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Application of Sasaki's numerical variational technique to the analysis of height and wind fields over Indian region

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सार — भारत के क्षेत्रों में भूविभव की ऊँचाई और वायु के विश्लेषण के लिए सस्की की संख्यात्मक विभेदक विश्लेषण तकनीक के आधार पर एक वस्तुनिष्ठ विश्लेषण पद्धति अपनाई गई है। आरम्भिक अथवा निवेश फील्डों की उपलब्धता के लिए एक विचर अनुकूलतम अंतर्वेशन पद्धति का प्रयोग किया गया है। तत्पश्चात इन फील्डों का विभेदक द्वारा समायोजन किया गया है।

4 से 8 जुलाई 1979 तक भारत और उसके निकटवर्ती क्षेत्रों में इस पद्धति का अध्ययन 850,700,500,300 और 200 हैक्टॉपास्कल स्तरों पर किया गया तथा इस पद्धति के प्रयोग से उपलब्ध हुए विश्लेषणों की तुलना एफ. जी. जी. ई. विश्लेषणों से की गई है।

ABSTRACT. An objective analysis method based on Sasaki's numerical variational analysis technique has been taken up for the analysis of geopotential height and wind over the Indian region. The univariate optimum interpolation (UOI) method is used to generate the initial or input fields. These fields are then adjusted by the variational method.

A study of this method over Indian and adjoining region for 850, 700, 500, 300 and 200 hPa levels is made from 4 to 8 July 1979 and the analyses obtained using this method are compared with the FGGE analyses.

Key words—Numerical Variational Objective Analysis (NVOA).

1. Introduction

Amongst the many kinds of objective analysis methods, at least two kinds of such methods produce analysed data in balance. One is multivariate optimum interpolation (MOI) method, which is multivariate version of the optimum interpolation method first formulated by Gandin (1963) and later it was developed and applied to real data objective analysis by Rutherford (1972), Schlatter (1975), Schlatter *et al.* (1976), Bergman (1979), Lorenc (1981), Dey and Morone (1985), Baker *et al.* (1987), DiMego (1988) and others. Over Indian region, MOI experiment was carried out by Sinha *et al.* (1992). The other method is numerical variational objective analysis (hereafter referred to as NVOA) which was first formulated by Sasaki (1958) by applying the technique of calculus of variations by subjecting the meteorological variables to dynamical constraints.

The variational optimization technique has been a common tool for solid mechanics problems (Lanczos 1970) for quite a long time. However, after Sasaki's initiation, the above technique has fascinated many research workers to use it for initializing and adjusting meteorological fields. The basic approach of this method is to optimize a function in a domain under certain constraints. The constraints may be diagnostic or prognostic relations. Sasaki used diagnostic equations such as geostrophic balance equations as constraints. Sasaki (1969, 1970) extended his original formulation to include time varying functions and categorised the constraints as "strong' or "weak" constraints. Strong

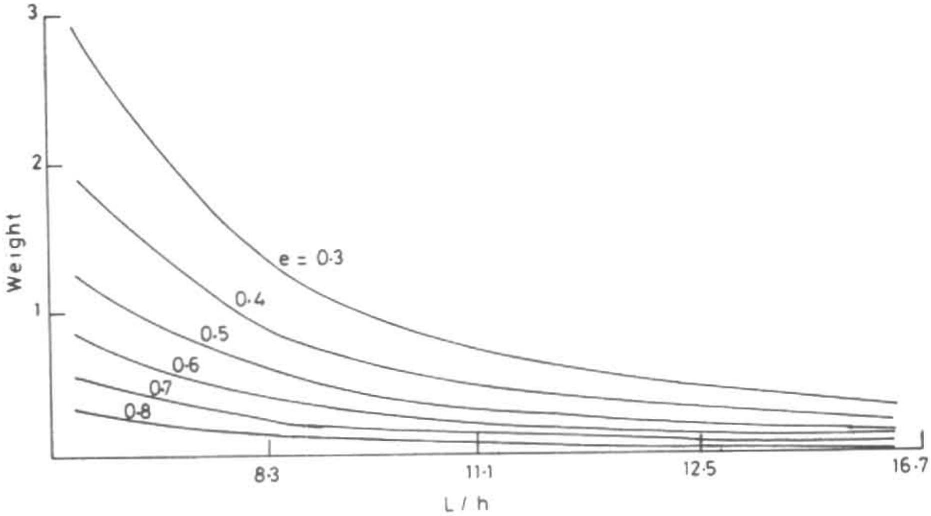


Fig. 1. Variation of weights, *W*, as a function of *L/h* for different values of parameter of accuracy (*e*)

constraint means that the relation is exactly fulfilled, and weak constraint means that the relation is approximately fulfilled in a sense of least square error.

Lewis and Grayson (1972) applied Sasaki's variational technique for the adjustment of surface wind and pressure. They have used horizontal momentum equations as dynamical constraints for the adjustment of sea level pressure and wind field. Their case study exhibited pronounced small scale features when the wind information was incorporated in the pressure field. Lewis (1972) has also applied variational optimization technique for the upper air analysis using hydrodynamical equation as constraints.

