

# Passive microwave atmospheric measurements : Part II- Absorption of microwaves by cloud and rain

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(Received 15 December 1979)

**ABSTRACT.** The paper describes a method for the computation of microwave absorption coefficients in the atmosphere due to rain and cloud. The absorption coefficients due to cloud at 19.35 GHz for different liquid water content of the cloud at different temperatures are presented. In the case of rain the dominant droplet size have been assumed to be small so as to neglect the effect of scattering. The absorption coefficient at the same frequency due to rain has been calculated for a Marshal-Palmer drop size distribution and empirical relationship between rainfall parameter and total water content of the rain mass.

## 1. Introduction

In a previous paper (Pandey *et al.* 1979) the absorption due to water vapour and oxygen molecule has been described for 19.35 GHz and 22.235 GHz frequencies. In this paper, we describe the microwave absorption due to cloud and rain for 19.35 GHz. Since the distribution of liquid water content in the cloud will be different for different types of clouds, the results of the absorption calculation has been presented for different temperatures of the liquid hydrometeors (-10, 0, 10, 20 and 30 °C). The absorption coefficients due to rain has been calculated based on the results of Mie parameters given by Paris (1971) and using a Marshal-Palmer distribution.

## 2. Theoretical approach

The expression for the extinction coefficient due to hydrometeors is given by (Wilheit *et al.* 1977)

$$Y_{\text{ext}} = \int_0^{\infty} N(a) Q_t da \quad (1)$$

where  $N(a)$  is the number density per unit size interval of droplets of radius  $a$  and  $Q_t$  is the extinction cross-section for a drop of diameter  $a$  in  $m^2$ . We assume that the droplets of clouds and rain have maximum diameters of .0001 and

.0060 metre respectively (Paris 1971). Hence Eqn. (1) can be written as

$$\begin{aligned} Y_{\text{ext}} &= \int_0^{a_{\text{max}}} N(a) Q_t da \\ &= \int_0^{0.0001} N(a) Q_t da + \int_{.0001}^{a_{\text{max}}} N(a) Q_t da \\ &= Y_{\text{cloud}} + Y_{\text{rain}} \end{aligned} \quad (2)$$

### 2.1. Absorption coefficient due to cloud

Taking first part of Eqn. (2) and applying Rayleigh approximations ( $a/\lambda \ll 1$ )  $Q_s$  becomes negligible compared to  $Q_a$  and we can write,

$$Y_{\text{cloud}} = \alpha_{\text{cloud}} = \int_0^{0.0001} N(a) Q_a da \quad (3)$$

where,

$$Q_a = \frac{\pi^2 a^3}{\lambda} I_m \left[ \frac{1 - m^2}{2 + m^2} \right]$$

and  $m = \text{complex index of refraction.}$

Eqn. (3) can be rewritten as :

$$\alpha_{\text{cloud}} = \frac{6 \pi \nu}{\rho_L C} I_m \left[ \frac{1 - m^2}{2 + m^2} \right] \cdot M_C \quad (4)$$

where,

$$M_C = \int_0^{a_{\text{max}}} \frac{\pi \rho_L}{6} \cdot a^3 N(a) da$$

is the liquid water content of the cloud, in  $\text{gm/m}^3$  and  $\rho_L$  is the density of liquid water in  $\text{gm/m}^3$ .

Eqn. (4) can be further simplified as follows :

$$\alpha_{\text{cloud}} = 0.0629 \cdot M_C \cdot \nu \cdot I_m \left[ \frac{1 - m^2}{2 + m^2} \right] \quad (5)$$

$\alpha_{\text{cloud}}$  in nepere per kilometre (NP/km) and  $\nu$  in GHz.

The complex index of refraction is given by

$$m = m' - jm'' = (E_1 - jE_2)^{1/2}$$

where,  $E_1$  and  $E_2$  are real and imaginary parts of the dielectric constant of water. The final expression for absorption coefficient due to clouds becomes :

$$\alpha_{\text{cloud}} = 0.0629 \left[ \frac{3E_2}{(2 + E_1)^2 + E_2^2} \right] M_C \nu \quad (6)$$

The expressions for  $E_1$  and  $E_2$  are given by :

$$E_1 = \frac{E_S - E_\infty}{1 + (2\pi \nu_H \tau)^2} + E_\infty \quad (7)$$

$$E_2 = \frac{(E_S - E_\infty) 2\pi \nu \tau}{1 + (2\pi \nu_H \tau)^2} + \frac{2\sigma}{\nu} \quad (8)$$

$$\nu_H = \nu \times 10^9$$

where  $\tau$  is the relaxation time in seconds,  $\sigma$  is the ionic conductivity in ESU,  $E_S$  is the static relative dielectric constant and  $E_\infty$  is the relative dielectric constant at very high frequencies. The values of these parameters derived from the results of Hollinger (1973) are :

$$E_S = 88.00 - 4.035 \times 10^{-1} T + 8.065 \times 10^{-4} T^2 \quad (9)$$

$$\tau = (18.70 - 5.489 \times 10^{-1} T + 5.758 \times 10^{-3} T^2) \times 10^{-12} \quad (10)$$

$$\sigma = (-8.570 \times 10^{-15} T + 2.996 \times 10^{-16} T^2) \times 10^{11} \quad (11)$$

$$E_\infty = 4.9 \quad (12)$$

where,  $T$  is the temperature of the hydrometeors in the cloud.

