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Normalized equivalent radar reflectivity factors for water and ice and their dependence on radar wavelength

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ABSTRACT. A new parameter, called the Normalized Equivalent Radar Reflectivity Factor (Z_e*) is introduced to enable proper appreciation of the effect of radar wavelength on the radar reflectivities of water and ice hydrometeors. Z_e * is defined as the value of the equivalent radar reflectivity factor (Z_e) obtained by using the normalized back-scatter cross-section $\sigma/\pi r^2$. This operation removes the effect of the geometric area of the back-scattering cross-section and enables us to make a more realistic assessment of the dependence of radar reflectivity on radar wavelength for any given hydrometeor. Values of Z_e* have been calculated, for four different radar wavelengths, for water and ice spheres in size-ranges common to hydrometeors. Certain aspects of the dependence of Z_e* on the nature and size of scatterers (hydrometeors) and on radar wavelength are discussed. On this basis, an effort is made to assess the relative merits of the different radar wavelengths commonly used in radar-meteorological operations and research.

1. Introduction

Wavelengths commonly used for weather radars are in the four bands, *viz.*, X, C, S and L bands, corresponding approximately to 3, 5, 10 and 23 cm. In general, the requirements of uniformity, standardization and comparability (especially in quantitative measurements) demand that one wavelength should be adopted for operational use. While the choice of appropriate wavelength will be influenced by various factors including the economic aspects, there are, nevertheless, certain basic parameters whose behaviour with change in radar wavelength has to be considered. One such important parameter is the Equivalent Radar Reflectivity Factor, Z_e, because this is the parameter, determined on the basis of the radar equation, for use in referencing echo intensity and rate of precipitation.

2. Equivalent radar reflectivity factor (Z_e)

The Rayleigh approximation of back-scattering holds good only for particles whose diameter, *D*, is small compared with the radar wavelength, λ, *i.e.*, when $\alpha << 1$; where α is the ratio of sphere perimeter to wave-length or, in other words, $\alpha = 2\pi r/\lambda$, where, $r = D/2$. For larger scatterers in the complex Mie-scattering region, the power received at the radar is no longer proportional to the Radar Reflectivity Factor, Z. In such cases it is convenient to express the radar echo reflectivity as an *equivalent* radar reflectivity factor, Z_e, which is a *fictitious* value of radar reflectivity factor that will yield the same value of received power as the observed reflectivity, had the Rayleigh approximation been applicable. For ease in computation and comparison, the scatterers are further assumed to comprise wholly of water. In other words, we can define Z_e of a radar echo as the Z of small (Rayleigh criterion) spherical water drops which would have the same total back-scatter as the observed echo.

Z_e is expressed as

$$Z_e = \frac{\sigma \lambda^4}{\pi^5 |K|^2} \times 10^6 \text{ mm}^6/\text{m}^3 \tag{1}$$

where, σ is the radar cross-section (cm²)

λ is the radar wavelength (cm)

and K is a function of the complex index of refraction.

3. Normalized back-scatter cross-section (σ_N)

The radar back-scatter cross-section is very much dependent on the geometric cross-section of the scatter and this creates difficulties in computation because the geometric cross-sections of

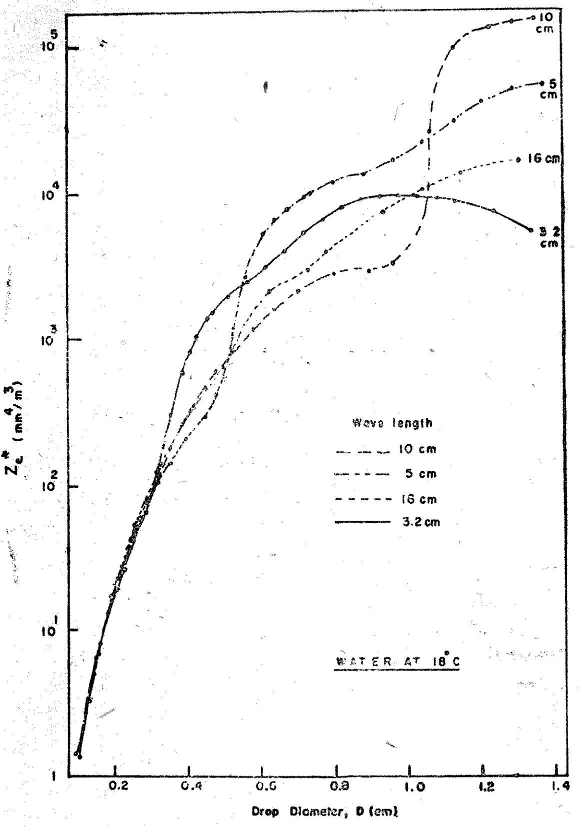

Fig. 1.

a large number of scatterers (hydrometeors) of numerous sizes have to be accounted for. It is therefore customary in radar-meteorology to define a *normalized* back-scatter cross-section, σ_N, which is the ratio of the radar cross-section of the scatterer to its geometric cross-section.

or $\sigma_N = \frac{\sigma}{\pi r^2}$ where σ_N is the normalized cross-section.

