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# A model of atmospheric tide with allowance for hourly variation of ozone density

F. B. A. GIWA and M. T. BABALOLA

*Department of Physics,*

*University of Ibadan, Nigeria*

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ABSTRACT. By using a simplified form of the photochemistry in the ozonosphere, the diurnal variation of ozone density is simulated in the atmosphere, and the tide generated from the resulting absorption of solar radiation is calculated.

It is shown that this tide differs significantly from that generated if a constant ozone density is assumed. In particular there is enchancement of the 24 hourly tide and a reduction of the 12 hourly tide.

## 1. Introduction

### (a) Background

According to modern view the world-wide atmospheric oscillation derives most of its energy from the absorption of solar radiation in the ozonosphere. The pioneering work along these lines started with Butler and Small (1963) who were able to account for over half of the semi-diurnal surface pressure amplitude. Other workers, example, Lindzen (1968), Chapman and Lindzen (1970) have done similar work. A peculiarity of all these works is that they neglected the hour to hour variation of ozone density.

There is ample evidence to show that ozone density in fact varies with time of the day. Firstly Dutsch (1962) using the data from Arosa, Khargian and Kouznetsov (1961) using the data from stations in Soviet Union and Khalek (1954) in Afghanistan have all reported significant diurnal ozone variation.

Secondly from theoretical calculations, Leovy (1964) showed that though at low levels the density of ozone is constant throughout the day, there is considerable diurnal variation above 45 km. This diurnal variation results in the existence of a secondary maximum at 70 km shortly before sunrise. After sunrise the secondary maximum disappears gradually.

Also in a similar work Hunt (1965, 1966a, 1966b) showed that a secondary maximum exists around 65 km and that the density of this secondary maximum varies with time of the day.

### (b) Statement of the problem

Now the total solar radiation absorbed at a particular point in the atmosphere can vary as a result of (*i*) the variation of solar zenith angle, (*ii*) the variation of total ozone above the point, and (*iii*) the variation of ozone density at the point. As the sun moves across the sky it causes photochemical action on the ozonosphere resulting in the variation with time of ozone density and hence a modification of the heating which would have been observed had ozone density remained constant. It is thus clear that the neglect of time variation of ozone is a simplification which may lead to a calculated tide different from the true picture. The purpose of this paper is, therefore, to investigate the implication of allowing for the time variation of ozone in atmospheric oscillation. To this end we shall start by discussing the photochemistry of the ozonosphere.

## 2. Photochemistry in the Ozonosphere

### (a) The Photochemical equations

The photochemistry in the ozonosphere involves over forty different reactions (*see* for example, Crutzen 1974). Our interest is not to
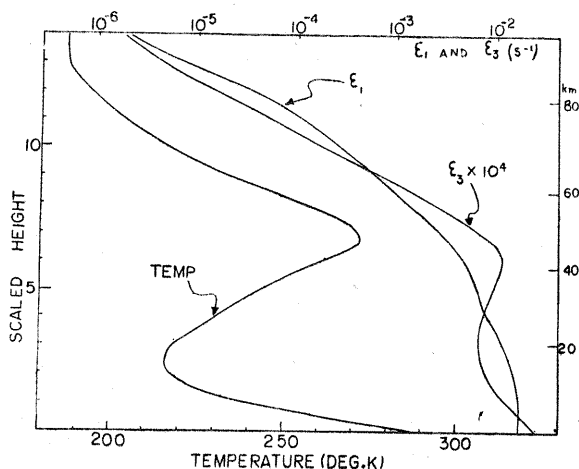
Fig. 1

study the details of these reaction rather *our interest is to investigate the modifications which ozone hourly variation has on the calculated tide.* In fact it would have been ideal if there was sufficient observation on the hour to hour variation of ozone density over the whole globe. Lacking such detailed information, we have to resort to producing a scheme which can simulate what is known so far about ozone spatial variation and hope that the same process will adequately produce the temporal variation.

We shall, therefore, use a set of simplified reactions and hope that by slight modifications of some parameters we can simulate the ozone distribution in the area of the stratosphere where most of the tidal energy is introduced.

An example of such simplified set of reactions can be found in Harwood and Pyle (1977) from which the following equations can readily be deduced.

$$\frac{\partial n_1}{\partial t} = 2J_2 n_2 + J_3 n_3 - k_2 n_1 n_2 n_m - k_4 n_1 n_3 - \epsilon_1 n_1 \quad (1)$$

$$\frac{\partial n_3}{\partial t} = k_2 n_1 n_2 n_m - J_3 n_3 - k_4 n_1 n_3 - \epsilon_3 n_3 \quad (2)$$

where $n_1$, $n_2$, $n_3$ and $n_m$ are respectively the number densities of atomic oxygen, molecular oxygen, ozone, and all the neutral particles; and we have neglected the advection terms normally associated with $\partial/\partial t$ because these terms are much smaller than the local rate of change term. $J_2$, $J_3$ are the number of photons absorbed per unit time per molecules of molecular oxygen and ozone respectively.

$$\epsilon_1 = k_5 [OH] + k_7 [HO_2] + k_9 [NO_2]$$

$$\epsilon_3 = k_8 [NO] \quad (3)$$

where expression of form $[X]$ means number density of molecule $X$.

