have :

$$F \downarrow = F_c \downarrow (1 + KN^2)$$

where  $K$  is a constant to be determined from field

observations, and  $F_c \downarrow$  stands for the downward radiation for a clear sky. For the present study  $K$  is taken to be of the same value as that of Girduk *et al.* (1973).

#### 4. Results

Hourly actinometric observations were recorded by four USSR research ships over the Bay of Bengal (Fig. 1) during the Monsoon 1977 experiment for the period 8-19 August 1977. As the day time observations were not reliable, we used only night time data in the present study. A total of 556 observations were recorded.

In addition to actinometric observations, hourly observations were made of air and sea surface temperature, total cloud cover, relative humidity and specific humidity.

TABLE 3

Mean absolute error of the effective longwave radiation ( $\text{ly.min}^{-1}$ )  
(Values in parenthesis are only for clear skies)

Cloud amount (okta)	Hastenrath and Lamb (1979)	Empirical relation					
		Girduk <i>et al.</i> (1973)	Swinbank (1963)	Geiger (1916)	Angstrom (1916)	Brunt (1932)	Mc Doland (1957)
8	0.0149	0.0143	0.0210 (0.0567)	0.0109 (0.0729)	0.0109 (0.0729)	0.0460 (0.0346)	0.0331 (0.0459)
7	0.0183	0.0123	0.0152 (0.0489)	0.0135 (0.0664)	0.0135 (0.0664)	0.0332 (0.0285)	0.0182 (0.0443)
6	0.0224	0.0130	0.0127 (0.0429)	0.0191 (0.0606)	0.0192 (0.0606)	0.0238 (0.0230)	0.0129 (0.0395)
5	0.0260	0.0165	0.0123 (0.0367)	0.0250 (0.0540)	0.0250 (0.0541)	0.0208 (0.0172)	0.0100 (0.0323)
4	0.0341	0.0273	0.0194 (0.0370)	0.0363 (0.0543)	0.0363 (0.0544)	0.0126 (0.0174)	0.0173 (0.0339)
3	0.0354	0.0313	0.0223 (0.0331)	0.0407 (0.0511)	0.0407 (0.0512)	0.0054 (0.0133)	0.0198 (0.0307)
2	0.0418	0.0392	0.0305 (0.0353)	0.0484 (0.0530)	0.0484 (0.0531)	0.0114 (0.0154)	0.0277 (0.0325)
1	0.0404	0.0393	0.0302 (0.0314)	0.0487 (0.0498)	0.0487 (0.0499)	0.0096 (0.0109)	0.0281 (0.0293)
Total	0.0234	0.0189	0.0191 (0.0461)	0.0219 (0.0633)	0.0219 (0.0633)	0.0292 (0.0256)	0.0229 (0.0402)

TABLE 4

Root mean square error of the effecting longwave radiation ( $\text{ly.min}^{-1}$ )  
(Values in parenthesis are only for clear skies)

Cloud amount (okta)	Hastenrath and Lamb (1979)	Empirical formulae					
		Girduk <i>et al.</i> (1973)	Swinbank (1963)	Geiger (1961)	Angstrom (1916)	Brunt (1932)	Mc Doland (1957)
8	0.0179	0.0178	0.0250 (0.0585)	0.0137 (0.0741)	0.0137 (0.0742)	0.0479 (0.0372)	0.0361 (0.0479)
7	0.0227	0.0149	0.0183 (0.0512)	0.0175 (0.0680)	0.0176 (0.0681)	0.0363 (0.0322)	0.0212 (0.0467)
6	0.0271	0.0176	0.0164 (0.0456)	0.0241 (0.0625)	0.0241 (0.0625)	0.0267 (0.0276)	0.0163 (0.0421)
5	0.0312	0.0226	0.0181 (0.0404)	0.0298 (0.0564)	0.0298 (0.0564)	0.0224 (0.0245)	0.0167 (0.0359)
4	0.0372	0.0310	0.0237 (0.0400)	0.0393 (0.0567)	0.0394 (0.0568)	0.0161 (0.0236)	0.0218 (0.0376)
3	0.0390	0.0322	0.0236 (0.0340)	0.0414 (0.0517)	0.0415 (0.0518)	0.0071 (0.0149)	0.0212 (0.0316)
2	0.0436	0.0410	0.0328 (0.0373)	0.0498 (0.0543)	0.0399 (0.0544)	0.0160 (0.0196)	0.4302 (0.0347)
1	0.0404	0.0400	0.0310 (0.0322)	0.0493 (0.0504)	0.0493 (0.0505)	0.0117 (0.0128)	0.0291 (0.0303)
Total	0.0281	0.0239	0.0247 (0.0491)	0.0281 (0.0653)	0.0281 (0.0653)	0.0347 (0.0301)	0.0272 (0.0428)

The influence of cloud cover on  $E$  is of considerable interest. Considering this aspect, the data were divided into eight groups in terms of the total cloud cover (in oktas).

