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Sodar studies of gravity waves in the planetary boundary layer at Delhi*

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ABSTRACT. Analysis of sodar echograms for a period of one year (May 19777-April 1978) shows that wavemotions, both simple and complex, occur in the planetary boundary layer at Delhi. The oscillations occur in the boundary layer under conditions of nocturnal radiation inversion, ascending inversion and frontal wind shear.

Auto correlation and power spectral density techniques of wave analysis have been applied to the observed wave motions to study their periodicity. The frequency spectra of the observed waves are reported. The time periods lying mostly within 1-15 minutes suggest them to be atmospheric gravity waves originating due to the stable restoring force of the density gradients. Wave lengths and scale sizes of the observed waves have been calculated. The scale size of the waves has been found to be small.

1. Introduction

During recent years gravity waves have been considered to play a major role in the observed dynamical processes in ionosphere (Hines 1960), in microwave communication (Stilke 1973), in hazardous aviation situations and in the scatter of atmospheric contaminants in the planetary boundary layer (Eymard 1978). The period of these waves (Keliher 1975) ranges from a few minutes to several hours. Waves below the Brunt Vaisala frequency are considered (Gossard *et al.* 1973) to have a component of propagation in the vertical direction which is responsible for the vertical redistribution of energy and momentum while waves above the Brunt Vaisala frequency are not able to propagate beyond the critical level in the lower atmosphere.

The remote sensing techniques of radars and acoustic echo sounders have further stimulated the studies of gravity waves. It is now being considered that gravity waves are mostly generated by shear instability (Gossard *et al.* 1975 and Mastrantonio *et al.* 1976) in the boundary layer flow while convective cells (Deardorff 1969) penetrating into stable atmosphere and storm systems and fronts (Curry and Murty 1974) may also generate gravity waves to some extent. The waves are superposed on the steady state of the atmosphere in the form of small scale perturbations with varying amplitude, wavelength and frequency.

The scope of the present work is restricted to remote sensing of waves of small scale in the statically stable atmospheric layers using acoustic echo sounding technique (Singal *et al.* 1978). A study of the various types of waves observed on sodar echograms under varying atmospheric conditions in Delhi are reported. An attempt has been made to decode some of the characteristic features of the observed waves by applying the correlation technique.

2. Method of analysis

The wave motions observed on the facsimile record have been manually digitized at an interval of Δt ($\sim$ 15-65 sec) and the digitized data forming a time-history record have been used for computation of the autocorrelation function and the power spectral density. The

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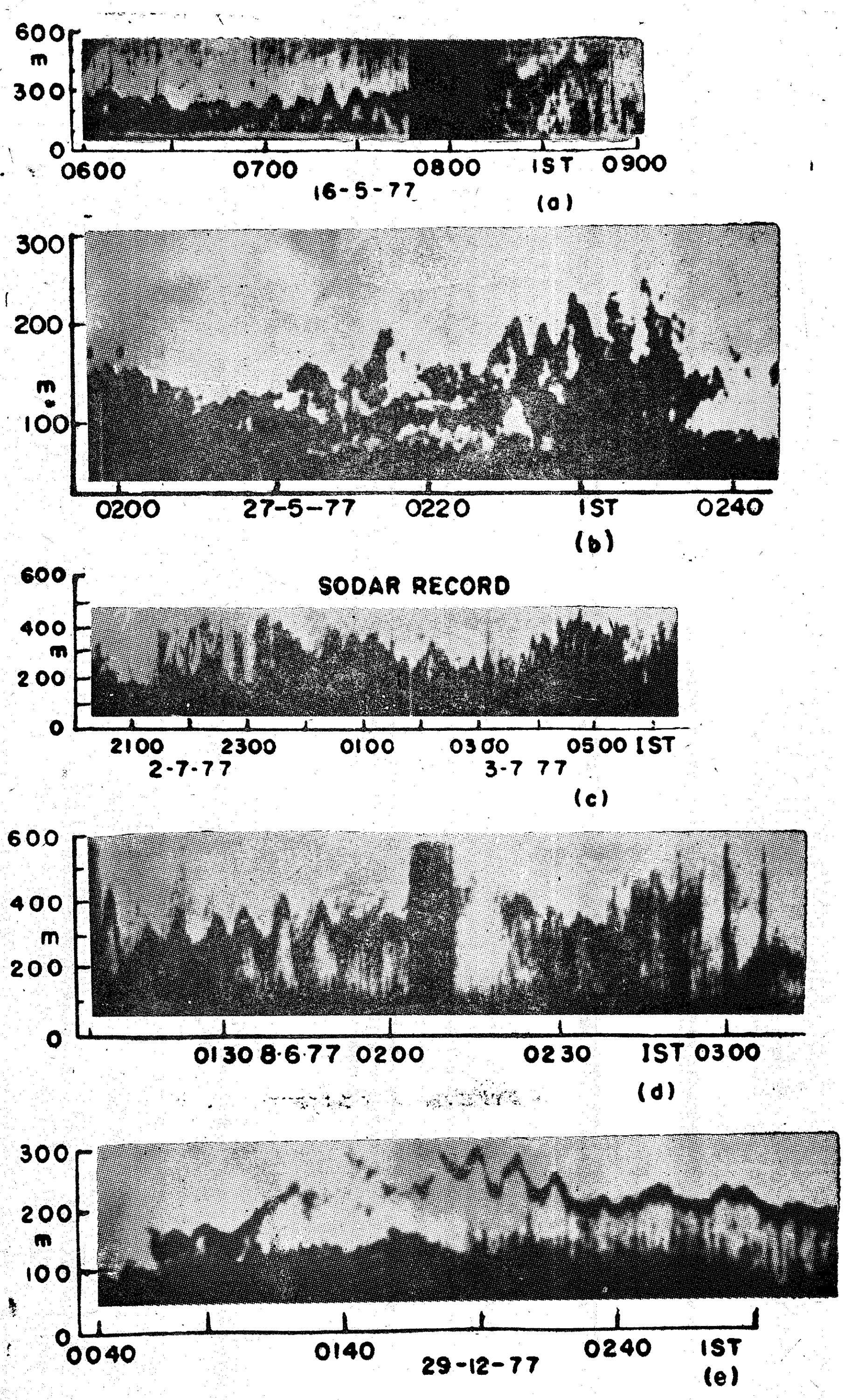