4. Normalized equivalent radar reflectivity factor (Z_e^*)

Since σ is dependent on the geometric cross-section, Z_e is also dependent on the geometric cross-section. It follows from Eqn. (1) that if this dependence on geometric cross-section can be eliminated from the equivalent radar reflectivity factor, it will be feasible to make a more realistic assessment of the inter-dependence of the remaining variables, *viz.*, radar reflectivity and radar wavelength for any given hydrometeor. This can be achieved by replacing σ by σ_N. In other words, use of *normalized* back-scatter cross-section in Eqn. (1) will remove the influence of the geometric cross section.

We may thus propose a new, very useful, radar-meteorological parameter, *viz.*, the *normalized* equivalent radar reflectivity factor, Z_e^*, which is expressed as

$$Z_e^* = \frac{\sigma_N \lambda^4}{\lambda^5/|K|^2} \times 10^6 \text{ mm}^4/\text{m}^3 \quad .. \qquad (2)$$

Since σ_N is a dimension-less factor, the change in the dimensions of Z_e^* (as compared to those of Z_e) is to be taken note of.

5. Computation of Z_e^* for different wavelengths

As explained above, Z_e^* will present the true dependence of radar reflectivity on radar wavelength for each hydrometeor size. Computations of Z_e^* have been made for drop diameters upto 1.4 cm and hail diameters upto 8 cm as these are approximately the largest sizes occurring generally.

Values of σ_N for different values of α (corresponding to various values of drop or hail diameter) were selected from the data of Herman and Battan (1959) and Stephens (1961) who have provided tables of back-scatter cross-sections for ice and water spheres of various sizes. Hailstones have been assumed to be dry and, for them, value of $|K|^2$ has been taken as 0.93.

Values of Z_e^* have been computed for four representative wavelengths, both for water and ice spheres. In the case of water drops, the wavelengths chosen were: 3.2, 5, 10 and 16 cm because values of σ_N were readily available for these wavelengths in the tables given by Stephens (1961). For hail, the wavelengths chosen were: 3.2, 4.7, 10.5 and 23 cm which correspond more approximately to the wavelengths currently in use in radar meteorolgy.

The results of these computations are presented graphically in Figs. 1 and 2.

6. Discussion of Z_e^* for raindrops

A comparative study of the values of Z_e^* obtained at different wavelengths for different sizes of raindrops (Fig. 1) brings out the following facts:

(*i*) Upto about 0.3 cm diameter drop-size, we are well within the Rayleigh regime and the values of Z_e^* for the various wavelengths are almost the same for any given drop-size upto this diameter.

(ii) when the drop size increases, the Rayleigh criterion begins to fail and we find that a change in the gradient of Z_e^* curve seems to occur when the value of D/λ approaches 0.05 to 0.07.

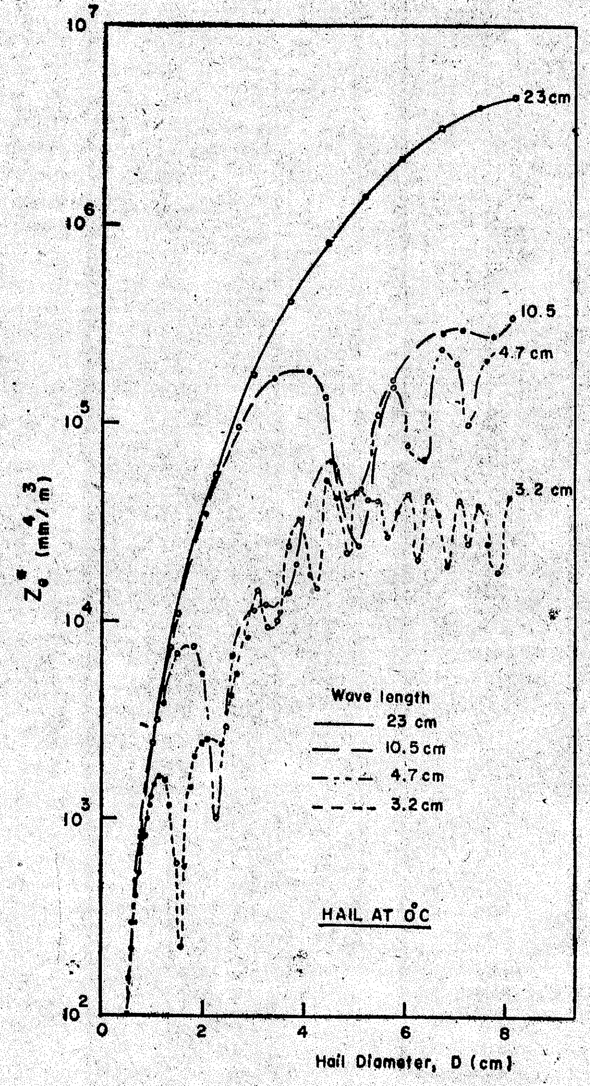

Fig. 2.

(*iii*) Soon after the value of D/λ approaches 0.1, each curve starts rising again and exhibits a value of Z_e^* higher than any other curve at that value of D. This condition is maintained until the next longer wavelength approaches the criterion $D/\lambda = 0.1$ and starts rising.