However, no work to the authors' knowledge has been carried out on the application of this variational analysis technique over Indian region. In this study, an attempt has been made to experiment with the data over Indian region by application of this method. The geostrophic relation has been used as diagnostic constraint.

2. Basic equation

2.1. *Variational scheme*

Based on the techniques of calculus of variations and the quasi-geostrophic relations as the functional constraints, the geopotential increments (ϕ') can be obtained by solving the following equation :

$$\phi'_{i+h,j}+\phi'_{i-h,j}+\phi'_{i,j+h}+\phi'_{i,j-h}-\left[4+\left(\frac{1-e}{e}\right)\left(\frac{2\pi h}{L}\right)^2\right]\phi'_{i,j}$$

$$=f h^2\zeta_0-[\phi_{0(i+h,j)}+\phi_{0(i-h,j)}+\phi_{0(i,j+h)}+\phi_{0(i,j-h)}-4\phi_{0(i,j)}] \qquad (1)$$

where, h, e and L are grid size, parameter of accuracy and wavelength respectively. Details are given in APPENDIX A.

2.2. *Decoupling of geostrophic approximation*

Near the equator, the geostrophic balance does not hold good. Consequently, this balance has to be decoupled in order to have realistic conditions. Hence the decoupling of the geostrophic balance with decreasing latitude is incorporated by means of a latitude (θ) dependent coupling factor 'K'(Mills and Seaman 1990), such that

$$K(\theta)=1.0, \qquad \theta>\theta_{max}$$

$$K(\theta)=\frac{\sin\theta-\sin(\theta_{min})}{\sin(\theta_{max})-\sin(\theta_{min})}, \qquad \theta_{min}\le\theta\le\theta_{max}$$

$$K(\theta)=0.0, \qquad \theta<\theta_{min}$$

where, θ_{max} and θ_{min} are 30 and 15 degrees.

Once the geopotential increments (ϕ') are obtained, the wind increments (u' and v') are computed by

$$u'=\left[-\frac{1}{f}\frac{\partial\phi'}{\partial y}-u_0-\frac{1}{f}\frac{\partial\phi_0}{\partial y}\right]K(\theta) \qquad (2)$$

$$v' = \left[\frac{1}{f}\frac{\partial \phi'}{\partial x} - v_0 + \frac{1}{f}\frac{\partial \phi_0}{\partial x}\right] K(\theta) \qquad (3)$$

Thus, through the decoupling factor $K(\theta)$, the wind component increments are completely geostrophically coupled to the geopotential increments polewards of θ_{max}, and the fields are completely decoupled equatorwards of θ_{min}, *i.e.*, south of 15°N. In other words analyses south of 15°N become univariate.

3. Data and synoptic situations

In numerical weather prediction techniques based on the quasi-geostrophic and thermal wind assumptions it was felt that the initial map should be constructed objectively by a method based on the same assumptions. In this study, objectively analysed values of height and wind fields by UOI method at 850, 700, 500, 300 and 200 hPa levels, 1200 UTC over the region from 1.875°N to 39.375°N and 41.250°E to 108.750°E were used as the input field. Persistency (analysis from FGGE for the corresponding previous day) was used as the initial guess field. Since the time scale of tropical systems is of the order of 2-3 days, the guess field certainly contains valuable information. However, in the case of rapidly developing system, this persistence field is severely limited. FGGE IIIb analyses of heights and winds at 850, 700, 500, 300 and 200 hPa levels from 4 to 8 July, 1979 were used for comparison.

We have carried out the NVOA experiments for the above levels for five consecutive, days, *viz.*, 4 to 8 July 1979, 1200 UTC to examine the variational scheme. During this period, on the first day, there was a low over north Bay with its central region near 20°N and 90°E. By 7 July, this system concentrated into a depression with its centre near 20°N and 88°E and by 8 July, the depression moved westward and crossed north Orissa coast. Thus we have different synoptic situations for making objective analyses and examining the performance of this scheme for different conditions.

