*Mausam*, (1993), **44**, 1, 1-8

551.509.313:551.513.1

# Implicit nonlinear normal mode initialization for a barotropic primitive equation limited area model

D. R. C. NAIR, B. CHAKRAVARTY and P. NIYOGI

*Indian Institute of Technology, Kharagpur*

(*Received 7 June 1991, Modified 12 May 1992*)

**सार** — अप्रत्यक्ष अरैखिक सामान्य बहुलक प्रारंभीकरण के एक सरल पाटांतर का उपयोग एक उष्णकटिबंधीय परिसर के सीमित क्षेत्र पर एक-स्तरीय पूर्वग समीकरण मॉडल के लिये किया गया है। मॉडल का संरूपण गोलीय निर्देशांक में उथले जल के समीकरणों पर आधारित है और विभव-एन्स्ट्राफी के संरक्षण को ध्यान में रखते हुए परिमित अंतर विधि का प्रयोग किया गया है। मॉडल का उपयोग बंगाल की खाड़ी के ऊपर निर्मित आदर्श मानसून अवदाब की गतिविधि के पूर्वानुमान के लिये किया गया है। उक्त विधि को अत्यन्त प्रभावशाली पाया गया, क्योंकि इसमें संहति और पवन क्षेत्रों के बीच संतुलन प्राप्त करने के लिये तीन पुनरावृत्तियों की ही आवश्यकता होती है। परन्तु एक-स्तरीय मॉडल की सीमाओं के कारण प्रस्तुत मॉडल भी अवदाब के गमन का पूर्वानुमान अत्यधिक सही रूप में प्राप्त नहीं कर सकता।

ABSTRACT. A simple version of implicit nonlinear normal mode initialization is applied to a limited area one-level primitive equation model over a tropical domain. The model formulation is based on shallow water equations in spherical co-ordinate and potential enstrophy conserving finite difference scheme is employed. The model is used for predicting the movement of a typical monsoon depression formed over the Bay of Bengal. The above scheme is found to be very effective as it requires only three iterations for attaining balance between the mass and wind fields. However this model is not able to predict the movement of the depression very accurately due to the limitations of such a one-level model.

**Key words** — Implicit normal mode initialization, Potential enstrophy, Shallow water model, $\beta$-term, Helmholtz equation, Nonlinear balance equation.

## 1. Introduction

Numerical weather prediction models based on the primitive equations generally give two types of solutions, *viz.*, slow moving meteorologically significant Rossby waves and fast moving gravity waves. The high frequency gravity wave oscillations are considered as 'noise' and they arise primarily from the initial imbalances between the wind and mass fields. Initialization is the process of adjusting the input data to the prediction model ensuring minimum noise. Several methods have been proposed to remove unwanted gravity oscillations including different versions of dynamic initializations and normal mode initializations.

Machenhauer (1977), Baer and Tribbia (1977) introduced nonlinear Normal Mode Initialization (NMI) which has become the most widely used initialization technique for many research as well as operational forecasts. Briere (1982) formulated NMI for a Limited Area Model (LAM) on a stereographic projection with constant Coriolis and map scale factors. Bourke and McGregor (1983) developed an initialization scheme where they used variable Coriolis parameter. Juvanon du Vachat (1986) showed that the normal modes of the model need not be found explicitly and the same can be treated as eigenfunction of an elliptic operator. However, this method is very similar to the second scheme suggested by Bourke and McGregor (1983). Following this, Temperton (1985, 1988) developed a powerful method called "Implicit Normal Mode Initialization" (INMI) applicable to both regional and global models. Lynch (1987) too developed a similar method suitable for models with semi-Lagrangian integration schemes.

INMI allows nonlinear normal mode initialization technique to be applied even when the linear system is not separable and computing the normal modes is very complicated. In this case the Coriolis and scale map factors can be treated as variables and the domain of integration need not be rectangular. The determination of the normal modes for a limited area model is very difficult (Kasahara 1982) and for applying the conventional NMI to a forecast model it is necessary to compute the modes coefficients explicitly and store them for further computations. In the case of a high resolution model it takes huge computer memory (Temperton 1989). By using INMI one can avoid such difficulties.

