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Temporal variation of rainfall intensity from convective cloud — A theoretical approach

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सार — संवहनी मेघों से वर्षा के दौरान वर्षा की तीव्रता में परिवंतन हो जाता है। इस शोधपत्र में अंतिम वेग v, बूंद का व्यास D और बूंद के आकार का वितरण N_D के रूप में वर्षा की तीव्रता के कालिक परिवर्तन के लिये गणितीय व्यंजकों का निगमन किया गया है और गणितीय रूप से वर्षा की तीव्रता की कालिक विभिन्नता के विभिन्न प्रकारों के लिये स्थिति उत्पन्न करने की और विभिन्न व्यंजकों की भौतिक सार्थकता को स्पष्ट करने की कोशिश की गई है। इससे पता चलता है कि वर्षण के दौरान उद्‌वाह की उपस्थिति या अनुपस्थिति धरती पर पहुंचने का एक प्रमुख नियंत्रण कारक है। इससे यह भी पता चलता है कि बूंद के आकार का वितरण भी महत्वपूर्ण भूमिका निभाता है।

ABSTRACT. The rainfall intensity from convective clouds vary during the period of rainfall. In the present paper a mathematical expression has been deduced for the temporal variation of rainfall intensity in terms of terminal velocity v, drop diameter D and drop size distribution N_D and an effort has made to lay down the condition for various types of temporal variation of rainfall intensity mathematically and the physical significance of various expressions were tried to explain. It is shown that the presence or absence of updraft during precipitation reaching ground is an important controlling factor. It has also been shown that the drop size distribution also plays a significant role.

1. Introduction

Rainfall from convective clouds are common during pre-monsoon months over northeast India. Large amount of precipitation occurs in a short spell with a high rate of rainfall. Although the rainfall duration from these convective clouds are brief the intensity of rainfall varies during the period. Considerable work has been done on the rate of precipitation. Kelkar (1959) gave drop size distribution with respect to rainfall intensity. Sivaramakrishnan (1961) related the rainfall rate R' with various other parameters like liquid water content (LWC), median volume diameter, radar reflectivity. The temporal and spatial variation of the rate of precipitation from thunderclouds can be explained, to a significant extent, according to Battan (1981), by the spatial and temporal variations of the vertical air motion. Bennetts and Bader (1981) described the enhancement of surface rainfall as the effect of merging two interacting cloud cells under certain conditions.

The author (Chakrabarty 1985) has done a study on the variation of rainfall intensity from convective clouds during pre-monsoon season over Gangetic West Bengal using 10 years data over Alipore and Dum Dum and has shown that the temporal variation of rainfall can be categorised into four types, *viz.,*

Type I — Rainfall intensity remains constant during the entire period of rainfall.

Type II — Rainfall intensity is maximum in the beginning and then gradually decreases.

Type III — Rainfall intensity is highest during the mid-portion of the total rainfall.

Type IV — Rainfall intensity is maximum in the beginning and reaches to minimum during mid-portion and again increases.

It was shown that majority of the cases fall under types II and III. It was shown that the type II cases resemble curve $y=a/\log x$ and type III cases resemble that of $y=ax^3 \ e^{-x}$.

In the present paper a mathematical expression has been deduced for the temporal variation of rainfall in terms of terminal velocity v, drop diameter D and drop size distribution N_D. The expression was analysed and mathematical conditions are laid down for various types of temporal variation of rainfall intensity.

2. Expression for temporal variation of rainfall intensity

The rainfall intensity is given by :

$$R' = 36\, qv \times 10^{-4} \ \text{cm hr}^{-1} \qquad (1)$$

where, q is the liquid water content (LWC) and v is the terminal velocity (Sivaramakrishnan and Selvam1967).

The liquid water content q is given by :

$$q = \frac{\pi}{6} \Sigma N_D D^3 \delta D \qquad (2)$$

where, N_D being the number of droplets in a volume of $1 m^3$ in the diameter range D and $D+\delta D$.