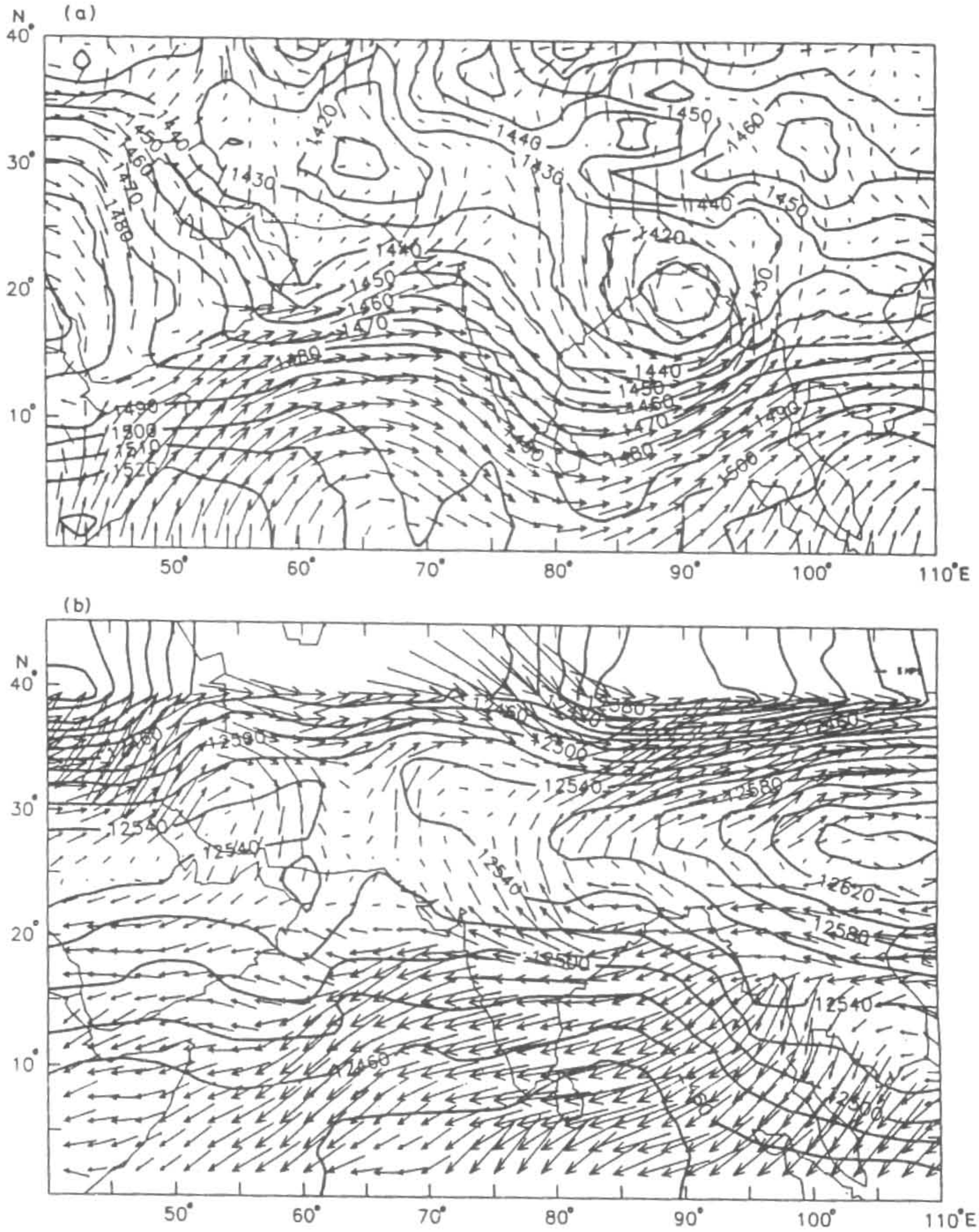
4. Numerical experiment and results

The UOI analysis method described by Gandin (1963) is the basic analysis procedure used to obtain the height and wind fields. The details of the UOI objective analysis technique has been described in Rajamani *et al.* (1983). As mentioned earlier α, β, γ in Eqn. (A17) play the dominant role in leading to the analysis of the meteorological field. We have considered here a two dimensional case study. Weighting factor W in Eqn. (A25) is determined by parameter of accuracy e and the ratio between the grid size and the wave length. The variation of weights (W) with L/h for different values of e is shown in Fig. 1. The second order partial differential equation in finite difference form (Eqn. 1) was solved with the boundary conditions as the observed values in the boundary. Relaxation technique is used to solve Eqn. (1).

A region bounded by 41.250°E and 108.750°E longitude and 1.875°N to 39.375°N latitude with a grid resolution of 1.875° was taken up for this experiment. NVOA experiment was carried out for 850, 700, 500, 300 and 200 hPa levels for all the five days. However, for illustration we present here the analyses for only two levels (850 and 200 hPa) of 7 July 1979.