The values of the reaction constants are taken from Harwood and Pyle (1977) and Crutzen (1974) and are given by

$$k_2 = 1 \cdot 05 \times 10^{-31} \exp(510/T), \quad k_5 = 3 \cdot 8 \times 10^{-11}$$
$$k_4 = 1 \cdot 9 \times 10^{-11} \exp(-2300/T), \quad k_7 = 2 \cdot 0 \times 10^{-11}$$
$$k_8 = 9 \cdot 0 \times 10^{-13} \exp(-1200/T), \quad k_9 = 9 \cdot 1 \times 10^{-12}$$

all in cm molecule sec units and $T$ is temperature in deg. K.

In Eqn. (1), for simplicity we have neglected the photons absorbed by $NO_2$ molecules on the grounds that these are much smaller than $J_2 n_2$ in the region of interest.

In our simplified set of equations we have not taken into account some other processes like eddy diffusion and so it was no surprise when in a preliminary investigation to solve Eqns. (1) and (2), the height of maximum ozone density increased from equator to the poles contrary to observations. The inclusion of these additional terms will make the solution of the equations very complicating and time consuming. We shall, therefore, parameterize the additional processes by modifying $\epsilon_1$ and $\epsilon_3$ in some suitable fashion to make calculation conform with observation. This procedure is similar to Harwood and Pyle (1977) in which the $NO_x$ profile was chosen so as to make values of $n_3$ for equinoxes agree with observation.

From Eqns. (1) and (2), it is easily deduced that a large $\epsilon_1$ and $\epsilon_3$ at high levels will reduce values of $n_3$ at these levels and thus lower the level of maximum ozone density.

After some judicious trial by error it was found that by scaling the equatorial values of $\epsilon_1$ and $\epsilon_3$ by a factor $F$ ($z$, Lat.) where Lat. = latitude, $z$ = altitude in km.

$$F(z, \text{Lat.}) = f(z) \sin^2 (\text{Lat.}) + \cos^2 (\text{Lat.})$$
$$f(z) = 10 \qquad\qquad z \geqslant 32$$
$$f(z) = 0.1 + 0.99\ (z-18) \quad 32 \geqslant z \geqslant 18$$
$$f(z) = 0.1 \qquad\qquad z \leqslant 18 \qquad (4)$$

it was possible to simulate ozone distribution which agreed fairly closely with observation.

Fig. 1 illustrates the values of $\epsilon_1$ and $\epsilon_3$ at the equator as function of scaled height $x$ (defined in Eqn. 9). To determine $\epsilon_1$ and $\epsilon_3$ at other latitudes it is only necessary to multiply by $F(z, \text{Lat.})$.

The number densities for the minor trace substances are essentially those *calculated* by Wofsy and McElroy (1974) with slight modification to ensure that ozone maximum density at the equator is not too small.

Having thus obtained the expression for time variation of $n_1$ and $n_3$ we require to investigate the variation of $J_2$ and $J_3$ as solar radiation passes through the depth of the atmosphere.

(b) *Absorption of solar radiation in the ozonosphere*

Let $J_\lambda\,\delta_\lambda$ be the number of photons per second between wavelengths $\lambda$ and $\lambda + \delta\lambda$ incident on a unit area at level $Z$ in the atmosphere and $J_{\lambda_\infty}\,\delta\lambda$ the corresponding values at the fringe of the atmosphere. If $\alpha_{\lambda 2}$ and $\alpha_{\lambda 3}$ are respectively the molecular coefficients of absorption for oxygen molecules and ozone then Beers law (on the assumption that these two gases are responsible for absorption in the wave band of interest) gives :

$$J_\lambda = J_{\lambda_\infty} \exp\left(-\alpha_{\lambda 2}\,\mu_2 - \alpha_{\lambda 3}\,\mu_3\right) \quad (5)$$

where the total number of oxygen and ozone particles along the path of solar rays of units cross sectional area are given by

$$\mu_2 = \int_z^\infty n_2 \sec\phi\,dz, \quad \mu_3 = \int_z^\infty n_3 \sec\phi\,dz \quad (6)$$

$\phi$ is the solar zenith angle.

In the calculation of $\mu_2$ and $\mu_3$ we shall for simplicity neglect the sphericity of the earth. This approximation is justifiable on the grounds that away from the poles the modification due to the sphericity of the earth is significant only at large zenith angle, *i.e.*, at sunrise and sunset during which period very little absorption of solar energy takes place and consequently what happens at these large zenith angles contributes little to the motion of interest. The extra complication of allowing for the sphericity of the earth is, therefore, a game which is not worth the candle.