Saha (1983) studied the movement of monsoon depression in a primitive equation barotropic model and concluded that such a model can predict the movement of monsoon depressions to a limited degree of accuracy. Singh and sugi (1986) applied a regional primitive multilevel model to the prediction experiment of a monsoon depression, and demonstrated that LAM is quite useful

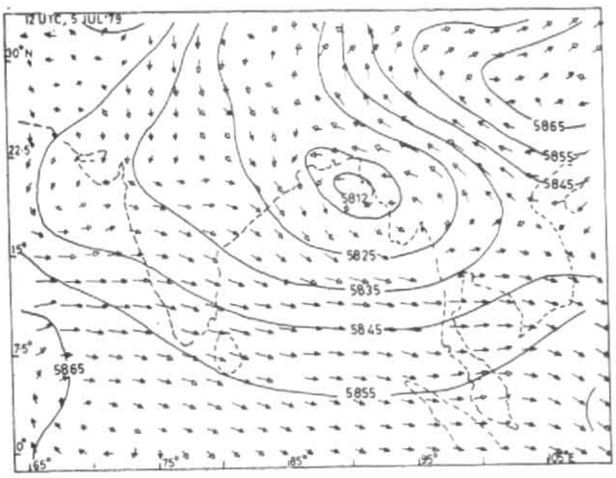


Fig. 1. Observed wind and height fields at 500 hPa on 5 July 1979 (12 UTC)

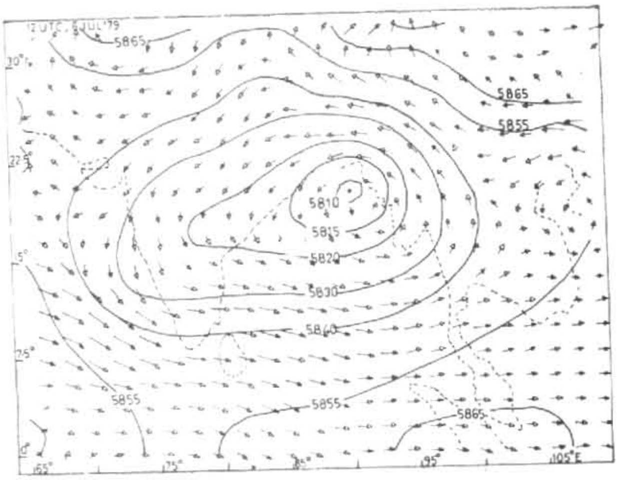


Fig. 2. Observed wind and height fields at 500 hPa on 6 July 1979 (12 UTC)

for predicting tropical disturbances. Recently Krishnamurti *et al.* (1990) used a semi-Lagrangian semi-implicit version of a high resolution regional model to study the different aspects of monsoon dynamics.

In this experiment we have implemented INMI to a simple one-level primitive equation limited area model. This model is integrated for 24 hours to predict the movement of monsoon depression that formed over the Bay of Bengal during the period of the summer monsoon (July 1979). In INMI scheme the Coriolis factor is treated as a variable. The model is based on shallow water equations in spherical co-ordinate with potential enstrophy conserving finite difference scheme.