5. Discussions

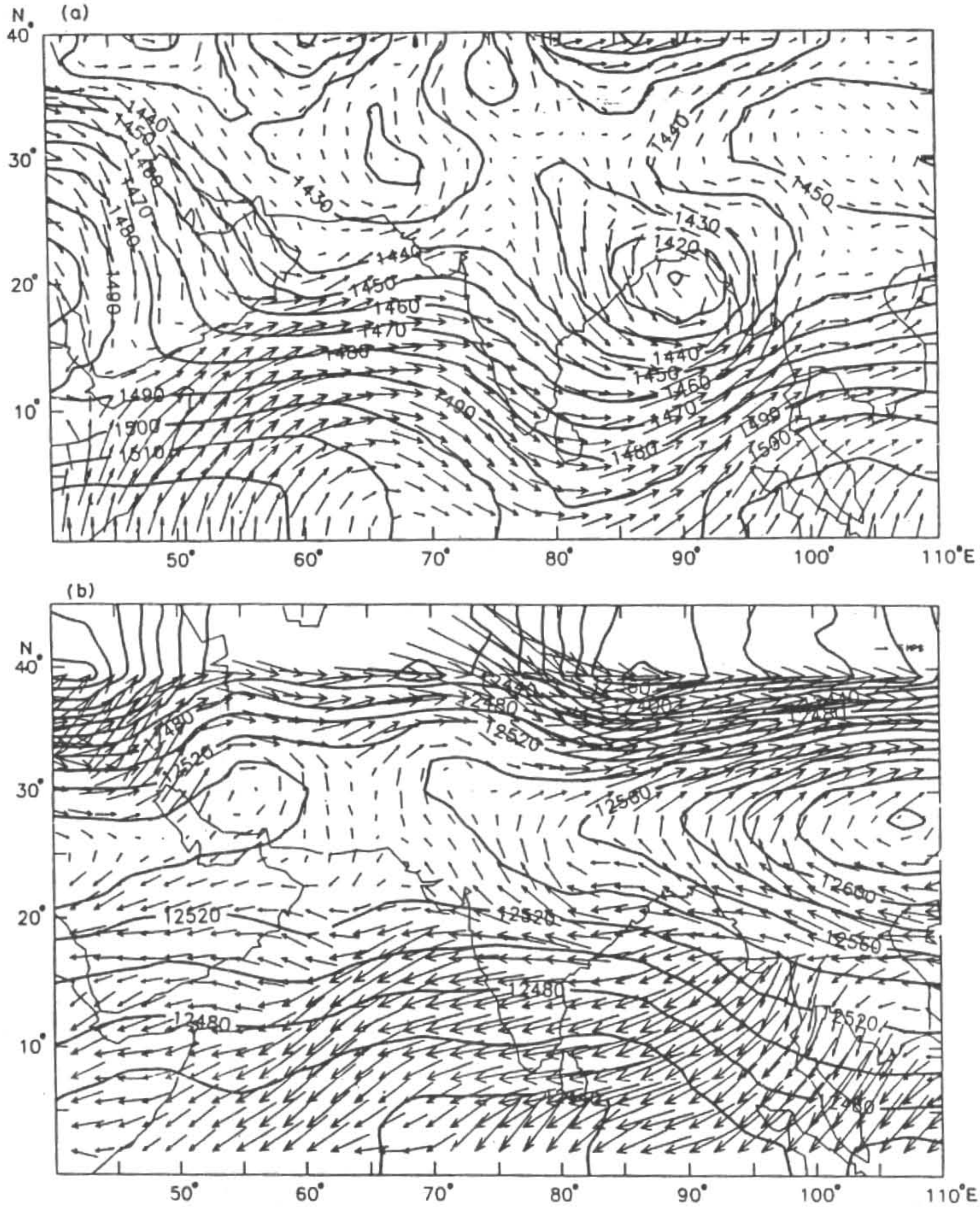
The objective of this study as mentioned at the beginning is to make the final analyses of height and wind fields to be in balance. In order to examine this feature, the wind analyses and the corresponding height analyses were plotted on the same charts. As can be seen from Figs 2(a&b) (which are for UOI schemes) there are a number of places where substantial cross isobaric flows are seen over a number of regions around 10°N, 65°E and 5°N, 90°E. Compared to this, the flows in the case of NVOA scheme are smoother being almost parallel to the isobars, Figs 3(a & b). This suggests that after the variational technique has been applied to the UOI analyses, the height and wind fields are in near balance. Although analyses of 7 July are shown we have found on examination that the analyses for other levels and days also show similar features, *i.e.*, the NVOA are better balanced than those of UOI analyses. The depression which centred over the Head Bay of Bengal (20°N, 88°E) on 7 July 1979 and which is seen in input wind field (Fig. 2) is well reflected in the NVOA analysis (Fig. 3).



Figs. 2 (a & b). Univariate wind analysis of (a) 850 hPa and (b) 200 hPa levels at 1200 UTC of 7 July 1979 plotted over univariate height analysis

Further in order to examine the new analyses (NVOA) with the FGGE analyses, we have plotted together the wind analyses and the corresponding height analyses from FGGE for 850 and 200 hPa levels. We found that in this case also wind flows are almost parallel to the isobars **Figs.** 4(a&b). This again suggests that the new analyses (NVOA) are comparable to FGGE analyses.