Thus, combining Eqns. (1) and (2) the rainfall intensity can be written as :

$$R' = 6\pi \times 10^{-4} \Sigma v N_D D^3 \delta D \qquad (3)$$

The above expression indicates that the rainfall intensity depends on both the drop size distribution and the terminal velocity of the rain drop.

Now, the rate of change of rainfall intensity with time may be given by :

$$R'' = \frac{dR'}{dt} = \frac{d}{dt}(Kqv)$$

where, K is constant depending on units.

$$\text{or} \quad R'' = K\left[v\frac{dq}{dt} + q\frac{dv}{dt}\right]$$

Now as both the liquid water content q and the terminal velocity v are dependent on drop diameter we may write :

$$\frac{dq}{dt} = \frac{dq}{dD} \times \frac{dD}{dt}$$

and

$$\frac{dv}{dt} = \frac{dv}{dD} \times \frac{dD}{dt}$$

Thus,

$$R'' = K\left[v\frac{dq}{dD} + q\frac{dv}{dD}\right]\frac{dD}{dt} \qquad (4)$$

Putting expression for q, Eqn. (4) can be expanded as :

$$R'' = K\sum\left[vD^2\left\{3N_D + D\frac{dN_D}{dD}\right\} + + N_D D^3\frac{dv}{dD}\right]\frac{dD}{dt}\delta D \qquad (5)$$

Thus, the temporal variation of rainfall intensity is a complicated function of various parameters.

To understand step by step let us go back to expression (4) for R'' :

$$R'' = K\left[v\frac{dq}{dD} + q\frac{dv}{dD}\right]\frac{dD}{dt}$$

Now, terminal velocity always increases with drop diameter, thus $(dv/dD) > 0$ always.

Thus, the sign of R'' will be positive or negative depending on the sign of dD/dt and sign and magnitude of dq/dD.

(1) If the drop diameter increases with time, *i.e.*, $dD/dt > 0$ then

(a) If $\frac{dq}{dD} > 0$ then $R'' > 0$

Therefore, R' increase with time.

TABLE 1

dD/dt	dq/dD	R' with time
(1) $\frac{dD}{dt} > 0$	(a) $\frac{dq}{dD} > 0$	Increases
	(b) $\frac{dq}{dD} < 0$ and $\left\lvert\frac{vdq}{dD}\right\rvert < \left\lvert\frac{qdv}{dD}\right\rvert$	Do.
	(c) $\frac{dq}{dD} < 0$ but $\left\lvert\frac{vdq}{dD}\right\rvert > \left\lvert\frac{qdv}{dD}\right\rvert$	Decreases
(2) $\frac{dD}{dt} < 0$	(a) $\frac{dq}{dD} > 0$	Do.
	(b) $\frac{dq}{dD} < 0$ but $\left\lvert\frac{vdq}{dD}\right\rvert < \left\lvert\frac{qdv}{dD}\right\rvert$	Do.
	(c) $\frac{dq}{dD} < 0$ but $\left\lvert\frac{vdq}{dD}\right\rvert > \left\lvert\frac{qdv}{dD}\right\rvert$	Increases

(b) if $\frac{dq}{dD} < 0$ but $\left\lvert\frac{vdq}{dD}\right\rvert < \left\lvert\frac{qdv}{dD}\right\rvert$

$R'' > 0$ so R' increases with time.

(c) If $\frac{dq}{dD} < 0$ and $\left\lvert\frac{vdq}{dD}\right\rvert > \left\lvert\frac{qdv}{dD}\right\rvert$

$R'' < 0$ so R' decreases with time.

(2) If the drop diameter decreases in time, *i.e.*, $\frac{dD}{dt} < 0$ then

(a) If $\frac{dq}{dD} > 0$, $R''<0$ and so R' decreases with time.

(b) If $\frac{dq}{dD} < 0$ but $\left\lvert\frac{vdq}{dD}\right\rvert < \left\lvert\frac{qdv}{dD}\right\rvert$

$R'' <0$ so R' decreases with time.

