MAUSAM, 49, 4 (October 1998), 439-442

551.515

Radius of influence of a vortex

O.P. SINGH*

Civil Aviation Training College, Allahabad, India

(Received 16 May 1996, Modified 16 May 1997)

सार — आवर्त ट्यूब में तथा उसके आस पास के दाब प्रोफाइल का उपयोग करते हुए आवर्त के प्रभाव की दूरी के लिए सरल सम्बंध का पता लगाया गया है। टॉरनेडो से हुए अधिकतम विनाश के क्षेत्र के मूल्यांकन के लिए इन परिणामों का अनुप्रयोग किया गया है। विश्लेषण से यह पता चलता है कि उष्णकटिबंधीय चक्रवात में चक्रवात के केन्द्र के आस पास के क्षेत्र की घटना एक द्रवगतिकीय परिघटना है।

ABSTRACT. Using the pressure profile in and around a vortex tube a simple relation for the radius of influence of the vortex has been derived. The results have been applied to assess the region of maximum devastation in a tornado. The analysis reveals that the occurrence of 'eye wall' region in a tropical cyclone is a hydrodynamical phenomenon.

Key words—Vortex, Spin velocity, Tornado, Eye wall.

1. Introduction

With certain modifications many concepts of hydrodynamical vortices could be applied to explain the observed behaviour of atmospheric vortices. Needless to say, the main factor that separates an atmospheric vortex from a hydrodynamical vortex is the earth's rotation. Due to this the meteorological vortices show some distinct characteristics. Yet several features of atmospheric vortices are derivable from the characteristics of hydrodynamical vortices. There are many non-disjoint features where Coriolis force makes little difference. One such example is the similarity between small scale atmospheric vortices (like tornadoes, dust devils etc.) and the hydrodynamical vortices. Up to a considerable distance from the centre, even a synoptic scale atmospheric system like tropical cyclone has several features of a hydrodynamical vortex.

Brand (1970) has applied certain characteristics of Rankine vortices to study the mutual interaction of tropical cyclones. In a recent study (under publication) the author has applied the cavitation phenomenon observed in hydrodynamical vortices to explain the occurrence of the 'eye' region in a tropical cyclone.

It is well known that a vortex affects the fluid beyond its boundary. The velocity and pressure distributions outside the vortex are determined by the spin and the size of the vortex. In the present study it has been established that the influence of a vortex extends upto a distance $\sqrt{2}\ R$ from the centre as far as pressure is concerned ('R' being the radius of the vortex). The pressure profile from the vortex centre to the distance $\sqrt{2}\ R$ shows that the pressure gradient between the region R to $\sqrt{2}\ R$ is the steepest. Thus, the observed structure of tropical cyclones which shows the existence of an 'eye wall' region having steepest pressure gradient and most fierce weather (Riehl 1954), conforms to the above hydrodynamical phenomenon.

2. Results and discussion

Consider a circular vortex tube of radius 'R' in an incompressible fluid with its axis of rotation perpendicular to xy - plane. The motion is two dimensional and the vorticity vector '$\vec{\zeta}$', is directed along z-axis, its value being constant within the tube and zero outside. From Stokes' theorem the circulation surrounding the tube is given by:

$$C = \int \vec{\mathbf{V}} . \vec{\mathbf{dr}} = \iint \text{curl } \vec{\mathbf{V}} . \vec{\mathbf{ds}} = \pi R^2 \zeta \qquad (1)$$

where, '$\vec{\mathbf{V}}$' is the tangential velocity.

There are two types of motion; spin motion inside the vortex tube and the induced motion outside the vortex tube which is irrotational. It is easy to verify that