The number of photons absorbed per unit time by an elementary depth $\delta z$ of air per unit area between wavelengths $\lambda$ and $\lambda + \delta\lambda$ is $\dfrac{\partial J_\lambda}{\partial z}\,\delta\lambda\,\delta\lambda$. This absorption takes place over a path of geometrical length equal to $\delta z \sec\phi$. Therefore the the total number of photons absorbed per unit volume per unit time over the same wavelength interval is

$$\frac{\partial J_\lambda}{\partial z}\,\delta\lambda\,\delta z/(\delta z \sec\phi)\,,$$

and substituting equations (5) and (6) the expression simplifies to

$$(\alpha_{\lambda_2}\,n_2 + \alpha_{\lambda_3}\,n_3)\,J_\lambda\,\delta\lambda$$

In this expression the part which contains $\alpha_{\lambda_2}$ represents that part of the photons which is absorbed by $O_2$ molecules and the other part is absorbed by ozone. Therefore the expressions for $J_2$ and $J_3$ which occur in equations (1) and (2) are given by :

$$n_2 J_2 = n_2 \int_{\lambda=0}^{\lambda=2423} d\lambda\,\alpha_{\lambda 2}\,J_{\lambda_\infty}\,\exp\left(-\alpha_{\lambda 2}\mu_2 - \alpha_{\lambda 3}\mu_3\right)$$

$$n_3 J_3 = n_3 \int_{\lambda=0}^{\lambda=11000} d\lambda\,\alpha_{\lambda 3}\,J_{\lambda_\infty}\,\exp\left(-\alpha_{\lambda 2}\mu_2 - \alpha_{\lambda 3}\mu_3\right) \quad (7)$$

where the upper limits of the integration in Angstrom unit are the critical wavelengths above which the photons cannot dissociate the particular molecule.

Similar relations exist for the energy $E$ absorbed per unit volume per unit time. In this case we note that where $h'$ and $c'$ are respectively Plancks constant and speed to light, the energy in a photon of wavelength $\lambda$ is $h'c'/\lambda$ and the integration is for all wavelengths. The counterpart of Eqn. (7) is, therefore,

$$E = h'c'\int_{\lambda=0}^{\lambda=\infty} \frac{d\lambda(\alpha_{\lambda 2}n_2 + \alpha_{\lambda 3}n_3)}{\lambda}J_{\lambda_\infty}\,\exp(-\alpha_{\lambda 2}\mu_2 - \alpha_{\lambda 3}\mu_3) \quad (8)$$

When this expression is substituted into the equation describing the vertical variation of the tide we have

$$\frac{d^2 y}{dx^2} + y\left\{\left(\frac{dH}{dx} + \frac{\gamma-1}{\gamma}H\right)\frac{1}{h} - \frac{1}{4}\right\} = \frac{R\,\triangle}{\gamma g h}e^{-\frac{x}{2}} \quad (9)$$

where $x = \int dz/H$

$ye^{x/2} =$ velocity divergence $- \triangle/T$

$T =$ mean temperature at the particular height

$H = RT/g$, $R$ is the atmospheric gas constant

$g =$ acceleration due to gravity

$\gamma = 1.4$

$h =$ Equivalent depth

$e^x =$ Undisturbed surface pressure/undisturbed pressure at a scaled height $x$

and $\rho\,c_p\,\triangle = E$, $\rho =$ undisturbed density and $c_p$ specific heat at constant pressure.

The set of Eqns. (1) to (9) form a complete set. We shall in the next section see how these equations can be solved.

### 3. Solution of the equations

(a) *The calculation of the fluxes*

The integration in Eqns. (7) and (8) are carried out with the whole spectrum within which ozone and molecular oxygen absorb energy, *viz.*, 1200 to 7700A° being divided into ninety intervals, the figure 90 having been chosen such that there is not much change between two successive intervals. The values of the intensity and absorption coefficients are taken from Houghton (1977).
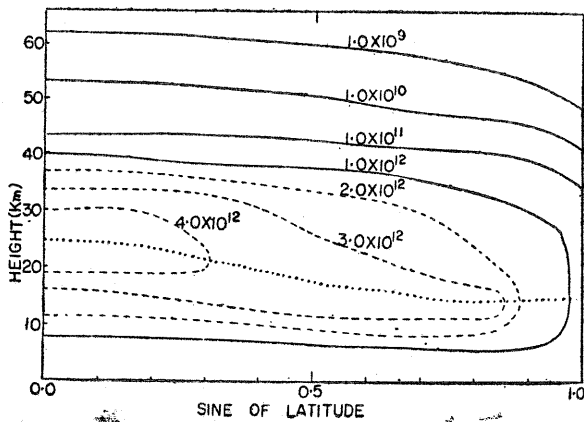
Fig. 2

In evaluating $J_2$, $J_3$ and $E$, it was found that the expression $J_3(u_2, u_3)$ can be separated in terms of $u_2$ and $u_3$ variations to give

$$J_3(u_2, u_3) = J_3(0,0) f_2(u_2) f_3(u_3) \qquad (10)$$

where $J_3(0,0)$ is the value of $J_3$ at the top of the atmosphere,

$$f_2(u_2) = J_3(u_2, 0)/J_3(0,0) \text{ and } f_3'(u_3) = J_3(0, u_3)/J_3'(0,0).$$

This relation is accurate to about 1 in $10^5$ from "top" of the atmosphere to about 5 km.