Fig. 1. Various types of undulations seen on the NPL sodar echograms (a) Ascending layer, (b) Nocturnal stability (c) After thunderstorm, (d) After severe dust storm and (e) During a fog storm

autocorrelogram gives the exact periodicity of undulations which has already been approximately estimated on the echogram and the power spectral density describes the general frequency composition of the data. The autocorrelation function for a time history record $x(t)$ has been defined as:

$$\rho(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, x(t+\tau)\, dt$$

where $\rho(\tau)$ is the autocorrelation function at time τ, $x(t)$ is the amplitude at time t, $x(t+\tau)$ is the amplitude after a lag time τ and T is the total time of the series. Spectral analysis of a data series has been carried out by the Fourier Transform alogrithm (Bendat and Piersol 1971, Brigham 1974) X_k of N equal to 2^6 or 2^7 data periods. The program handles $2n$ data points where n is an inetger. The computations have been done by a suitable computer programme using Fortran IV.

3. Analytical studies

Waves and perturbations (Fig. 1) have been seen many a times on the sodar echograms under stable atmospheric conditions. These undulations are mostly quasi-periodic (semi-sinusoidal) in structure of both simple and complex nature and have large periods of the order of minutes. The amplitude of these waves is generally around 100 metres peak to peak. They have been observed both at the top of the surface based layer or superposed on the elevated layer occurring mostly during the early morning hours when the atmosphere is highly stable. To a lesser extent undulations have also been observed in the ascending inversion layer, in the early part of the night and during dense fog or cloud covered day.

Sodar echograms were obtained over day and night for 140 days during the year May 1977-April 1978 spread evenly over all the seasons. The operational time was about 3500 hours. An analysis of the occurrence percentages of the various structures has shown that waves occur only for 4 per cent of the total time of observations. Their distribution over the year shows that they occur most often during pre-monsoon period (58 per cent of the time of waves), the least during the post-monsoon period (6 per cent), while during monsoon and winter waves occur for 26 per cent and 10 per cent of the total time of their existence respectively. Duration period of the undulations has also been studied. It is seen that very often undulations have a short life span of a couple of hours only while waves of durations as long as 10 hours have also been seen to exist. Undulations exist at any height level within the sodar probing range although most often they occur within 250 m of the lower atmosphere.

Three categories of oscillating phenomena have been observed as also reported by Eymard (1978) in his analysis of a large number of wave motions observed on the tripple Doppler sodar:

(*i*) *Oscillations of nocturnal radiation inversion*

Nocturnal radiation inversion layers, surface based or elevated, have often shown oscillations superposed over them. These oscillations appear in the form of almost parallel and oblique bars in the manner of herring bone structure as shown in Fig. 1(b). They have periods of the order of a few minutes, their peak to peak amplitude is about 100 m and their life span can be of the order of few hours. Sometimes, oscillations in the form of successive wave trains are also seen existing almost throughout the night. They may disappear for a part of the night from a few minutes to several hours.

(*ii*) *Oscillations of ascending inversion*

The ascending inversion layer obsereved in the morning time lasting for about 2-3 hours after sun rise has also been seen to show undulations. (Fig. 1a). These oscillations in the form of peaks persist upto the disappearance of the echo layer, have magnitudes of 100-150 m from peak to peak and are of periods of a few minutes which seem to increase at the end when the echo layer is at a higher altitude and is dissipating.

(*iii*) *Oscillations due to a front*

The passage of a duststorm, thunderstorm, tornado and a severe fog cloud etc, has also been seen to be accompanied with undulations on the sodar echograms (Figs. 1c & 1d). These oscillations can be both regular and irregular with their peak to peak amplitude of about 100 m. They have a period of the order of a few minutes and may exist for quite long periods.