(*iv*) The variation of Z_e^* with drop-size is very nearly smooth in the case of L-band wavelength for drop sizes generally occurring in nature. This is because the above mentioned criterion of $D/\lambda = 0.1$ is not reached.

(*v*) Since raindrops larger than 1.0 cm diameter seldom, if ever, occur in nature, the discussion of the behaviour of Z_e^* for drop diameters larger than 1.0 cm becomes more or less academic.

(*vi*) It is seen from Fig. 1 that for drop-diameters of 0.3 to 0.55 cm, Z_e^* at 3 cm wavelength not only has the highest value of all the four wavelengths, but it is more sensitive to changes in drop-size. On these considerations, the X-band radar would have been ideal for use in weather detection. However, the attenuation suffered by X-band wavelengths overrides, to a large extent, the advantage demonstrated by the Z_e^* curve.

(*vii*) C-band wavelength is seen to have a disadvantage when drop sizes are between 0.3 and 0.5 cm in diameter. This is in addition to the fact that C-band wavelength also suffers significant rain attenuation, especially in tropical rains. This study of Z_e^*—D relationships, therefore, shows that C-band wavelengths should be least advantageous particularly in tropical regions.

(*viii*) As long as the size of raindrops is limited to about 0.52 cm, there is not much to choose between 10 and 16 cm wavelengths. But if the drops are larger, the 16 cm wavelength gains an advantage because of a higher Z_e^* in addition to the inherent advantage of less attenuation.

7. Discussion of Z_e^* for hail

Continuing the discussion on the same lines as adopted above for raindrops, the following facts are apparent for hail on the basis of Fig. 2:

(*i*) Upto about 1 cm diameter hail, there is no appreciable variation of Z_e^* with wavelength.

(*ii*) When the value of D/λ is of the order of 0.3, the Z_e^* curve starts a dip which continues till D/λ approaches 0.5 when the Z_e^* curve starts rising again.

(*iv*) When D/λ approaches unity, there is a small secondary dip in the Z_e^*—D curve. The variation of Z_e^* associated with this dip is very small compared to the first dip and may, for the most part, be neglected.

(*iv*) When D/λ approaches unity, there is another dip in the Z_e^* curve which, although smaller than the first dip, is nevertheless significant at that particular drop size.

(*v*) It will be seen from Fig. 2 that in the range of hail sizes such as we may expect in nature, the S-band curve exhibits only the first (major) and the second (negligible) dips. The L-band curve is quite smooth and shows no dips whatsoever upto hail sizes of 8 cm in diameter.

(*vi*) As a result of these variations in Z_e^* with change in diameter, X-band radars have greater Z_e^* values than C-band radars in the hail size ranges 2.1 to 2.5 cm, 2.9 to 3.2 cm and 3.6 to 4.0 cm. In the size range 5.0 to 5.3 cm, Z_e^* on X-band wavelength, although less than that on C-band, is greater than that on S-band. These advantages in certain discrete hail size ranges are not appreciable and certainly cannot compensate for the attenuation on X-band.

(*vii*) Fig. 2 shows that C-band wavelengths also do not have any advantage. This curve remains appreciably below the one corresponding to 10.5 cm wavelength except in the narrow range of 4.6 to 5.4 cm diameter.

(*viii*) The S-band curve maintains its advantage over the shorter wavelengths for hail sizes upto 4 cm diameter. The curve then suffers a serious dip whereas the L-band (23 cm) curve maintains its smooth trend throughout the range of hail sizes normally expected to occur in nature.

(*ix*) Although all the curves in Fig. 2, on the average, tend to become less sensitive to hail size with the increase in diameter; the change in the Z_e^* gradient is much less on L-band than on S-band or shorter wavelengths. A choice of a wavelength of the order of 15 cm would do away with most of the abrupt changes in Z_e^* values which are still evident at 10.5 cm wavelength.

8. Conclusions

A new radar-meteorological parameter Z_e^*, the *normalized* equivalent radar reflectivity factor, has been defined and its variation with the size of scattering hydrometeors has been studied for four representative wavelengths and both for rain drops and hail. Relative merits of each of Z_e^* with D is concerned, have been discussed representative wavelength, as far as the variation in the preceding two sections. This study supports the author's earlier findings (Kulshrestha 1968, Kulshrestha 1971) that a choice of radar wavelength of the order of 15 cm is likely to prove advantageous especially in the tropics and for detection of hail. Such a choice may however lead to some sacrifice in beam width and will tend to limit the useful range of the radar by reducing the returned power received at the radar receiver. These effects would not be serious within a range of 200 km.

Till the time such a wavelength is available commercially and can be used operationally, it appears essential that the existence of the abrupt perturbations in the Z_e^* curve is recognized for all quantitative echo-intensity measurements and the corresponding estimation of rainfall or hail size. A significant improvement in the accuracy of these measurements would require the use of different Z-R relationships for different drops or hail sizes, not only in different climatic zones, not only in different types of precipitation, but even in different parts of the same echo area.

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