The quantities $f_2(u_{2n})$ and $f_3(u_{3n})$ are stored in the computer memory for

$$u_{2n} = 1 \cdot 0 \times 10^{16} r^n \quad u_{3n} = 1 \cdot 0 \times 10^{13} r^n$$

and for $n = 0, 1, 2, 3 \ldots 20$ and with $r = \sqrt{10}$. Whenever it was required to calculate, $say$, $f_3$ for a given $u_3$, the two values of $u_{3n}$ closest to $u_2$ are determined, and by linear interpolation $f_3$ for the particular $u_3$ is determined.

Similar result holds for $J_2$, $E_2$ and $E_3$; the latter being the power absorbed per molecule of molecular oxygen and ozone respectively.

By this process we were able to save considerable amount of computer time. Naturally the process of interpolation involved some approximation. In a test it was found that in the worst case the fractional error by this time saving procedure was less than one in a thousand.

### (b) Solution of the photochemical equations

Using a slightly modified backward in time implicit scheme, where $X_t$ and $Y_t$ are the finite difference approximations to $n_1$ and $n_3$ at time $t$, the finite difference forms of Eqns. (1) and (2) with time step $\delta t$ are

$$X_{t+\delta t} - X_t = \{(2J_2 n_2)_t + (J_3 Y)_t - P_t X_{t+\delta t}\} \delta t$$

$$Y_{t+\delta t} - Y_t = \{k_2 n_2 n_m)_t X_t - Q_t Y_{t+\delta t}\} \delta t$$

where, $P_t = (k_2 n_2 n_m + k_4 Y + \epsilon_1)_t$ and
$$Q_t = (J_3 + k_4 X + \epsilon_3)_t$$

Thus,

$$X_{t+\delta t} = \{X_t + (2J_2 n_2)_t \delta t + (J_3 Y)_t \delta t\}/\{1 + \delta t P_t\}$$
$$\qquad (11)$$

$$Y_{t+\delta t} = \{Y_t + (k_2 n_2 n_m)_t X_t \delta t\}/\{1 + \delta t Q_t\} \qquad (12)$$

Eqns. (11) and (12) are unconditionally stable and approximate the true solution if,

$$\delta t Q_t \ll 1 \text{ and } k_2 n_2 n_m x_t \ll Y_t \qquad (13)$$

A minimum time step of 5 minutes was found to be a good compromise between satisfying condition (13) at all levels and minimizing computer time. In the interest of details, it is pertinent to point out that the use of 5 minutes as minimum time step has the effect of neglecting $\partial n_1/\partial t$ in Eqn. (1) at low levels.

Eqns. (11) and (12) are solved for different values of latitude using 21 steps between the poles and 71 uniform steps of $\delta z/H$ equal to 0.15 between the surface and about 80 km.

Starting from the top of this system of grids, with $J_2$ and $J_3$ assumed to take values for $u_2 = u_3 = 0$, we scan from pole to pole and progressively come to the surface.

At each grid point, the equations are solved starting with some reasonable guess for $J_3$ and $J_2$. The resulting $n_3$ and $u_3$ modify $J_3$ and $J_2$ and these modified values are used to obtain an improvement on the calculated $n_3$ and $n_1$. The whole process is repeated until the differences between $(n_3, n_1)_{t=0}$ and the end values $(n_3, n_1)_{t=24}$ hours are less than $10^5$ per cc.

### (c) Calculation of the tide

The computed values of the fluxes of energy $E$, were analysed into 24 hours and its harmonics. These harmonics were further analysed into the appropriate Hough functions. Eqn. (9) was then solved with the boundary condition proposed by Wilkes (1949), using U.S. Standard Atmosphere (1976) temperature profile.

### 4. Results of the calculation

#### (a) Ozone density

In Fig. 2 we have illustrated the calculated ozone density at 12 : 00 noon as function of latitude and height. Since the calculation is for equinox there is symmetry about the equator.

The level of maximum ozone density decreases from a height of 26 km at the equator to 18 km at 64° latitude. Ozone maximum concentration at the equator is $4.4 \times 10^{12}$ per cc. All these agree very closely with observaion—see for example U. S. Standard Atmosphere (1976) or London and Park (1974) who quoted personal communicated set of data from Dutsch.