During the year of our reported observations (May 1977-April 1978) about 40 wave systems in the above three categories of oscillating phenomena were seen on the facsimile echograms. Analysis by both autocorrelations as well as power spectral density techniques has been carried out to determine the dominant frequency modes for the observed waves. Sampling was done manually. Undulations which were complex in structure and were thus difficult to trace were not analysed reducing the analysis to 23 cases. December 1974 large amplitude wave motion echogram which had earlier shown a major tropospheric upheaval in the atmosphere, hazardous for microwave propagation (Mitra *et al.* 1977) has been, however, included in the analysis.

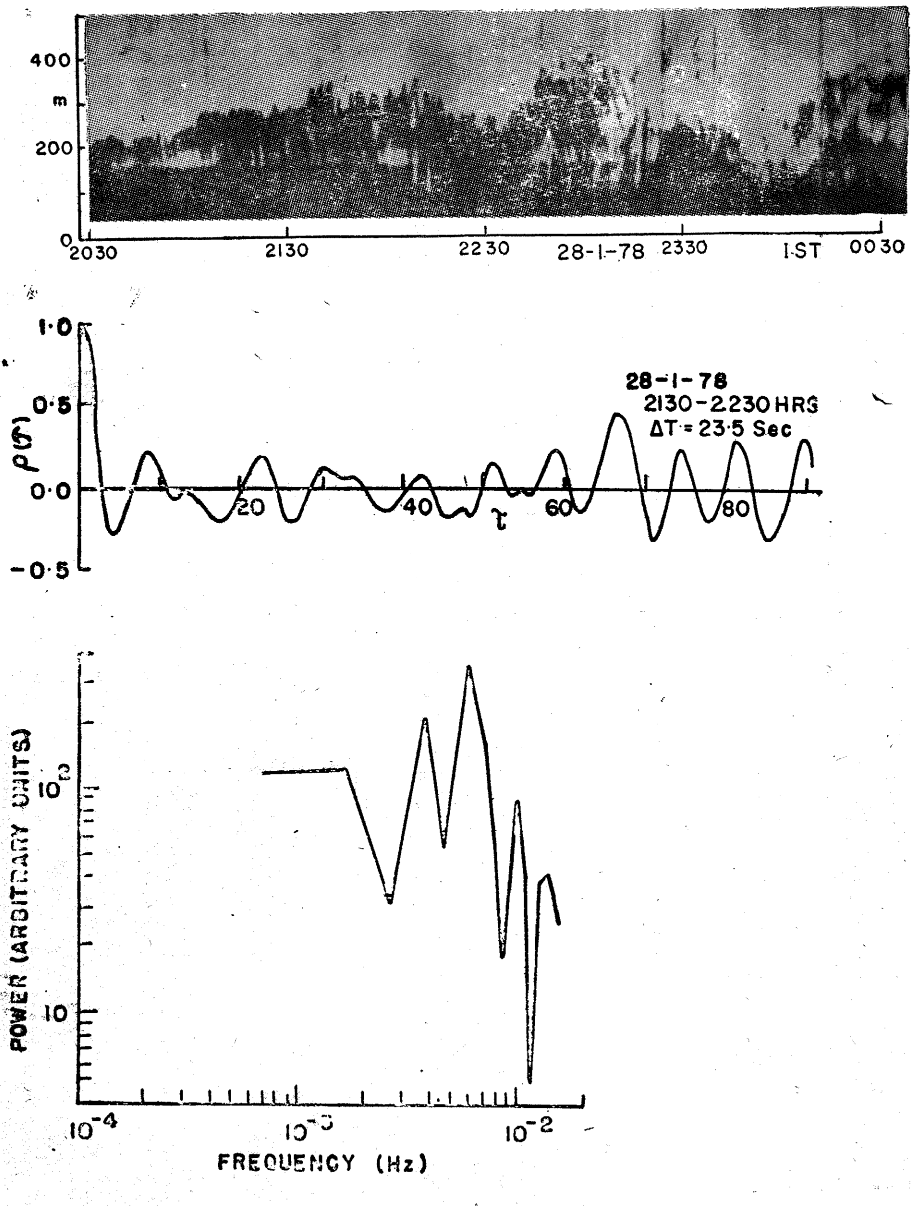

Fig. 2. Sodar echogram and plot of autocorrelation function and power spectral density for the undulations observed on 28-29 January 1978. The analysis has been performed during the time interval 2100-2230 hours