(c) If $\frac{dq}{dD} < 0$ also $\left\lvert\frac{vdq}{dD}\right\rvert > \left\lvert\frac{qdv}{dD}\right\rvert$

$R'' > 0$ so R' increases with time.

Table 1 gives a consolidated picture of the above six inferences.

3. Discussion

Let us now try to understand the physical significance of each equation.

At first the condition (1) states $dD/dt>0$. This means that the drop diameter is increasing with respect to time. Obviously this is the condition within the cloud mass and is possible only in the presence of updraft. Thus, $dD/dt>0$ means the presence of updraft. On the other hand if there is no updraft present in the cloud mass then the drop diameter will decrease with reference to time or $dD/dt < 0$ which is stated in Eqn. (2).

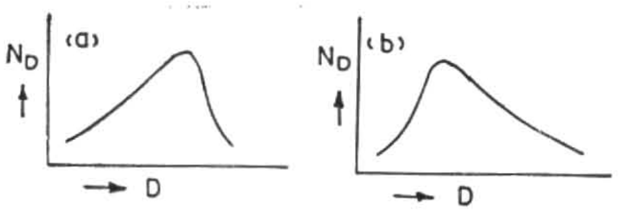

Fig. 1 (a & b)

Now let us try to understand the meaning of 1(a), *i.e.*, $dq/dD > 0$

Now,

$$q = \frac{\pi}{6} \sum N_D D^3 \delta D$$

$$\frac{dq}{dD} = \frac{\pi}{6} \sum \left[3N_D D^2 \delta D + \frac{dN_D}{dD} D^3 \delta D \right] \quad (6)$$

Now dq/dD will be positive only when $\Sigma[(dN_D/dD)\,\delta D] > 0$, *i.e.*, the drop diameter distribution is such that $\Sigma[(dN_D/dD)\,\delta D]$ should be positive. This is only possible when the drop diameter distribution is like that shown in Fig. 1(a). Thus, if the drop diameter distribution is such that the skewness is towards higher diameter then $\Sigma[(dN_D/dD)\,\delta D]$ will be positive and the rate of precipitation will increase with time. Thus, condition 1(a) means that during the presence of updraft if the drop diameter distribution is such that the concentration of the drops are towards higher diameters as shown in Fig. 1(a) then the rate of precipitation will increase with time. Kelkar (1959) showed that for higher precipitation rate the concentration of drop size is towards higher range of diameter, *i.e.*, as in Fig. 1(a).

On the other hand $dD/dt < 0$, as indicated in condition (2), can be attributed to absence of the updraft and under such circumstances the same drop size distribution would lead to decrease in precipitation rate as in condition 2(a).

Thus, we get an important conclusion that the presence or absence of updraft is an important feature for increase or decrease of rate of precipitation.

Battan (1981) in his paper on variable nature of thunderstorm updraft and precipitation stated that the observations of rain and hail falling from thunderstorm show a high degree of variability over time period of the order of minutes. Measurements of the water loading and vertical motion profile by means of techniques having approximately small spatial resolution, of the order of 100 m or less show a high degree of variability over space scales of the order of about a km or less. He argued that the temporal variation of precipitation are related, to a significant degree, to the spatial variations in draft velocities.

Let us now try to understand the condition 1(b). Condition (1) indicates presence of updraft, *i.e.*, $dD/dt > 0$.

The second condition is that $dq/dD < 0$.

From Eqn. (6) we see that the first term on the r.h.s. is always positive so to have $dq/dD < 0$, the second term should be negative and its magnitude should be more than that of the first term.

$$\text{Therefore, } \sum \frac{dN_D}{dD} D^3 \delta D < 0 \quad (7)$$

From Fig. 1(b) it is evident that if the drop size distribution is such that the concentration of drops takes place in the lower diameter range then $\Sigma\,[(dN_D/dD)\, D^3\, \delta D]$ will be negative.