## TABLE 1

### Fourier Hough components of pressure oscillation

(Amplitude in microbar, times of first maximum in hours local mean time)

| Height (km) | 24 hr component $h=-12.13$ $A_{11}$ $\tau_{11}$ | 24 hr component $h=0.69$ $A_{13}$ $\tau_{13}$ | 12 hr component $h=7.85$ $A_{22}$ $\tau_{22}$ | 12 hr component $h=2.11$ $A_{24}$ $\tau_{24}$ | 8 hr component $h=12.88$ $A_{33}$ $\tau_{33}$ | 8 hr component $h=5.08$ $A_{35}$ $\tau_{35}$ | 6 hr component $h=15.36$ $A_{44}$ $\tau_{44}$ | 6 hr component $h=7.29$ $A_{46}$ $\tau_{46}$ |
|---|---|---|---|---|---|---|---|---|
| 67.9 | 1  17.9 | 1  2.6 | 2  2.8 | 0  7.1 | 0  2.1 | 0  6.2 | 0  4.5 | 0  4.3 |
| 59.7 | 3  17.9 | 1  9.6 | 4  2.8 | 0  8.9 | 1  2.1 | 0  6.3 | 0  4.5 | 0  4.3 |
| 52.9 | 7  17.9 | 2  14.8 | 8  2.8 | 0  9.9 | 2  2.1 | 0  6.4 | 1  4.5 | 1  4.3 |
| 45.8 | 16  17.9 | 4  19.5 | 13  2.8 | 0  11.4 | 3  2.1 | 0  6.7 | 1  4.5 | 1  4.3 |
| 39.0 | 23  17.9 | 6  20.6 | 16  2.9 | 1  2.4 | 3  2.1 | 0  6.8 | 1  4.5 | 1  4.4 |
| 35.7 | 18  18.0 | 1  21.5 | 15  2.9 | 0  4.2 | 3  2.0 | 1  6.7 | 1  4.5 | 1  4.4 |
| 32.5 | 12  18.0 | 10  8.2 | 12  2.9 | 1  7.9 | 4  2.0 | 1  6.7 | 1  4.5 | 1  4.4 |
| 29.4 | 8  18.0 | 19  8.2 | 6  2.9 | 3  8.0 | 4  2.0 | 1  6.8 | 1  4.5 | 1  4.3 |
| 26.4 | 6  18.1 | 19  8.0 | 1  8.7 | 4  8.1 | 5  2.0 | 1  6.9 | 2  4.5 | 0  4.4 |
| 23.4 | 5  18.1 | 3  6.8 | 13  8.8 | 5  8.0 | 6  2.0 | 0  7.1 | 2  4.5 | 0  1.3 |
| 20.5 | 4  18.2 | 25  20.3 | 32  8.8 | 5  8.0 | 7  2.0 | 0  0.2 | 2  4.5 | 1  1.4 |
| 17.7 | 2  18.5 | 48  20.3 | 57  8.8 | 4  7.9 | 8  2.0 | 0  2.2 | 2  4.5 | 3  1.4 |
| 14.8 | 1  5.4 | 51  20.2 | 93  8.8 | 2  7.8 | 10  2.1 | 1  2.6 | 2  4.5 | 5  1.4 |
| 12.0 | 4  5.8 | 23  20.2 | 142  8.8 | 2  2.2 | 12  2.1 | 2  2.7 | 3  4.5 | 8  1.4 |
| 9.1 | 9  5.9 | 5  8.2 | 206  8.8 | 5  2.1 | 16  2.1 | 3  2.8 | 4  4.5 | 11  1.4 |
| 3.6 | 18  6.0 | 49  8.2 | 359  8.8 | 12  2.1 | 27  2.1 | 5  2.8 | 6  4.5 | 20  1.4 |
| 0.0 | 25  6.0 | 76  8.2 | 497  8.8 | 16  2.1 | 37  2.1 | 7  2.8 | 8  4.5 | 28  1.4 |

There is a small decrease in the maximum concentration as we approach the poles from the equator. Observation shows that in fact maximum concentration at the poles is larger than at the equator. We may note, however, that since most of the energy driving the tide is from the tropics, the small discrepancy near the poles will hardly have any noticeable effect on the calculated tide which is our main interest.

The significant features of the hourly variations illustrated in Fig. 3 are :

(i) Both at sunrise and at sunset, there is a sudden change of density of ozone.

(ii) The night time density is high and constant. The day time density is low, nearly constant with a little rise from sunrise to sunset. This diurnal variation becomes more prominent at altitudes greater than 40 km.