Our computational algorithm is based on Eqns. (6) to (12) and the results are given in Tables 1 and 2.

TABLE 1

Real and imaginary parts of dielectric constants at 19.35 GHz: at different temperatures using regression equation by Hollinger (1973)

Temp. (°C)	$E_1$	$E_2$
-10	13.5593	26.0828
0	18.3615	30.6177
10	25.6649	34.8201
20	35.2137	36.9630
30	44.4897	35.7059

## 2.2. Absorption due to rain

In order to calculate absorption due to rain from Eqn. (2) we shall assume that for rainfall rates (1-20  $\text{mm h}^{-1}$ ), the extinction/attenuation of electromagnetic radiation is caused mainly by absorption and scattering is a perturbation term (Wilheit *et al.* 1977). Hence to a first approximation

$$\alpha_{\text{rain}} = \int_0^{a_{\text{max}}} N(a) Q_a da \quad (13)$$

0.0001

The Marshall-Palmer drop size distribution is given by :

$$N = N_0 \exp(-b \cdot a) \quad (14)$$

where,

$$b = 4100 (R)^{-0.21} \text{ m}^{-1} \quad (15)$$

$$N_0 = 8.0 \times 10^6 \text{ m}^{-4} \quad (16)$$

and  $R$  = rainfall intensity  $\text{mm h}^{-1}$

The maximum drop diameter for a given rainfall intensity is calculated from (Stephen 1962) :

$$a_{\text{max}} = 0.0023 (R)^{0.213} \text{ m} \quad (17)$$

It should be remembered that  $R$ , the rainfall rate is a convenient parameter to specify a drop size distribution for the further calculation of the radiative properties. The actual precipitation rate is controlled by the dynamics of the storm and especially by the up and downdrafts.

The final expression for calculating absorption coefficients due to different rainfall rates becomes:

$$\alpha_{\text{rain}} = 8.0 \times 10^6 \int_0^{a_{\text{max}}} Q_a \exp \left[ -4100 \times (R)^{-0.21} \cdot a \right] da \quad (18)$$

0.0001

where the value of  $Q_a = (Q_i - Q_s)$  is obtained from the table given by Paris (1971) for 19.35 GHz and at different temperatures (0, 10, 20°C). The temperature distribution in the rain enters into the absorption calculation through  $Q_a$  which is a function of complex index of refraction. The results of the computation for different temperature distribution of rain drops are presented in Table 3 for 19.35 GHz.

TABLE 2

Absorption coefficient due to cloud at 19.35 GHz for different liquid water content of the cloud at different temperatures

$M_C$ (gm/m <sup>3</sup> )	Absorption coefficient (NP/km)				
	$T = -10^\circ\text{C}$	$T = 0^\circ\text{C}$	$T = 10^\circ\text{C}$	$T = 20^\circ\text{C}$	$T = 30^\circ\text{C}$
0.1	1.0325 E-02	8.2687 E-03	6.4284 E-03	4.9058 E-03	3.7941 E-03
0.2	2.0650 E-02	1.6537 E-02	1.2857 E-02	9.8116 E-03	7.5883 E-03
0.3	3.0975 E-02	2.4806 E-02	1.9285 E-02	1.4717 E-02	1.1382 E-02
0.4	4.1299 E-02	3.3075 E-02	2.5714 E-02	1.9623 E-02	1.5177 E-02
0.5	5.1624 E-02	4.1344 E-02	3.2142 E-02	2.4529 E-02	1.8971 E-02
0.6	6.1949 E-02	4.9612 E-02	3.8570 E-02	2.9435 E-02	2.2765 E-02
0.7	7.2274 E-02	5.7881 E-02	4.4999 E-02	3.4341 E-02	2.6559 E-02
0.8	8.2599 E-02	6.6150 E-02	5.1427 E-02	3.9246 E-02	3.0353 E-02
0.9	9.2924 E-02	7.4418 E-02	5.7856 E-02	4.4152 E-02	3.4147 E-02
1.0	1.0325 E-01	8.2687 E-02	6.4284 E-02	4.9058 E-02	3.7941 E-02
1.1	1.1357 E-01	9.9224 E-02	7.0712 E-02	5.3964 E-02	4.1736 E-02
1.2	1.2390 E-01	9.0956 E-02	7.7141 E-02	5.8870 E-02	4.5530 E-02
1.3	1.3422 E-01	1.0749 E-01	8.3569 E-02	6.3775 E-02	4.9324 E-02
1.4	1.4455 E-01	1.1576 E-01	8.9998 E-02	6.8681 E-02	5.3118 E-02
1.5	1.5487 E-01	1.2403 E-01	9.6426 E-02	7.3587 E-02	5.6912 E-02
1.6	1.6510 E-01	1.3230 E-01	1.0285 E-01	7.8493 E-02	6.0706 E-02
1.7	1.7552 E-01	1.4057 E-01	1.0928 E-01	8.3399 E-02	6.4501 E-02
1.8	1.8585 E-01	1.4884 E-01	1.1571 E-01	8.8304 E-02	6.8295 E-02
1.9	1.9617 E-01	1.5711 E-01	1.2214 E-01	9.3210 E-02	7.2089 E-02
2.0	2.0649 E-01	1.6537 E-01	1.28507E-01	9.8116 E-02	7.5883 E-02