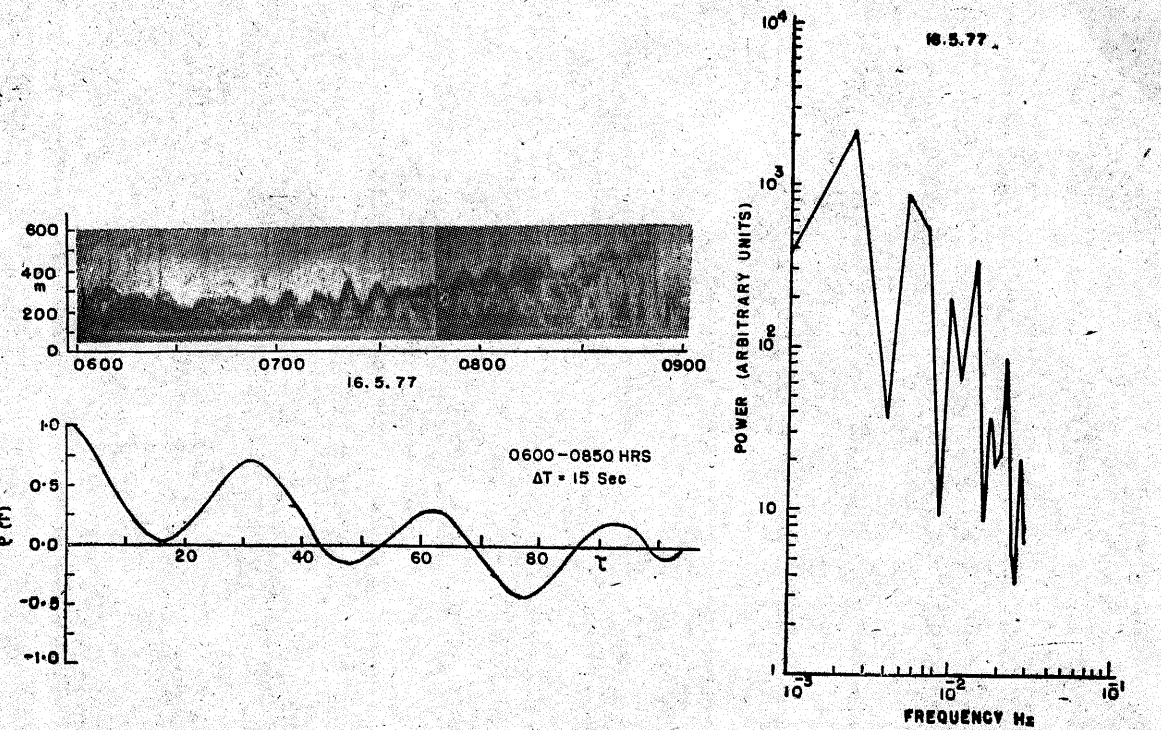

Fig. 3. Sodar echograpm of the oscillating ascending inversion layer observed on 16 May 1977 during the time interval 0600-0900 hours. The auto correlogram and power spectral density plots of the observed oscillations are also shown. The auto correlogram shows strong periodicity

The results of analysis grouped in the three categories of the oscillating phenomena are given in tabular form Table 1(a-c). Values of the wave periods noted from sodar echograms, autocorrelograms and power spectral density plots are listed in these tables for comparison. Assuming the propagation velocity of the waves (Eymard 1978) equal to the wind velocity at the level of observation of waves (taken from the radiosonde data observed at Aya Nagar, Delhi), wavelength of the waves and scale size of the perturbed atmosphere have been computed and are also given in these tables. It is seen from these tables that the periods determined by the various techniques agree to a rough approximation and further that it may not always be possible to estimate the periods from the echogram and auto-correlograms but the special density plot is always able to offer the required information.

4. Case studies

A few particularly distinct cases of the three categories of the oscillating phenomena have been selected to elucidate the physical characteristics of the sodar echograms, auto-correlograms and power spectral density plots. In the following we describe these cases:

(*i*) *Oscillations of nocturnal radiation inversion-echogram of 28-29 January* 1978

Fig. 2 shows the echogram and spectral analysis of the oscillations seen on 28 January 1978 from 2130 to 2230 hours. We observe on the facsimile record several oscillations of period about 5 minutes. The waves propagate between 175-350 m altitude with the oscillating layer seeming to vary in height slightly (60 m) during the course of oscillations. The oscillations are centered around 225 m on an average and have peak to peak amplitude of about 150 m. The auto-correlogram shows a clear combination of random noise and a periodic structure. It is difficult to estimate the wave period from the correlation curve. On the energy spectrum plot, existence of a peak centred at the frequency 4×10^{-3} Hz is seen which gives a wave period of 250 seconds. The spectrum on either side of this frequency behaves as blank noise because the peaks do not emerge from the