### (b) The calculated tide

Table 1 gives the variation with height of each of the Fourier components of pressure variation analysed into the appropriate Hough function when allowance is made for temporal variation of ozone. The corresponding equivalent depths are given at the top of each column. The first set of figures are the amplitudes, $A_{s,n}$ and the second set are the phases represented by $\tau_{s,n}$ time of first maximum local mean time in hours.
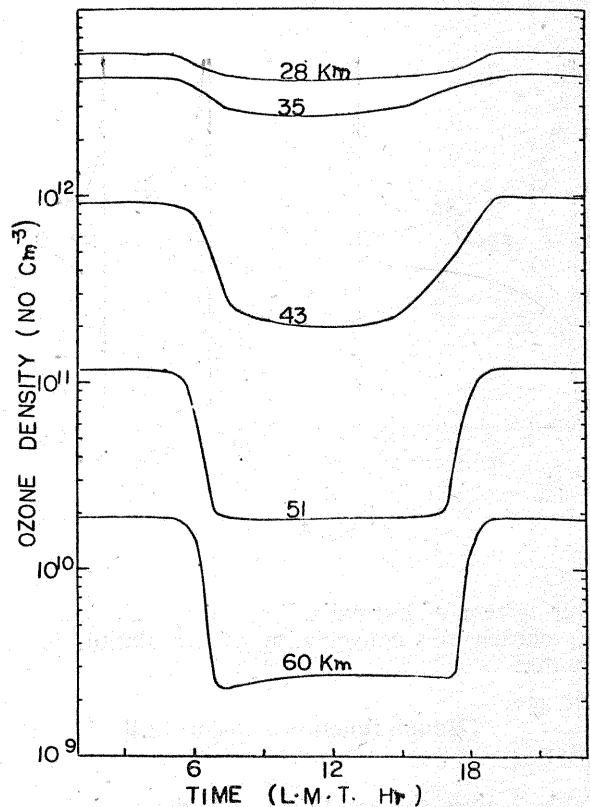


Fig. 3

## TABLE 2

**Calculated surface pressure variation (i) with allowance for ozone hourly variation (upper figures) and (ii) assuming that throughout the day ozone density takes a constant value equal to the noon value (lower figures)**

| Sine of Lat. | Lat. (deg.) | Surface pressure amplitude microbar | | | | Times of first max. (of surface pressure) in hours local mean time | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 24 hr | 12 hr | 8 hr | 6 hr | 24 hr | 12 hr | 8 hr | 6 hr |
| 0.0 | 0.0 | 116 | 595 | 37 | 35 | 20.4 | 8.8 | 1.9 | 4.4 |
| | | 108 | 647 | 30 | 34 | 20.7 | 8.9 | 1.9 | 4.5 |
| 1.1 | 5.7 | 99 | 578 | 37 | 31 | 20.5 | 8.8 | 1.9 | 4.4 |
| | | 92 | 627 | 29 | 31 | 20.7 | 8.9 | 1.9 | 4.5 |
| 0.2 | 11.5 | 54 | 528 | 36 | 20 | 20.7 | 8.8 | 2.0 | 4.4 |
| | | 50 | 571 | 29 | 20 | 21.0 | 8.9 | 2.0 | 4.5 |
| 0.3 | 17.5 | 8 | 452 | 34 | 5 | 1.7 | 8.8 | 2.1 | 4.5 |
| | | 8 | 488 | 28 | 5 | 2.1 | 9.0 | 2.0 | 4.5 |
| 0.4 | 23.6 | 41 | 361 | 32 | 11 | 7.4 | 8.8 | 2.1 | 7.3 |
| | | 39 | 387 | 26 | 10 | 7.5 | 9.0 | 2.0 | 7.4 |
| 0.5 | 30.0 | 60 | 266 | 28 | 22 | 7.6 | 8.9 | 2.2 | 7.3 |
| | | 56 | 283 | 23 | 21 | 7.7 | 9.0 | 2.1 | 7.4 |
| 0.6 | 36.9 | 59 | 176 | 23 | 27 | 7.5 | 8.9 | 2.3 | 7.3 |
| | | 55 | 187 | 19 | 26 | 7.6 | 9.1 | 2.1 | 7.4 |
| 0.7 | 44.4 | 47 | 101 | 17 | 24 | 7.2 | 8.9 | 2.3 | 7.3 |
| | | 45 | 106 | 14 | 24 | 7.3 | 9.1 | 2.2 | 7.4 |
| 0.8 | 53.1 | 35 | 45 | 10 | 16 | 6.7 | 9.0 | 2.4 | 7.4 |
| | | 33 | 47 | 8 | 15 | 6.8 | 9.1 | 2.2 | 7.4 |
| 0.9 | 64.2 | 25 | 11 | 4 | 5 | 6.3 | 9.0 | 2.4 | 7.4 |
| | | 24 | 11 | 3 | 5 | 6.3 | 9.1 | 2.4 | 7.4 |
| 1.0 | 90.0 | 0 | 0 | 0 | 0 | — | — | — | — |
| | | 0 | 0 | 0 | 0 | — | — | — | — |
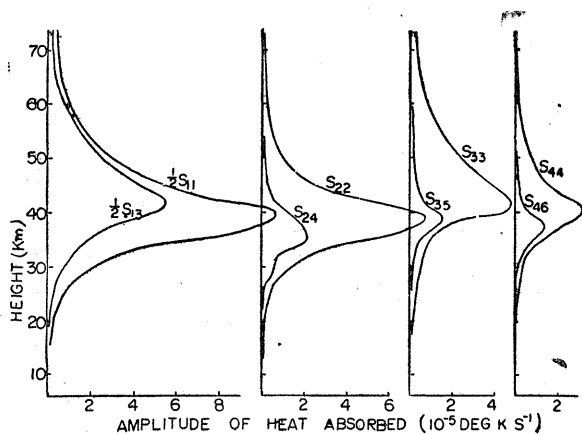