$$\zeta = 2\,\omega \qquad (2)$$

**Present affiliation*: Meteorological Centre, Bhubaneswar, India.

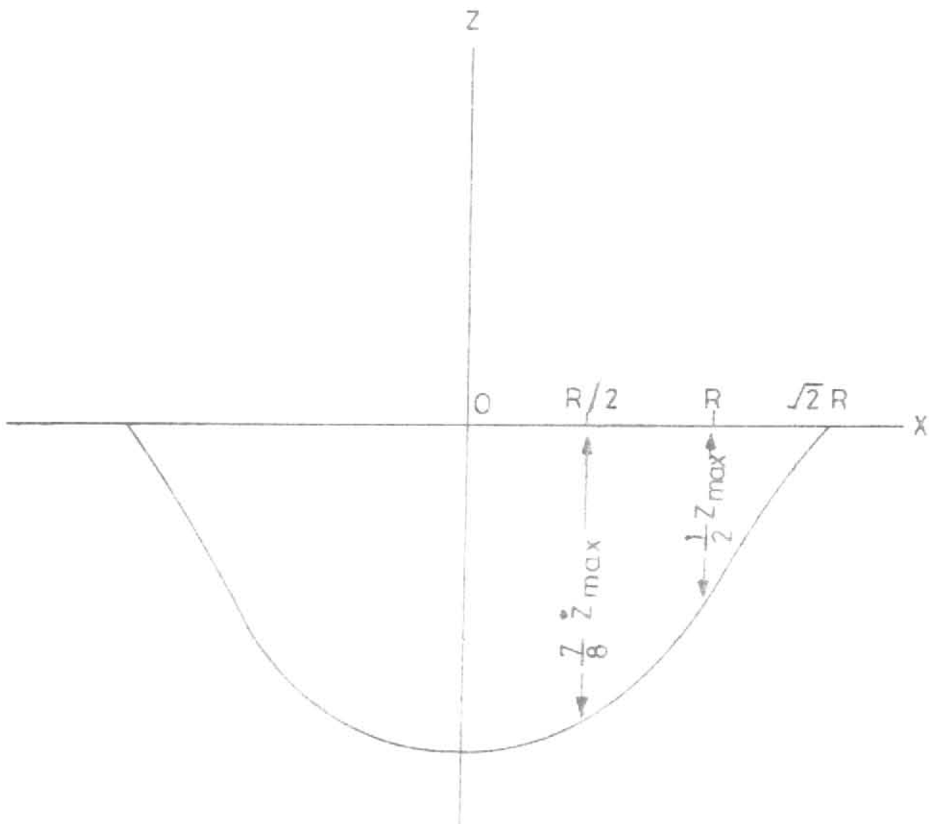

Fig.1. Pressure profile in and around a vortex tube

where, 'ω' is constant spin velocity within the vortex tube. Applying Eqn. (1) to a point inside the vortex located at a distance 'r' from the centre ($r < R$) we get :

$$2\pi r v = \pi r^2 \zeta = 2\pi r^2 \omega \quad (3)$$

or $v = \omega r$

Similarly, for a point outside the vortex ($r > R$) we have

$$v = \omega R^2/r \quad (4)$$

Thus, the existence of a finite vortex in a fluid implies the existence of a velocity field outside it. Eqns.(3) and (4) show that the velocity field is continuous at the boundary of the vortex.

2.1. *Pressure profile*

Due to rotation, each fluid element inside the vortex experiences a centripetal force which constitutes a pressure as follows:

$$dp = (\rho \, dr)\, \omega^2 r \quad (5)$$

On integration we get,

$$\frac{p}{\rho} = \frac{1}{2}\omega^2 r^2 + \frac{p_0}{\rho} \quad (6)$$

where 'p' is the pressure at a distance r ($<R$) from the centre, 'ρ' is the fluid density and 'p_0' is the central pressure. Similarly, the pressure equation for the induced motion outside the vortex is given by (Sharma and Sachdeva 1973)

$$\frac{p}{\rho} = \frac{P}{\rho} - \frac{1}{2}\frac{\omega^2 R^4}{r^2} \quad (7)$$

where 'P' is the pressure at infinity. 'p_0' could be eliminated in Eqn.(6) by considering the continuity of pressure at the boundary of the vortex (at $r = R$) and Eqn.(6) could be written as:

$$\frac{p}{\rho} = \frac{1}{2}\omega^2 r^2 + \left(\frac{P}{\rho} - \omega^2 R^2\right) \quad (8)$$

If the vortex is moving irrotationally under the action of gravity then the potential 'gz' could be introduced in Eqns.(7) and (8) as an external forcing as follows:

$$\frac{p}{\rho} = \frac{P}{\rho} - \frac{1}{2}\ \frac{\omega^2 R^4}{r^2} - gz \quad \text{where } r > R \qquad (9)$$

and

$$\frac{p}{\rho} = \frac{1}{2}\ \omega^2 r^2 + \left(\frac{P}{\rho} - \omega^2 R^2\right) - gz \quad \text{where } r < R \quad (10)$$

where, 'g' is acceleration due to gravity and 'z' is the height. From Eqns.(9) and (10) we can write the equations of free surfaces (isobars) by putting p = constant, and by taking the origin at usual level of isobars (*i.e.*, $z = 0$ when $r \to \infty$). Thus the depth of depression (dimple) of isobars could be written as:

$$z = \frac{1}{2}\ \frac{\omega^2 r^2}{g} - \frac{\omega^2 R^2}{g} \qquad (11)$$

Eqn.(11) describes the pressure profile in and around the vortex (in xz-plane). At the centre of the vortex, i.e., at r=0 we get from Eqn.(11)

$$z = -\frac{\omega^2 R^2}{g} \qquad (12)$$

which is the maximum depression of isobars z_{max}. At the boundary of the vortex, *i.e.*, $r = R$ we have

$$-z = \frac{1}{2}\ \frac{\omega^2 R^2}{g} = -\frac{1}{2}\ z_{max} \qquad (13)$$

The depth of depression of isobars vanishes at $r = \sqrt{2}\,R$, which is obtained by putting $z = 0$ in Eqn.(11). The pressure profile can now be constructed, which is a parabola (Fig.1).