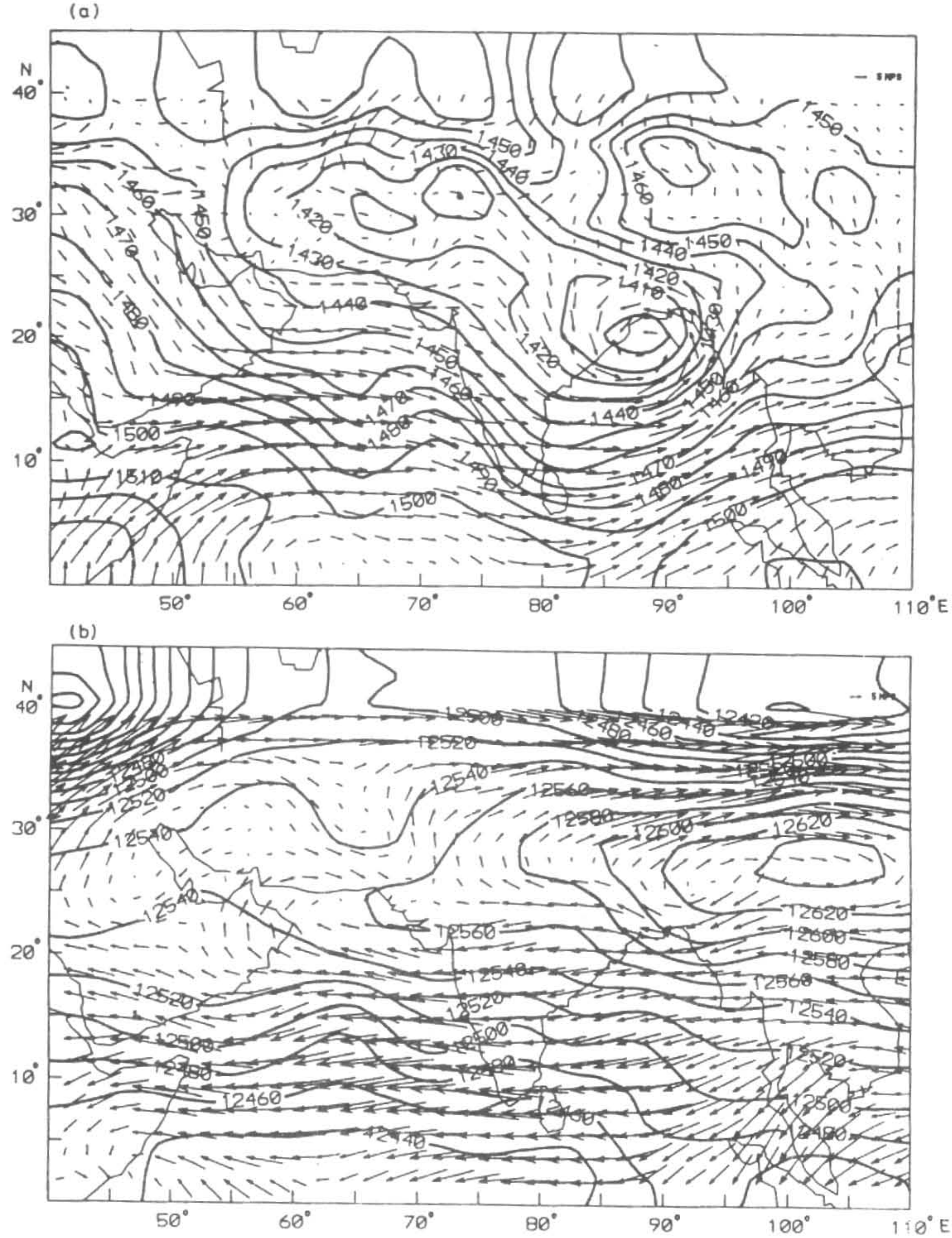
The NVOA analyses which are subjected to decoupling show large amount of cross isobaric flow especially near the equator. However, the centres of the system from wind analyses agree with the centres of the height analyses. In the case of univariate analyses the centres of the wind analyses are slightly to the west of that of height analyses. The cross isobaric flow is definitely required,



Figs. 3 (a & b). NVOA of wind field of (a) 850 hPa and (b) 200 hPa levels at 1200 UTC of 7 July 1979 plotted over NVOA of height field

otherwise, the energy conversion and the generation of kinetic energy will not take place or at least it will be minimum. Hence it is necessary to allow the cross isobaric flow to some reasonable extent. In view of the above, the analyses of NVOA with decoupling factor can be adjudged to be more realistic. A detailed study should be undertaken as to how much decoupling to be allowed for Indian region.

For quantitative evaluation, root mean square (RMS) error is calculated by comparing the NVOA fields with the FGGE analyses and is given in the Table 1 for different levels on different days. The plots between RMS errors and levels for different days are shown in Figs. 5(a-c). It could be inferred that the analyses become more balanced after the application of NVOA technique.



Figs. 4 (a & b). FGGE wind field of (a) 850 hPa and (b) 200 hPa levels at 1200 UTC of 7 July 1979 plotted over FGGE height

6. Conclusion

The analysis from UOI scheme is found to have cross isobaric flow over many regions. When variational technique is applied, the cross isobaric flow is smoothed out. However, in order to have more realistic cross isobaric flow permitted in the analysis, one should use the modified NVOA, *viz.*, decoupling of this constraint near equator as we have carried out here. Further work is needed to specify how much decoupling would be required.