## 2. The model

The model is formulated by using shallow water equations in spherical co-ordinates (Williamson 1976) and the horizontal discretization of the model equations are based on Sadourny's (1975) potential enstrophy conserving scheme. The staggering of the variables are done over the Arakawa-C grid. The model equations can be written in semi-discretized form for a unit sphere ($a=1$) as follows :

$$\frac{\partial u}{\partial t} = \frac{-1}{\mu}\,\delta_{\lambda}\phi + \overline{\eta}^{\theta}\,\overline{V}^{\lambda\theta} - \frac{1}{\mu}\,\delta_{\lambda}K \qquad (1)$$

$$\frac{\partial v}{\partial t} = -\,\delta_{\theta}\phi - \overline{\eta}^{\lambda}\,\overline{U}^{\lambda\theta} - \delta_{\theta}K \qquad (2)$$

$$\frac{\partial \phi}{\partial t} = -\,\Phi\,\frac{1}{\mu}\left(\delta_{\lambda}u + \delta_{\theta}v\mu\right) - \frac{1}{\mu}\,\delta_{\lambda}\left(\overline{\phi_1}^{\lambda}u\right) - \frac{1}{\mu}\,\delta_{\theta}\left(\overline{\phi_1}^{\theta}v\mu\right) \qquad (3)$$

where, $u$ and $v$ are the velocity components of the wind along zonal and meridional directions respectively. $\phi$ is the geopotential field and $\lambda$, $\theta$ are longitude and latitude respectively, $f$ is the Coriolis parameter and $t$ is the time. Now the other variables in the above system are defined as $\mu=\cos\theta$, $K=(\overline{u^2}^{\lambda}+\overline{v^2}^{\theta})/2$, $\phi_1=\phi-\Phi$,

where, $\Phi$ is the mean geopotential $V=\overline{\phi}^{\theta}v$, $U=\overline{\phi}^{\lambda}u$, and the potential vorticity $\eta$ is defined as:

$$\eta = \left[\frac{1}{\mu}\left(\frac{\partial v}{\partial \lambda} - \frac{\partial(u\mu)}{\partial \theta}\right) + 2\,\Omega\sin\theta\right] \Big/ \overline{\phi}^{\lambda\theta}.$$

The independent variables $\lambda$ and $\theta$ are discretized as $\lambda_i=i\triangle\lambda$, $i=1\ldots.m$, $\theta_j=j\triangle\theta$, $j=1\ldots.n$ where, $m$ and $n$ are the number of grid along latitude and longitude. The operator $\delta$ denotes central difference quotient and which is defined as follows :

For any variable $F$,

$$\delta_{\lambda}(F) = \frac{F_{i+1/2}-F_{i-1/2}}{\triangle\lambda}\;;\;\delta_{\theta}(F) = \frac{F_{j+1/2}-F_{j-1/2}}{\triangle\theta}$$

The over bars appearing in Eqns. (1)-(3) denote average quantities of the variables indicated with respect to $\lambda$ or $\theta$ which are defined as follows :

$$\overline{F}^{\lambda} = (F_{i+1/2}+F_{i-1/2})/2;\;\overline{F}^{\theta} = (F_{j+1/2}+F_{j-1/2})/2$$

and $\overline{F}^{\lambda\theta}$ denotes the successive averaging with respect to $\lambda$ and $\theta$, and this operation is commutative ($\overline{F}^{\lambda\theta}=\overline{F}^{\theta\lambda}$).

### 3. Domain, data and boundary conditions of the model

The grid point data of geopotential and wind components have been extracted from FGGE-IIIb data sets of European Centre for Medium Range Weather Forecast (ECMRWF) available in 1.875° Lat./Long. grid points. The data of a typical monsoon depression, 5 July 1979 has been chosen as the basic input of this study. In order to obtain the initial balanced data of the model, geopotential field is computed from the nonlinear balance equation (Krishnamurti 1969).

$$\nabla^2 \phi = f \nabla^2 \psi + \frac{\partial \psi}{\partial \theta} \quad 2\Omega \mu + \frac{2}{\mu} J \left( \frac{1 \partial \psi}{\mu \partial \lambda}, \frac{\partial \psi}{\partial \theta} \right) \quad (4)$$

where, $\psi$ the stream function can be calculated from the vorticity field $\zeta$ through the relation $\nabla^2 \psi = \zeta$, $\nabla^2$ is the horizontal Laplacian operator in spherical co-ordinate (unit sphere), $J$ denotes the Jacobian, $f$ is the Coriolis parameter and $\Omega$ is the angular velocity of the earth. The boundary conditions of the Eqn. (4) are the observed values of the geopotential at the boundaries of the domain.