The radius of influence of the vortex, R_i is given by (Fig.1),

$$Ri = \sqrt{2}\,R \qquad (14)$$

It is evident from Fig.1 that the pressure gradient is steepest just beyond the boundary of the vortex (between the region R to $\sqrt{2}\,R$).

The pressure gradient becomes very slack towards the central region. In fact depth of depression of isobars varies from $1/2z_{max}$ to $7/8z_{max}$ from R to 1/2R towards the centre whereas it varies from $7/8z_{max}$ to z_{max} from 1/2R to the vortex centre. Thus, the pressure gradient remains very flat in the immediate vicinity of the vortex centre (upto the distance R/2 from the centre).

The pressure gradient in the region 1/2R to R is three times stronger than that observed in the region from the vortex centre to 1/2R. The pressure gradient in the region from R to $\sqrt{2}\,R$ is 4/3 times stronger than that in the region from 1/2R to R and about 4 times stronger than that in the region from vortex centre to 1/2R. This type of pressure gradient profile gives valuable insight into the structure of a tropical cyclone.

2.2. *Applications to atmospheric systems*

2.2.1. *Tornadoes*

Due to their small scale the tornadoes come very close to the vortices described in the foregoing section. Thus, the maximum winds would occur just at the boundary of a tornado from Eqns.(3) & (4). The steepest pressure gradient would occur just after the boundary of the tornado.

2.2.2. *Tropical cyclones*

As mentioned earlier the pressure profile in an around a hydrodynamical vortex provides a theoretical background for the observed structure of a tropical cyclone in the vicinity of its centre. If we consider the fields of velocity and pressure close to the cyclone centre, then these variables have similar features as observed in small scale atmospheric vortices, like tornadoes and dust devils. The flow in this part of a tropical cyclone is cyclostrophic due to the negligible Coriolis force as compared to the pressure gradient and the centrifugal forces. Now, if we apply the pressure gradient distribution of section 2.1 to a tropical cyclone it would imply that there is a central region in a cyclone where pressure is minimum, but the pressure gradient is flat and hence the winds are gentle or calm. This region extends approximately upto a distance of about 1/2R on either side of the centre. This appears to be the 'eye' of the cyclone as observed in actual structure of tropical cyclones. It may be pointed out that here'R' is the radius of maximum winds. Thus, theoretical results show that the 'eye' radius in a tropical cyclone is about half of the radius of maximum winds.

The ring 1/2R to $\sqrt{2}\,R$ would probably qualify for the 'eye wall' region as observed in tropical cyclones. The theoretical 'eye wall' region, 1/2 R to $\sqrt{2}\,R$ to satisfies all the properties of velocity and pressure fields that are associated with the actual 'eye wall' region, namely strongest winds and the steepest pressure gradient. The theoretical results clearly bring out a region around the vortex with the characteristics of the 'eye wall' region of a tropical cyclone. Another application of the theoretical results presented here would be to deduce that in a tropical cyclone the flow is approximately cyclostrophic upto the distance $\sqrt{2}\,R$ from the cyclone centre whereas it is gradient beyond $\sqrt{2}\,R$.

3. Conclusions

The following deductions could be made from the theoretical results of the study:

(*i*) The theory predicts the existence of an 'eye wall' region in a tropical cyclone which agrees well with the observed structure of tropical cyclones. That the occurrence of an 'eye wall' region in a tropical cyclone is a hydrodynamical phenomenon is proved beyond doubt.

(*ii*) The theoretical results show that 'eye' radius in a tropical cyclone is approximately half of the radius of maximum winds, *i.e.*,

$$R_0 \approx 1/2\, R$$

where R_0 and R are radii of the 'eye' and maximum winds respectively.

(*iii*) The theory predicts the existence of a super 'eye wall' (R to $\sqrt{2}\,R$) within the 'eye wall' ($1/2R$ to $\sqrt{2}\,R$). Thus, the most fierce weather would occur in the region R to $\sqrt{2}\,R$ in a tropical cyclone. Similarly, in tornadoes maximum devastation would occur in the region R to $\sqrt{2}\,R$, It is difficult to verify this from the observational evidence at present.

(*iv*) In a tropical cyclone the flow appears to be approximately cyclostrophic upto the distance $\sqrt{2}\,R$ from the centre and beyond $\sqrt{2}\,R$ it is gradient. This is in accordance with the well known assumption which is probably based on some observational evidence.

References

Brand, S., 1970, "Interaction of binary tropical cyclones of the western north Pacific ocean", *J. Appl. Meteor.*, **9**, 3, 433-411.

Riehl, H., 1954, "Tropical storms", *Tropical Meteorology*.

Sharma, B. and Sachdeva, B.K., 1973, "Vortex motion", *Elementary Hydrodynamics*.