In addition the condition:

$$\left| \sum \frac{dN_D}{dD} D^3 \delta D \right| > 3 \sum N_D D^2 \delta D \quad (8)$$

also should be fulfilled to have $dq/dD < 0$. Condition 1(b) also requires :

$$\left| \frac{vdq}{dD} \right| < \left| \frac{qdv}{dD} \right|$$

Expanding terms for dq/dD and q we can write by taking negative sign for $(dN_D/dD)\, D^3\, \delta D$:

$$v \frac{\pi}{6} \sum D^2 \left[D \frac{dN_D}{dD} \delta D - 3 N_D \delta D \right] > \frac{\pi}{6} \left[\sum N_D D^3 \delta D \right] \frac{dv}{dD}$$

or

$$v \sum D^2 \left[D \frac{dN_D}{dD} D - 3 N_D \delta D \right] - \left[\sum N_D D^3 \delta D \right] \frac{dv}{dD} > 0 \quad (9)$$

The conditions (8) and (9) are for discrete values of drop diameters. Now if we assume that the drop distribution is continuous from zero to a critical value D_c and the drops outside this range are negligible then we can assume that $N_D = f(D)$ and then conditions (8) and (9) can be rewritten as :

$$\left| \int_0^{D_c} D^3 f'(D)\, dD \right| > 3 \left| \int_0^{D_c} f(D)\, D^2\, dD \right| \quad (10)$$

and

$$v \int_0^{D_c} D^2 (D f'(D) - 3 f(D))\, dD - \frac{dv}{dD} \int_0^{D_c} D^3 f(D)\, dD > 0$$

or

$$\int_0^{D_c} \int_0^{D_c} D^2 [D f'(D) - d f(D)]\, dD\, dD - \log v_c \int_0^{D_c} D^3 f(D)\, dD > 0 \quad (11)$$

where v_c is the critical velocity for drop with diameter D_c.

Thus the condition 1(b) is a combination of conditions (10) and (11) in the presence of updraft. Obviously this is quite complicated one and depends very much upon the spectrum of drop diameters.

The conditions for 1(c) can be very easily drawn by just changing the sign of (11), *i.e.*,

$$\log v_c \int_0^{D_c} D^3 f(D)\, dD - \int_0^{D_c}\int_0^{D_c} D^2\, [D\, f'(D) - 3\, f\,(D)]\, dD\, dD > 0 \qquad (12)$$

the other conditions as in (10) and $dD/dt > 0$ remains unchanged.

Let us now try to understand the conditions in (2) of the Table 1. As discussed earlier the condition $dD/dt < 0$ means absence of updraft. The second parts (a), (b) and (c) can be analysed on the same line as discussed earlier in association with those 1(a), (b) and (c). The conditions as stipulated in (11) and (12) are also applicable. However, the result is reversed from those of (1) of Table 1.

Thus, we see that presence or absence of updraft and the drop diameter distribution plays very important role in determining the rate of change of precipitation intensity. It may be concluded that if the drop diameter distribution is like that of Fig. 1(a), *i.e.*, concentration of drops towards higher diameters then the precipitation intensity will increase with reference to time during the presence of updraft and will decrease when the updraft ceases to exist in the cloud mass. On the other hand, if the drop diameter distribution is such that the concentration of drops is towards smaller diameter range as shown in Fig. 1(b) then, the precipitation intensity may increase or decrease in time depending upon the complicated conditions laid down in (11) and (12). Moreover, N_D being a function of D for any type of drop diameter distribution the condition (11) or (12) will be satisfied depending on $f(D)$ and thus, it is quite possible, that the drop diameter distribution may change after a little while of precipitation and the rate of change of rainfall intensity may reverse dependng upon whether condition in (11) or (12) is satisfied. Thus, possibility of change in drop distribution is also supported by the work of Bennetts (1981) who described the enhancement of surface rainfall as the effect of merging two interacting cloud cells under certain conditions.

4. Conclusion

Various types of rainfall intensity pattern of the precipitation from convective clouds is mathematically justified by finding out expression for temporal variation of precipitation intensity and laying down conditions for the various types of temporal variation of precipitation intensity. It has been shown mathematically that the presence or absence of updraft in the convective clouds and the drop diameter distribution plays an important role in temporal variation of precipitation intensity.

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