Fig. 4

Our system of normalization of Hough function, chosen for convenience of calculation is such that

$$\int_{\text{S. pole}}^{\text{N. pole}} (\text{Hough function})^2 \, d[\sin(\text{Lat.})] = 1$$

Table 2 gives the amplitude and phase of the resulting surface pressure changes as function of latitude. For the purpose of comparison with "static ozone" theory we also present in this table the tide that would be generated if ozone density assumed the noon value throughout the day.

In the calculation of the latitudinal distribution of pressure amplitudes we summed up the components of Hough function up to the sixth order beyond which further contributions was negligible. We may also note that since the calculation is for equinox, the amplitudes of all antisymmetric Hough components are zero.

For the purpose of discussion of the pressure changes we have illustrated in Fig. 4 the amplitude of some of the Fourier Hough components of heat energy absorbed as function of height. Half the amplitude of the 24 hourly components are plotted because they are much larger than the others and it is desirable to accommodate the four harmonics on the same diagram.

### (i) 24 hourly component

Though most of the heat input is in the 24-hourly components, the surface pressure amplitude of 0.12 mb is much smaller than the corresponding figure for 12 hourly components. The small amplitude for $S_{1,3}$ is a consequence of the vertical oscillatory nature of this component. As pointed out by Green (1965), this oscillatory nature makes absorption of energy in the ozone region an inefficient mechanism.
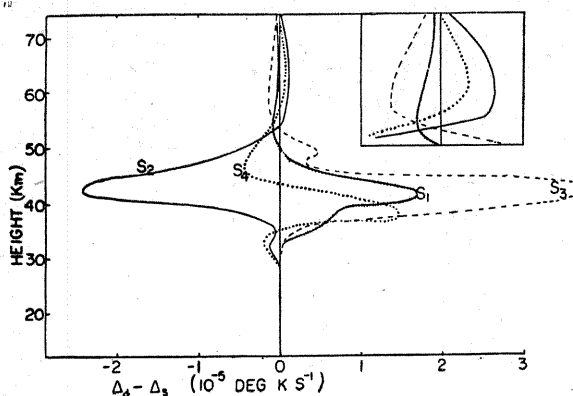
Fig. 5

$S_{1,1}$ however, is not oscillatory in the vertical. In fact it is strongly exponential. In order to explain the small amplitude for $S_{1,1}$ we note that if the solution for the homogeneous part of Eqn. (9) is of form

$$y_0 = \exp(\pm kx)$$

then where $Q = \triangle R \exp(-x/2)/\gamma gh$, using method of variation parameters the expression for $y$ at the surface is

$$y_{(x=0)} = (k - H/h + \tfrac{1}{2})^{-1} \int Q \, dx \, \exp(-kx) \quad (14)$$

where the integration is carried through the region for which energy is absorbed and $H$ is the scale height at $x=0$.

If $k$ is imaginary $\exp(-kx)$ is oscillatory and consequently the quantity under the integral sign will change sign if the energy is absorbed over a depth greater than half the vertical wavelength. This is the mathematical basis of the argument for the explanation of $S_{1,3}$ amplitude.

If, however, $k$ is real it is clear from Eqn. (14) that the higher the region of energy input, the less effective it is for generating tide at the surface. The equivalent depth for $S_{1,1}$ is negative and so $k$ in this equation has a value slightly more than a half. This means that since the absorption of solar energy in the ozonosphere, is significant from about $x=5$, to higher levels the effective energy is reduced by a factor of less than $e^{-2.5}$.

This is one reason why as pointed out by Lindzen (1967) water vapour at lower levels generates more diurnal tide than ozone at the high levels.

The calculated heat energy input is very similar to the results of Butler and Small—maximum energy input is at about 40 km in both calculations. Butler and Small had a maximum value of $1.30 \times 10^{-4}$ deg. K per second as against our figure of $1.10 \times 10^{-4}$ in same units.

The amplitude of 0.12 mb for dynamic ozone tide and 0.11 mb for static ozone tide at the equator are much larger than that calculated by Butler and Small. The difference can be explained by the difference in temperature profile and the difference in values of equivalent depths used for $S_{1,3}$. We note that since $S_{1,3}$ is highly oscillatory in the vertical, a change in wavelength which can be caused by changes in either temperature profile or equivalent depth can modify considerably the amplitude of pressure oscillation at the surface.

(ii) 12 hourly tide

The surface pressure amplitude for the 12 hourly tide at the equator was found to be 0.59 mb when ozone variation is allowed for and 0.65 mb with ozone assuming the 12 o'clock value throughout the day.