TABLE 1

Root mean square errors for height (m) and wind (m/s) fields compared with FGGE analysis for NVOA

Levels (hPa)	Height of wind fields	Date (July 1979)				
		4	5	6	7	8
850	*z*	17.5	16.3	11.0	12.3	17.4
	u	3.3	3.3	3.0	3.1	3.2
	v	2.9	2.3	2.3	2.7	3.3
700	*z*	11.4	14.8	9.2	9.2	15.6
	u	3.6	3.3	3.0	2.7	2.9
	v	3.2	2.6	2.8	2.6	3.4
500	*z*	15.9	18.9	12.8	15.2	20.7
	u	3.1	3.8	3.5	3.7	3.6
	v	3.1	4.1	4.3	4.0	4.3
300	*z*	25.3	18.1	25.2	19.5	28.5
	u	4.8	5.5	5.3	4.5	5.2
	v	4.5	4.7	5.0	4.1	4.8
200	*z*	24.6	26.0	27.6	27.7	32.1
	u	7.5	6.2	5.9	5.6	6.8
	v	5.5	4.9	4.6	4.8	4.8

APPENDIX A

In regard to variational optimization of meteorological parameters the functional constraints are considered by Sasaki (1958) as quasi-geostrophic and thermal wind equations. Let ϕ_0, u_0, v_0 and T_0 denote the observed geopotential height, wind components and temperature and ϕ, u, v, and T be the modified values. Then in x, y, p coordinate system following Sasaki (1958) :

$$u=-\frac{1}{f}\frac{\partial\phi}{\partial y},\ v=\frac{1}{f}\frac{\partial\phi}{\partial x} \qquad \text{(A1)}$$

$$\frac{\partial u}{\partial p}=\frac{R}{pf}\frac{\partial T}{\partial y},\ \frac{\partial v}{\partial p}=-\frac{R}{pf}\frac{\partial T}{\partial x} \qquad \text{(A2)}$$

where, f represents the coriolis parameter, R is the gas constant and the hydrostatic equilibrium relation is,

$$\frac{RT}{p}=-\frac{\partial\phi}{\partial p}. \qquad \text{(A3)}$$

Let us introduce a new variable p^* such that

$$p^*=-R\ln\left(\frac{p}{\mathbf{P}}\right)$$

P is the pressure at some reference level. Equations (A1), (A2) and (A3) then can be written as

$$u=-\frac{1}{f}\frac{\partial\phi}{\partial y},\ v=\frac{1}{f}\frac{\partial\phi}{\partial x} \qquad \text{(A4)}$$

$$\frac{\partial u}{\partial p^*}=-\frac{1}{f}\frac{\partial T}{\partial y},\ \frac{\partial v}{\partial p^*}=\frac{1}{f}\frac{\partial T}{\partial x} \qquad \text{(A5)}$$

$$T=\frac{\partial\phi}{\partial p^*}. \qquad \text{(A6)}$$

According to Sasaki (1958), the observed values need not satisfy the Eqns (A4), (A5) and (A6). Let us now define the deviation or difference between observed and modified values by the primed quantities

$$u'=u-u_0$$
$$v'=v-v_0$$
$$\phi'=\phi-\phi_0$$
$$T'=T-T_0 \qquad \text{(A7)}$$

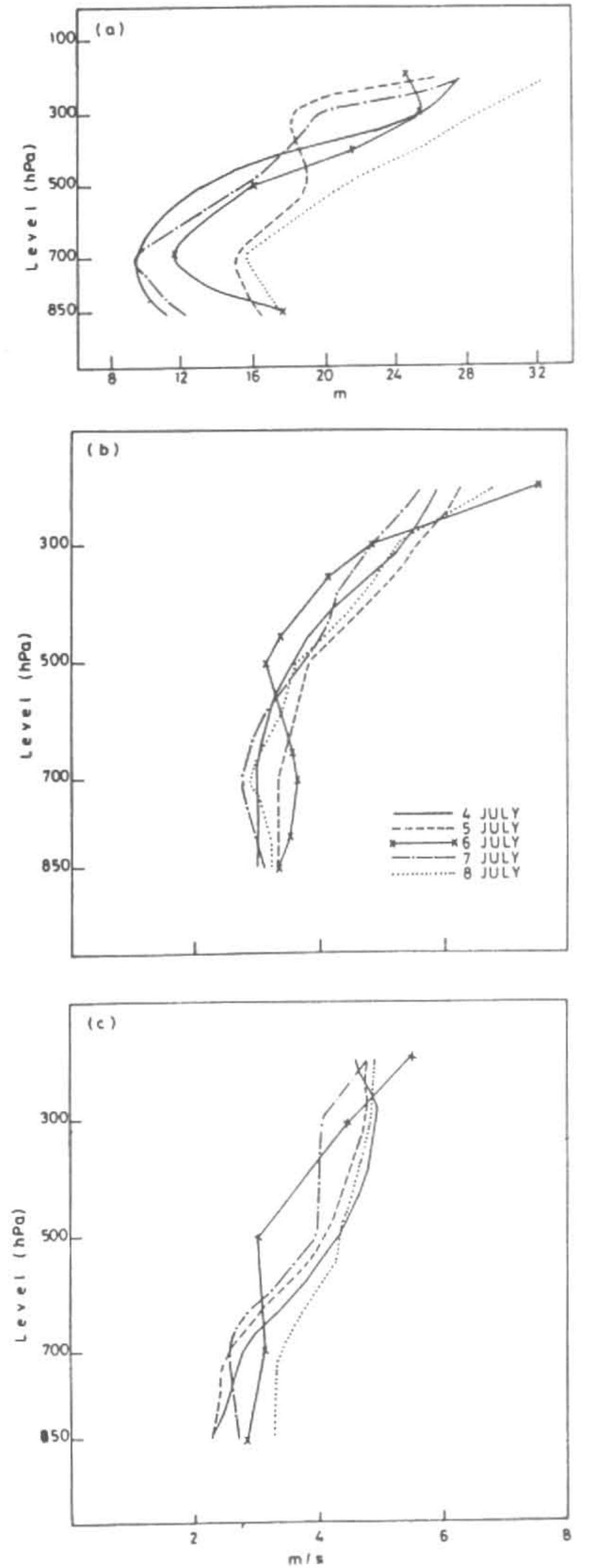
substituting (A7) in Eqns (A4), (A5) and (A6) we get