In Table 1 we indicate the observed values of  $E$  for different cloud amounts, and for comparison we also indicate the values of  $E$  computed by the seven empirical relations Eqns. (3.1) to (3.9). The values in parenthesis are those applicable for clear skies. We note that the observed values of  $E$  increase as the cloud cover decreases. This is to be expected because the counter radiation decreases with lesser amounts of cloud.

Further, the mean values of  $E$ , computed with suitable adjustment in Eqns. (3.5) to (3.9) to include cloudiness, decrease considerably and approach the corresponding observed values. But, it is found that for an overcast sky (3.5) and (3.9) give an underestimated value of  $E$ . Eqn. (3.8) yields a negative value of  $E$  which is unusual for this period.

In Table 2 we have indicated the standard deviation of the effective long wave radiation both by empirical relations, and also as observed by USSR research vessels. The interesting fact emerges that the standard deviation shows wide variations with different amounts of cloud cover. One cannot discern a systematic variation in standard deviation by categorising the data in terms of cloud cover.

We have defined a mean absolute error by

$$\epsilon = \frac{1}{n} \sum_{i=1}^n \left| \left( E_{\text{observed}} - E_{\text{computed}} \right) \right| \quad (4.1)$$

The mean absolute error ( $\epsilon$ ) in terms of the deviation of the effective computed long wave radiation and the observed values shows considerable decrease by the inclusion of cloudiness in (3.5)-(3.9). A few exceptions were observed for Eqn. (3.8) of Brunt, with a cloud cover of more than 4 oktas.

Without stratification of data according to cloudiness, (3.2) by Girduk *et al.* provides the least value of  $\epsilon$ . But, the stratified data do not reveal the merit of any single empirical relation over the entire cloud spectrum. In general, (3.2) gives better results with a cloud cover of more than 5 okta, but (3.5) and (3.9) yield minimum absolute errors for 5 and 6 okta of cloud cover. Brunt's formula gives minimum error when the cloudiness does not exceed 4 okta.

The root mean square error of the effective long wave radiation computed by empirical relations is shown in Table 4.

We observe that the computations by Geiger and Angstrom show the smallest r.m.s. errors

for overcast skies, but as the cloud amount decreases these relations do not give satisfactory results.

As in Table 3, the r.m.s. error of  $E$  for the entire data set without stratification, is minimum for the formula suggested by Girduk *et al.* (1973).

Similar to the mean absolute error, the r.m.s. error was considerably less for the formula by Girduk *et al.* when cloud exceeded 5 okta. The effective longwave radiation by McDolond's formula (3.9) shows least r.m.s. error with a cloudiness 5-6 oktas, while Brunt's formula gives minimum values of r.m.s. error for a cloud cover of less than 4 okta.

The statistics presented in Tables 1-4 confirm the improvement in estimates of effective outgoing longwave radiation from a sea surface, when adjustments for cloudiness are made in Eqns. (3.5)-(3.9).

## 5. Conclusions

The estimation of  $E$  by the different empirical relations, using the Indo-USSR Monsoon-77 data lead us to the following conclusions :

- (i) During the southwest monsoon more than 75 per cent of the observations were with 5 okta or more of cloud. This emphasizes the importance of cloud cover on the effective longwave radiation.
- (ii) There is merit in applying the correction of Bolz to clear sky values of the estimated counter radiation from the atmosphere ( $F^{\downarrow}$ ).
- (iii) Difference of air and water temperature over the Bay of Bengal during monsoon period is small, and does not exceed 1.0 deg. C, while the relative humidity remains almost constant at an approximate value of 85 per cent. Therefore, it is desirable to stratify the data only according to cloud cover. This brings out the influence of cloudiness on effective longwave radiation.
- (iv) For overcast sky, or a cloud cover exceeding 5 okta, the semi-empirical relation (3.2) by Girduk *et al.* (1973) may be considered to be most suitable. But, for lesser cloud cover not exceeding 4 okta, Brunt's formula may be recommended for estimating  $E$ .

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