TABLE 3

Absorption coefficients due to rain at 19.35 GHz

Rainfall rate (mm/hr)	Total water content (gm/m <sup>3</sup> )	Max. drop diameter (m)	Attenuation in NP/km		
			0°C	10°C	20°C
1	0.8890 E-01	0.2306 E-02	0.1380 E-01	0.1349 E-01	0.1352 E-01
2	0.1591 E-00	0.2673 E- 2	0.2850 E-01	0.2875 E-01	0.2969 E-01
3	0.2237 E-00	0.2914 E-02	0.4416 E-01	0.4512 E-01	0.4712 E-01
4	0.2849 E-00	0.3098 E-02	0.5951 E-01	0.6124 E-01	0.6435 E-01
5	0.3436 E-00	0.3249 E-02	0.7572 E-01	0.7821 E-01	0.8242 E-01
6	0.4005 E-00	0.3378 E-02	0.9161 E-01	0.9487 E-01	0.1002 E-00
7	0.4558 E-00	0.3490 E-02	0.1077 E-00	0.1117 E-00	0.1180 E-00
8	0.5099 E-00	0.3591 E-02	0.1239 E-00	0.1286 E-00	0.1360 E-00
9	0.5629 E-00	0.3682 E-02	0.1402 E-00	0.1456 E-00	0.1540 E-00
10	0.6150 E-00	0.3766 E-02	0.1566 E-00	0.1627 E-00	0.1721 E-00
11	0.6663 E-00	0.3843 E-02	0.1731 E-00	0.1798 E-00	0.1901 E-00
12	0.7168 E-00	0.3915 E-02	0.1896 E-00	0.1970 E-00	0.2082 E-00
13	0.7667 E-00	0.3982 E-02	0.2053 E-00	0.2132 E-00	0.2254 E-00
14	0.8159 E-00	0.4045 E-02	0.2219 E-00	0.2304 E-00	0.2434 E-00
15	0.8646 E-00	0.4105 E-02	0.2386 E-00	0.2476 E-00	0.2614 E-00
16	0.0128 E-00	0.4162 E-02	0.2543 E-00	0.2639 E-00	0.2785 E-00
17	0.9605 E-00	0.4216 E-02	0.2710 E-00	0.2811 E-00	0.2965 E-00
18	0.1008 E-01	0.4268 E-02	0.2866 E-00	0.2972 E-00	0.3134 E-00
19	0.1055 E-01	0.4317 E-02	0.3034 E-00	0.3144 E-00	0.3313 E-00
20	0.1101 E-01	0.4365 E-02	0.3190 E-00	0.3305 E-00	0.3482 E-00

### 3. Results and discussion

Results presented in Table 2 shows that at 19.35 GHz, the absorption coefficient due to cloud depends upon total liquid water content of cloud as well as temperature distribution in the cloud. However, for rain, the dependence of absorption coefficient due to rain on temperature distribution is weak and depends mainly on the total liquid water content of the rainmass characterised by drops size distribution. These conclusions are in agreement with that of Olsen *et al.* (1978) and Staelin (1966).

It may be mentioned here that while introducing clouds into the atmospheric models, the relative humidity corresponding to the cloud thickness is adjusted 100 per cent and the absorption due to water vapour is calculated accordingly. Similarly for introducing rain into the models, effect of cloud with certain liquid water content distribution is always taken into account. When rain is present its contribution to the total absorption coefficient is more than the contributions from water vapour, oxygen and clouds.

The above studies will be useful while analysing the 19.35 GHz satellite microwave radiometer data from India's second satellite *Bhaskara* to get the information about rainfall rates over oceans.

### Acknowledgement

The authors are grateful to Dr. T.A. Hariharan, Head, Meteorology Division for his constant encouragement and guidance in the present work. Useful comments from Dr. Pranav Desai and Dr. M.S. Narayanan are thankfully acknowledge.

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