The boundary conditions for solving the model equations are taken as follows (Krishnamurti *et al.* 1990). The prognostic variables $u$, $v$ and $\phi$ are held fixed with time along the boundary. In order to avoid the wave reflection caused by such a boundary condition a Laplacian type smoother is applied near the boundary. A similar smoothing is applied over the entire domain for which the value of the smoothing coefficient increases from the centre to the boundaries of the domain.

The domain of integration is from equator to 31.875°N and 65.625°E to 108.75°E, and consists of 24×18 grid points. The domain is selected in such a manner that the depression comes almost at the centre so that the influence of the boundary on the depression will be reduced.

### 4. Integration of the model

For this numerical experiment we assume $\triangle \lambda = \triangle \theta = 1.875°$ and the time step $\triangle t = 4$ minutes and the model is integrated for 24 hours. The integration begins with a "smooth start" consisting of a forward time step of length $\triangle t/2$, followed by a centred step of length $\triangle t$ before using regular leapfrog (central) time steps. In order to avoid the computational mode associated with leapfrog scheme a Robert time filter is used as follows :

For any time dependent variable $F = F(t)$, $\bar{F}_t = F_t + \gamma (F_{t+1} - 2F_t + \bar{F}_{t-1})$ where, $t-1$, $t$, $t+1$ indicate different time levels and the coefficient $\gamma$ is taken as 0.05.

### 5. Implicit nonlinear normal mode initialization

The shallow water equations in spherical coordinates can be expressed in vorticity and divergence form as shown below :

$$\frac{\partial \zeta}{\partial t} = -2\, \Omega \sin \theta\, D + N_{\zeta} \quad (5)$$

$$\frac{\partial D}{\partial t} = 2\, \Omega \sin \theta\, \zeta - \nabla^2 \phi + N_D \quad (6)$$

$$\frac{\partial \phi}{\partial t} = -\Phi\, D + N_{\phi} \quad (7)$$

where, $D$ is the divergence, $\zeta$ is the vertical component of vorticity $N_\zeta$, $N_D$ and $N_\phi$ are the terms containing nonlinear and remaining linear terms ($\beta$-terms). For simplicity the $\beta$-terms are omitted in the linear system, though these terms are important in determining the fast modes and are related to the evolution of the slow modes. Now the linearized system can be expressed in matrix form.

$$\frac{\partial}{\partial t} \begin{bmatrix} \zeta \\ D \\ \phi \end{bmatrix} = \begin{bmatrix} 0 & -f & 0 \\ f & 0 & -\nabla^2 \\ 0 & -\Phi & 0 \end{bmatrix} \begin{bmatrix} \zeta \\ D \\ \phi \end{bmatrix} \quad (8)$$

where, $\Phi = gH$, $H$ is the mean depth of the fluid and $g$ is the gravity.

Temperton (1985, 1988) and Juvanon du Vachat (1986) have developed an initialization method equivalent to the normal mode method. They decomposed the state vector $\mathbf{X} = (\zeta, D, \phi)^T$ into slow (Rossby) and fast (Gravity) orthogonal components such that $\mathbf{X}_O = \mathbf{X}_R + \mathbf{X}_G$, where, the suffixes $O$, $R$ and $G$ denote "observed" (total), Rossby and gravity terms respectively. The vectors $\mathbf{X}_O = (\zeta_O, D_O, \phi_O)^T$, $\mathbf{X}_R = (\zeta_R, D_R, \phi_R)^T$ and $\mathbf{X}_R = (\zeta_R, D_R, \phi_R)^T$ where the superscript $T$ indicates the transpose of the vector. Similarly the time derivative (.) of the fields can be expressed as $\dot{\mathbf{X}}_O = \dot{\mathbf{X}}_R + \dot{\mathbf{X}}_G$. Temperton (1988) derived INMI using the Machenhauer's (1977) algorithm, *i.e.,* at time $t=0$, $\dot{\mathbf{X}}_G = 0$ and considering the following two properties of the normal modes :