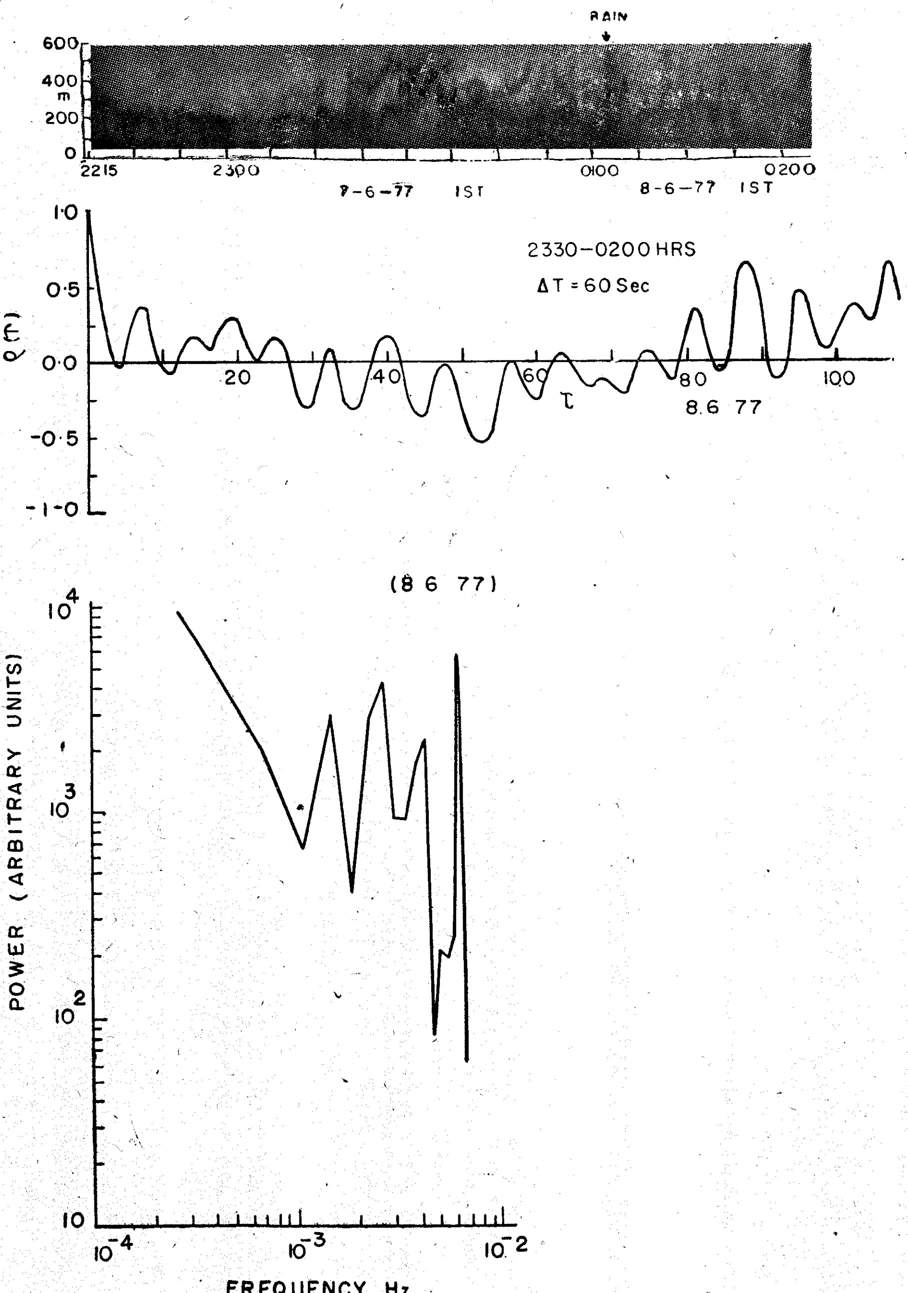

Fig. 4. Sodar eccogram of the oscillations observed on 7-8 June 1977 during the time interval 2330-0200 hours after a severe duststorm was experienced in the evening at 1900 hours. Cold winds were blowing throughout the night. The autocorrelation and power spectral density plots are also shown in the diagram. The aut-o correlogram shows a definite periodicity