$$u'=-\frac{1}{f}\frac{\partial\phi'}{\partial y}-u_0-\frac{1}{f}\frac{\partial\phi_0}{\partial y} \qquad \text{(A8)}$$

Similarly,

$$v'=\frac{1}{f}\frac{\partial\phi'}{\partial x}-v_0+\frac{1}{f}\frac{\partial\phi_0}{\partial x} \qquad \text{(A9)}$$

$$T'=\frac{\partial\phi'}{\partial p^*}-T_0+\frac{\partial\phi_0}{\partial p^*} \qquad \text{(A10)}$$



Figs. 5 (a-c). Root mean square errors of (a) height field, (b) zonal component of the wind and (c) meridional component of the wind at different levels

Let us now express the sum of the squares of these deviations as

$$\xi^2=\alpha^2 u'^2+\alpha^2 v'^2+\beta^2\phi'^2+\gamma^2 T'^2 \quad \text{(A11)}$$

where α, β, γ are weighting factors.

The volume integral over the volume V in (x, y, p^*) space can be written as follows:

$$I=\iiint_V \xi^2 \mathrm{d}x\mathrm{d}y\mathrm{d}p^* \quad \text{(A12)}$$

i.e.,

$$I=\iiint_V\left[\alpha^2\left(\frac{1}{f}\frac{\partial\phi'}{\partial y}+u_0+\frac{1}{f}\frac{\partial\phi_0}{\partial y}\right)^2+\alpha^2\left(\frac{1}{f}\frac{\partial\phi'}{\partial x}-v_0+\frac{1}{f}\frac{\partial\phi_0}{\partial x}\right)^2+\beta^2\phi'^2+\gamma^2\left(\frac{\partial\phi'}{\partial p^*}-T_0+\frac{\partial\phi_0}{\partial p^*}\right)^2\right]\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}p^* \quad \text{(A13)}$$

The problem is now to determine the modified values of the parameters, *viz.*, ϕ, u, v and T objectively. For this we require I to be minimum. Based on the techniques of calculus of variations,

$$\delta I=0 \quad \text{(A14)}$$

where, δ is the variational operator.

Let the functional relation F be given by

$$F(x,y,p^*,\phi',q,r,s)=\left[\alpha^2\left(\frac{1}{f}\frac{\partial\phi'}{\partial y}+u_0+\frac{1}{f}\frac{\partial\phi_0}{\partial y}\right)^2+\alpha^2\left(\frac{1}{f}\frac{\partial\phi'}{\partial x}-v_0+\frac{1}{f}\frac{\partial\phi_0}{\partial x}\right)^2+\beta^2\phi'^2+\gamma^2\left(\frac{\partial\phi'}{\partial p^*}-T_0+\frac{\partial\phi_0}{\partial p^*}\right)^2\right] \quad \text{(A15)}$$

where,

$$q=\frac{\partial\phi'}{\partial x},\quad r=\frac{\partial\phi'}{\partial y},\quad s=\frac{\partial\phi'}{\partial p^*}.$$

The Euler-Lagrange equation following Daley (1991) is given by,

$$\frac{\partial^2 F}{\partial q\,\partial x}+\frac{\partial^2 F}{\partial r\,\partial y}+\frac{\partial^2 F}{\partial s\,\partial p^*}-\frac{\partial F}{\partial\phi'}=0. \quad \text{(A16)}$$