These amplitudes are less than those obtained with the temperature profile used by Butler and Small. The main reason for the difference can be accounted for by the larger value of temperature used by the latter in the 50 km region. As shown by Jacchia and Kopal (1952) and also by Giwa (1968) the temperature in this region has considerable influence on the magnification for equivalent depths close to 8 km. Indeed for 7.85 km the magnifications with Butler and Small temperature profile and with U.S. Standard Atmosphere are respectively 2.2 and 1.4.

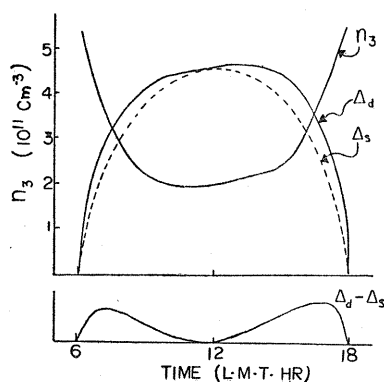The time of maximum amplitude is 8.8 hour local mean time.

Fig. 6

The characteristic phase change of half a period at 25 km can be observed in Table 1.

### (iii) The 8 hourly component

The surface pressure amplitude at the equator for dynamic ozone tide is 37 microbar and 30 microbar for static ozone.

### (iv) The 6 hourly component

The surface pressure amplitudes at the equator are 35 and 34 microbar for dynamic and static ozone respectively.

It may be noted from Fig. 4 that though most of energy for this Fourier component is in the $S_{4,4}$, mode from Table 1 we can see that the contribution of $S_{4,6}$ to pressure change is larger. The equivalent depth of $S_{4,4}$ is 15.4 km and so the variation in the vertical is strongly exponential.

### (c) Comparison of dynamic and static ozone tides

In Fig. 5 we have illustrated as function of height, the difference, energy absorbed with dyamic ozone *minus* that absorbed with static ozone for $S_{1,1}$, $S_{2,2}$, $S_{3,3}$ and $S_{4,4}$. In the diagram the data at the higher levels are repeated in the upper right hand corner where they have been magnified by a factor of 5 to reveal the details in this section of the atmosphere.

From the figure it can be seen that the effect of using constant ozone density at high levels is to increase energy absorbed for the 24 hourly and 8 hourly tides and to decrease energy absorbed in the 12 and 6 hourly tides.

Between 30 km and 50 km where most of the tidal energy resides, using constant ozone density decreases the energy input for 12 hourly tides and increases the energy for other components.

In order to explain the above result it is necessary to show in greater detail the hourly variation of ozone density. This is illustrated in Fig. 6 for 43 km level at the equator. The corresponding energy absorbed in arbitrary units both for dynamic and static ozone are indicated.

From the figure we can see that at 43 km ozone density is minimum at noon. Thus before noon and afternoon, the energy absorbed (which is proportional to ozone density), is larger with dynamic than static ozone. At noon the two are very nearly equal. This difference in energy is as shown in the lower diagram in Fig. 6. It can easily be seen that the 24 hourly Fourier component of this difference in energy is maximum at noon and so it is in phase with 24 hourly component of static ozone energy input; hence the increase of 24 hourly dynanmic ozone energy at this level.

Similar argument shows that the 12 hourly harmonic of the difference has a *minimum* at noon, i.e., out of phase with 12 hourly static ozone tidal energy. Hence photochemical changes reduce 12 hourly component of energy at this level.

At higher levels from Fig. 3, we note that ozone density attains its maximum daytime value at 12 noon. Therefore using the same argument we see that photochemical changes increases 12 hourly component and decreases the 24 hourly components of energy.

Now ozone density is not exactly symmetric about noon. This anti-symmetry and the general shape of the time variation modifies the 8 hourly and 6 hourly energy input.

Since most of the energy absorbed is between 30 to 50 km in which region the energy absorbed in the 12 hourly components is reduced by the time variation of ozone, we can readily see why

the pressure change is less for 12 hourly components and more for the other components if we allow for ozone variation.

## 5. Conclusion

We have shown that the inclusion of diurnal variation of ozone modifies the result of the calculated tide. Depending on what is referred to as the mean ozone density, the use of static ozone in the theory of tides may result in an error of more than 10 per cent in the calculated tide near the equator.

In fact in a preliminary investigation in which a slightly different definition of mean ozone was used, and in which $\epsilon_1$ and $\epsilon_3$ in Eqns. (1) and (2) were set to zero, the tide generated by dynamic ozone was larger than static ozone by 12 per cent for 12 hourly tide.

It is also pertinent to point out that a decrease in the minor trace constituents of the atmosphere (represented by $\epsilon_1$ and $\epsilon_3$) resulted in production of more ozone and consequently greater amplitude of the 12 hourly tide.

The implication of all these is that there is need for closer study of ozone behaviour in order to be able to describe adequately the influence of ozonosphere on the oscillation of the atmosphere.

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