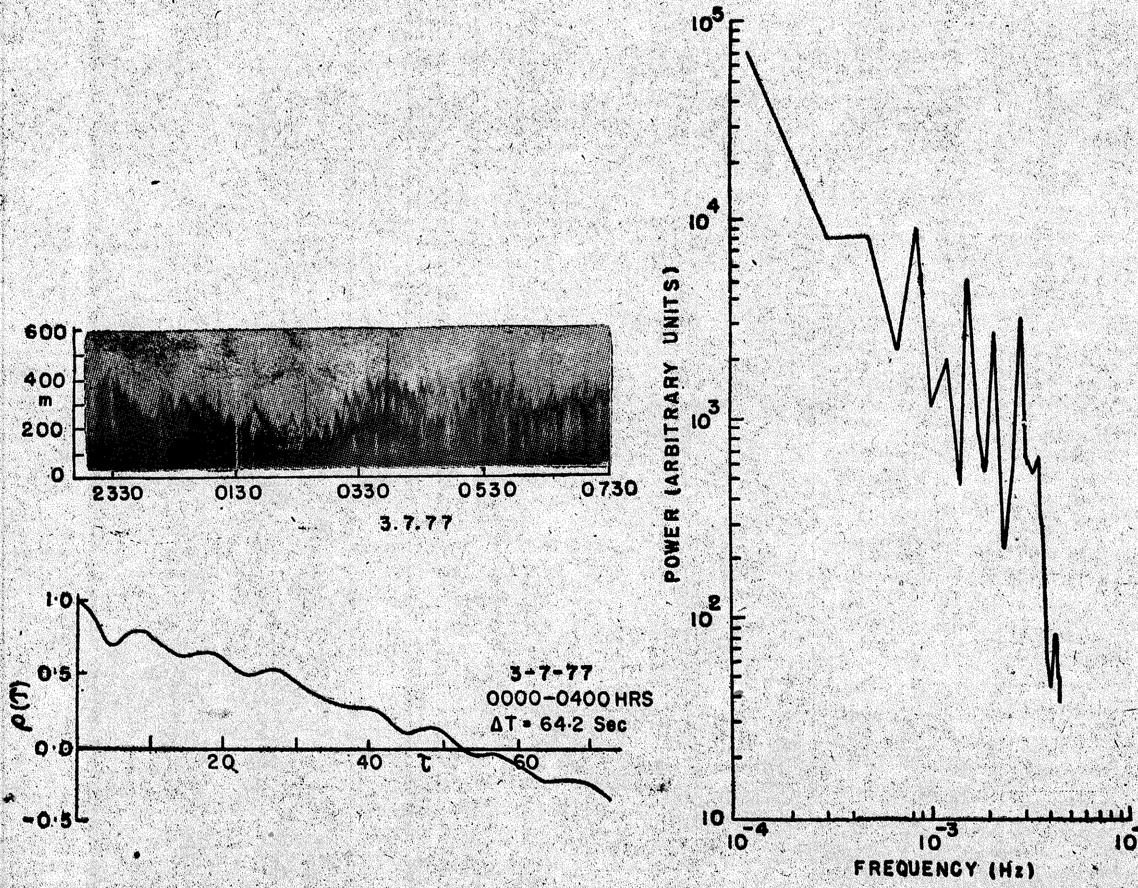

Fig. 5. Sodar echogram of the oscillations observed in the night of 2-3 July 1977. A thunderstorm with a few showers of rainfall was experienced during the previous day. The autocorrelation and power spectral density plots show the formation of a complex wave

noise. We consider a frequency spectra to be significant only if the ratio between its energy level and the noise level is greater than 2.

(*ii*) *Oscillations of ascending inversions-echogram of* 16 *May* 1977

The examination of the facsimile in Fig. 3 shows that the surface based echo layer is situated to a mean height level of 250 m at 0700 hours. It starts rising regularly and goes to an altitude of 400 m by 0830 hours, then disappears gradually while still rising further. The layer seems to be lifted by the underlying convection developed due to solar heating of the surface of the earth. The facsimile shows the presence of oscillations of amplitude 140 m peak to peak throughout the record. The oscillations seems to be generally periodic of period about 7 minutes with irregularities in the beginning.

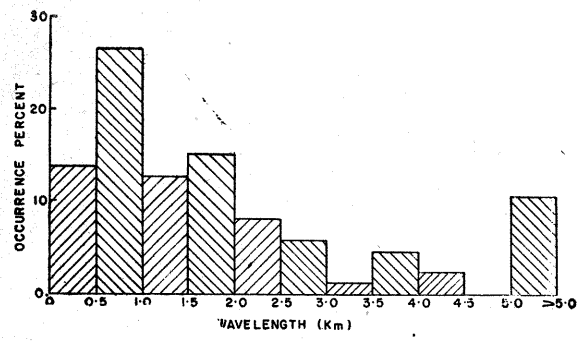

Fig. 6. Hystogram of occurrence percentages over ranges of wave length for the observed waves

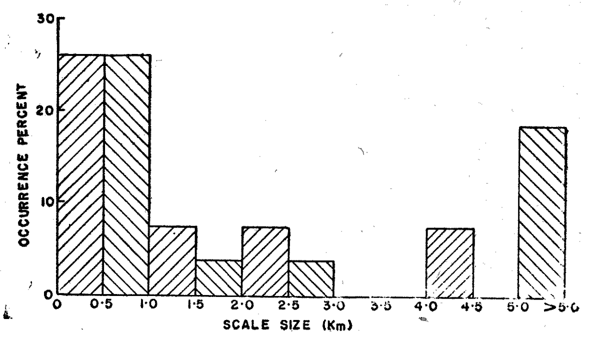

Fig. 7. Hystogram of occurrence percentages and scale sizes of turbulence (random) for the observed waves

The autocorrelogram of the oscillations made during the period 0600-0850 hours shows a perfect periodicity of period 450 seconds. However, the power spectral density plot shown on the same diagram (Fig. 3) gives two distinct close peaks at 5.5×10^{-3} and 2.5×10^{-3} Hz. The two frequencies may represent a harmonic of each other since both the sodar record and the correlogram show a simple wave of apparent period of oscillations close to 7 minutes. The plume structure underneath the ascending inversion is not very clear, so it is not possible to compare the analysed time periods with the period of the convective cells.

(*iii*) *Oscillations of layers due to fronts*

Here in we discuss two cases (i) after a severe duststorm of 7-8 June 1977 and the (ii) after a thundershower of 2-3 July 1977.