Thus, from Eqns (A15) and (A16), we have

$$\alpha^2\left(\frac{1}{f^2}\frac{\partial^2\phi'}{\partial x^2}-\frac{1}{f}\frac{\partial v_0}{\partial x}+\frac{1}{f^2}\frac{\partial^2\phi_0}{\partial x^2}+\frac{1}{f^2}\frac{\partial^2\phi'}{\partial y^2}+\frac{1}{f}\frac{\partial u_0}{\partial y}+\frac{1}{f^2}\frac{\partial^2\phi_0}{\partial y^2}\right)$$

$$+\gamma^2\left(\frac{\partial^2\phi'}{\partial p^{*2}}+\frac{\partial^2\phi_0}{\partial p^{*2}}-\frac{\partial T_0}{\partial p^*}\right)-\beta^2\phi'=0$$

or,

$$\left[\frac{\partial^2\phi'}{\partial x^2}+\frac{\partial^2\phi'}{\partial y^2}+\left(\frac{\gamma}{\alpha}f\right)^2\frac{\partial^2\phi'}{\partial p^{*2}}\right]-\left(\frac{\beta}{\alpha}f\right)^2\phi'$$

$$=f\left(\frac{\partial v_0}{\partial x}-\frac{\partial u_0}{\partial y}\right)+\left(\frac{\gamma}{\alpha}f\right)^2\frac{\partial T_0}{\partial p^*}$$

$$-\left[\frac{\partial^2\phi_0}{\partial x^2}+\frac{\partial^2\phi_0}{\partial y^2}+\left(\frac{\gamma}{\alpha}f\right)^2\frac{\partial^2\phi_0}{\partial p^{*2}}\right]$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\gamma f}{\alpha}\right)^2\frac{\partial^2}{\partial p^{*2}}\right]\phi'-\left(\frac{\beta f}{\alpha}\right)^2\phi'$$

$$=f\zeta_0+\left(\frac{\gamma f}{\alpha}\right)^2\frac{\partial T_0}{\partial p^*}$$

$$-\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\gamma f}{\alpha}\right)^2\frac{\partial^2}{\partial p^{*2}}\right]\phi_0$$

In short this may be expressed as

$$\nabla^2\phi'-\left(\frac{\beta f}{\alpha}\right)^2\phi'=f\zeta_0+\left(\frac{\gamma f}{\alpha}\right)^2\frac{\partial T_0}{\partial p^*}-\nabla^2\phi_0 \qquad \text{(A17)}$$

where,

$$\zeta_0=\frac{\partial v_0}{\partial x}-\frac{\partial u_0}{\partial y} \qquad \text{(A18)}$$

is the relative vorticity of the observed wind and the three dimensional Laplacian operator is given by

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\gamma}{\alpha}f\right)^2\frac{\partial^2}{\partial p^{*2}}. \qquad \text{(A19)}$$

For two dimensional case, Eqn. (A17) reduces to

$$\nabla^2\phi'-\left(\frac{\beta}{\alpha}f\right)^2\phi'=f\zeta_0-\nabla^2\phi_0 \qquad \text{(A20)}$$

where,

$$\zeta_0=\frac{\partial v_0}{\partial x}-\frac{\partial u_0}{\partial y} \qquad \text{(A21)}$$

and

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \qquad \text{(A22)}$$

Solution of Eqn. (A20) gives the value of ϕ' which is the deviation of geopotential height. The modified value (analysed value) ϕ, is obtained from Eqn. (A7). This modified value may be used in numerical prediction techniques based on quasi-geostrophic assumption. For practical use Eqn. (A20) is expressed in the form of finite differences. For a grid size h, Eqn. (A20) is approximated as follows :

$$\left(\phi'_{i+h,j}+\phi'_{i-h,j}+\phi'_{i,j+h}+\phi'_{i,j-h}\right)$$

$$-\left[4+\left(\frac{\beta}{\alpha}fh\right)^2\right]\phi'_{i,j}=fh^2\zeta_0$$

$$-\left[\phi_{0(i+h,j)}+\phi_{0(i-h,j)}+\phi_{0(i,j+h)}+\phi_{0(i,j-h)}-4\phi_{0(i,j)}\right]. \qquad \text{(A23)}$$

Following Sasaki (1958) let us now define e, the parameter of accuracy

$$e=\frac{1}{1+\left(\frac{\beta}{\alpha}f\right)^2\frac{L^2}{4\pi^2}} \qquad \text{(A24)}$$

where, L is the wave length.

In terms of parameter of accuracy (e), the weighting function $W=\left(\frac{\beta}{\alpha}fh\right)^2$ can be expressed as

$$W=\left(\frac{\beta}{\alpha}fh\right)^2$$

$$W=\left(\frac{1-e}{e}\right)\left(\frac{2\pi h}{L}\right)^2 \qquad \text{(A25)}$$

which shows the weighting factor W may be determined by the parameter of accuracy e and the ratio between the grid size and the wave length. Rewriting Eqn. (A23) using Eqn. (A25), we get Eqn. (1).

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