(*i*) *Echogram of* 7-8 *June* 1977

Fig. 4 shows the echogram of oscillations observed during the night of 7-8 June 1977. Oscillations start soonafter 2330 hours and continue upto 0200 hours. The preceding evening had experienced a severe duststorm at 1900 hours followed by strong cold winds (fronts). The surface temperature had shown a steep fall during the period of oscillations. The oscillations were seen at a mean altitude of 250 m having a peak to peak amplitude of about 200 m. Before the oscillations had started, a elevated layer at a mean height of 200 m was present which had later burst into oscillations. Intermittent oscillating multiple layers were also seen. The undulations are very regular and seem to have a period of 6 minutes.

The autocorrelogram plotted on the same diagram (Fig. 4) shows a definite periodicity of a combination of at least two periods. The random nature of the correlogram is also very clear. The power spectral density plotted on the same diagram suggests two significant frequencies at 2.5×10^{-3} Hz and 2.5×10^{-4} Hz. The period of waves of frequency 2.5×10^{-3}Hz is close to the period of oscillations seen on the echogram.

(*ii*) *Echogram of* 2-3 *July* 1977

Fig. 5 shows another echogram of oscillations observed during the night of 2-3 July 1977 after a spell of rainfall during the day. The preceding day was cloudy and had experienced intermittent thunder showers for brief intervals. It had not, however, rained during the night but it was cloudy. Winds were medium strong in the evening but had become low in the late night. Oscillating structures wihin a height range of 200-500 m were being seen with their mean level at 300 m. Oscillations were generally irregular but became very much periodic during the short interval 0230-0330 hours. The period of the regular oscillations is around 635 seconds with their peak to peak amplitude of about 150 m.

The autocorrelation and power spectral density plots of the oscillations observed during the time interval 0000-0400 hours are also shown on Fig. 5. The auto correlogram suggests the presence of a periodic structure mixed with random noise. The structure may also have more than one period. The power spectral density plot marks four significant spectral peaks at 3×10^{-3}, 1.5×10^{-3}, 8×10^{-4} and 2×10^{-4} Hz. The multiplicity of spectra shows that the structure has a very complex wave form.

TABLE 1

Date	Duration of waves (hr)	Height range (m)	Wave amplitude peak to peak (Mean) (m)	Approx. wave period (sec)	Dominant frequency		Wind speed (Radio sonde) (m/s)	Wave-length (km)	Auto corre-lation time width (Random) *ts* (sec)	Scale size (km)	Remarks
					Auto-correlation	Power spectra					
(a) **Analytical results of wave motions observed during nocturnal radiation inversion**											
19.12.74	2245-0045	100-350	100	1500	5.5×10^{-4}	3×10^{-3}	5.0	1.7	330	1.6	Light* winds simple wave
27.5.77	0200-0400	125-300	120	132	7.9×10^{-3}	1.5×10^{-3}	3.0	2.0	276	0.828	Moderate* to heavy* winds diffused structure
29.7.77	0000-0300	225-275	50	900	1.5×10^{-3}	2.3×10^{-3}	5.0	2.2	120	0.6	Humid weather simple wave
4.11.77	0400-0700	175-250	50	625	2×10^{-3}	2.6×10^{-3}	10.0	3.8	231	2.3	Moderate wind complex wave
28.1.78	2100-2230	175-350	150	225	3.5×10^{-3}	4.0×10^{-3}	2.0	0.4	35	0.7	Clear weather calm conditions
29.1.78	0000-0130	275-425	100	300	2.6×10^{-3}	2.8×10^{-3} 9.2×10^{-3}	4.0	1.4	106	0.4	Light wind complex structure
29.1.78	0430-0630	250-350	70	514	3.9×10^{-3}	3.3×10^{-4} 4×10^{-3}	4.0	1.0	752	3.0	Light wind complex structure.
16.2.78	2230-0200	275-400	—	480	—	6.8×10^{-4} 1.7×10^{-3}	3.0	1.7	814	2.4	Light wind complex structure
14.4.78	0100-0500	150-250	75	626	—	6.8×10^{-4}	5.0	7.3	—	—	Moderate wind complex structure
16.4.78	0100-0600	200-300	—	720	2.6×10^{-3}	2.5×10^{-3}	5.0	2.0	83	0.4	Moderate wind simple wave structure
26,27.4.78	2300-0200	150-200	50	500	2.3×10^{-3}	5×10^{-4}	5.0	10.0	125	0.6	Moderate wind simple wave structure
(b) **Analytical results of wave motions associated with ascending inversion**											
4-2-77	0940-1120	150-200	50	415	1.9×10^{-3}	2×10^{-3}	4.5	2.25	120	0.540	Cloudy weather simple wave
28-2-77	0845-1045	175-350	80	547	1.1×10^{-3}	2.2×10^{-3}	4.0	1.818	189	0.756	Light winds diffused wave structure

TBBLE 1—contd.

Date	Duration of waves (hr)	Height range (m)	Wave Amplitude peak to (peak mean) (m)	Approx. wave period (sec)	Dominant frequency		Wind speed (Radio sonde) (m/s)	Wave-length (km)	Auto corre-lation time width (Random t_s (sec)	Scale size (km)	Remarks
					Auto-correlation	Power spectra					
16-5-77	0600-0850	200-500	140	410	2.2×10^{-3}	2.5×10^{-3} 5.5×10^{-3}	3.0	1.20 0.545	142.5	0.427	Regular wave motion
15-6-77	0600-0845	150-325	100	—	—	2.7×10^{-3} 6.5×10^{-3}	4.0	1.48 0.615	156.5	0.624	Irregular wave motion
16-2-78	0900-1100	275-550	75	500	2.1×10^{-3}	3.1×10^{-3}	3.0	0.967	140.1	0.420	Complex wave structure
4-5-78	0600-0800	175-300	75	820	2.2×10^{-3}	1.5×10^{-3}	5.0	3.33	220.8	1.104	Light winds irregular wave motion
(c) Analytical results of wave motions associated with fronts											
8-6-77	2330-0200	175-400	200	415	—	2.5×10^{-4} 2.5×10^{-3}	12	— 1.2	120	1.440	Severe dust storm in the evening
3-7-77	0000-0400	200-500	150	635	—	1.5×10^{-3} 3.0×10^{-3} 8×10^{-4} 2×10^{-3}	3.0	1.0 2.0 15.0 3.7	1992	5.977	Rain squall at night —
6-7-77	1230-1500	80-200	75	514	1.7×10^{-3}	2.2×10^{-3} 5.3×10^{-3}	5.0	22.7 0.9	189	0.946	Rainfall squall during the day
28-12-77	1245-1645	250-500	150	400	—	4.5×10^{-3} 2.5×10^{-3} 9.5×10^{-4}	2.5	1.0 0.5 2.6	1745	4.36	Dense fog layer during the day
29-12-77	0000-0240	75-250	100	540	—	2.7×10^{-3} 5×10^{-4}	2.0	0.7 4.0	2226	4.45	Dense fog layer
17, 18-3-78	1830-0200	upto 150	100	—	—	1×10^{-3} 4×10^{-4}	9.0	9.0 —	876	7.88	Tornado in the evening (University area)

*Light winds=0-5 m/s Moderate winds=5-8 m/s Heavy winds=8 m/s onwards

5. Discussions

The analysis of the wave systems shows that most of the time (75 per cent) the wave systems are regular and periodic with their wave periods of the order of a few minutes. For the rest of the cases more than one period (not harmonic multiple) has been found to be present showing the wave forms to be complex in that case. According to Beer (1974) uniform sinusoidal oscillations lying within the narrow bands of period 5-10 minutes should be essentially tropospheric gravity waves, irregular fluctuations with periodicities lying between 15 minutes to one hour can be due to convection and influence of nearby weather systems while waves with periods less than 5 minutes can be sonic in origin and high frequency, generally non-periodic noise, can be due to turbulence.

Grouping the time periods of the observed wave forms, according to Beer (1974) in the three categories of periods, i.e., less than 5 minutes, between 5-15 minutes and more than 15 minutes, their occurrence percentages within these three categories have been calculated. It is seen that for about 50 per cent of the time observed waves on the sodar echograms have time periods between 5 & 15 minutes showing the presence of tropospheric gravity waves in the lower atmosphere for most of the time as per the analysis of Beer. For more than 90 per cent of the time, the observed waves have periods between 1 & 15 minutes as also shown by Eymard (1978).

Assuming that the waves travel with the velocity of the medium at the mean position of observation of waves as also shown experimentally by Eymard, wavelengths of the observed waves have been calculated and given in Table 1(a-c). The occurrence percentage of wavelengths in various length ranges, have been plotted in the hystogram of Fig. 6. It is seen that a large number of the observed waves have wavelengths within 2 km while the single largest group lies in the wavelengths range 0.5-1.0 km.

From the time elapsed for 1/e width of the autocorrelation functions, the scale sizes of turbulence (random) for the observed waves have also been calculated and given in Table 1(a-c). The hystorgram (Fig. 7) of occurrence percentages of the various scale sizes shows that the distance range of turbulence (scale size) mostly lies within 1.0 km indicating that the waves can be mostly grouped in the small scale size as also shown by Eymard. To a small extent waves also have a medium scale size.

From the above it is thus seen that the physical characteristics of wave systems observed on sodar echograms are associated with the atmosphere in the planetary boundary layer. They may have been formed due to wind shear, convective cells or frontal movements. The amplitude of the waves generally lies around 100 m peak to peak. They may be surface based or elevated, simple or complex with their period mostly lying within 1-15 minutes suggesting them to be atmospheric gravity waves.

The capability of the acoustic sounder for the study of waves in the planetary boundary layer has been shown. The addition of Doppler sodar facility can give further information of the propagation velocity of the waves. From these measurements it will be possible to know exactly the wave length of waves and scale size of turbulence, an information which can be further used for sudies connected with atmospheric modelling, to find scattering range of atmospheric contaminants and to derive effective range of turbulence in hazardous situations of aviation and communication.

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