(*i*) The slow modes are stationary and non-divergent, *i.e.,*

$$\nabla^2 \phi_R = \zeta_R\, f \; ; \; D_R = 0 \quad (9)$$

(*ii*) The fast modes have zero linearized potential vorticity, *i.e.,*

$$\Phi\, \zeta_G = f\, \phi_G \quad (10)$$

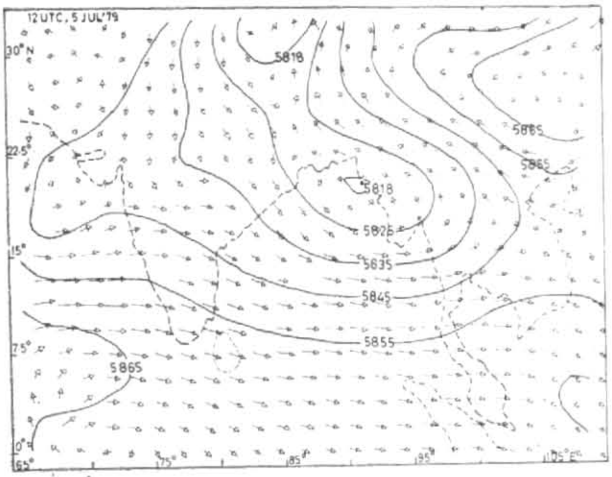


Fig. 3. Initialized input of the model at 500 hPa on 5 July 1979 (12 UTC)

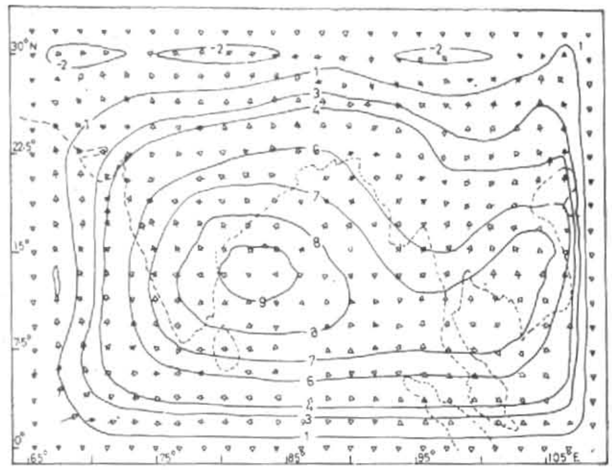

Fig. 5. The difference between the initialized and observed fields at 500 hPa on 5 July 1979 (12 UTC)

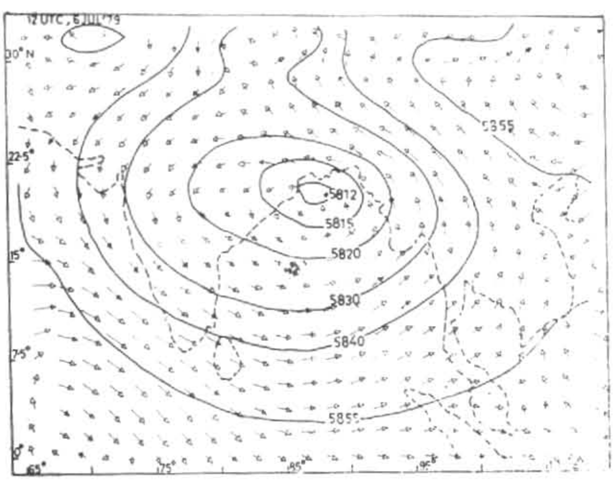


Fig. 4. Prediction with initialization at 500 hPa on 6 July 1979 (12 UTC)

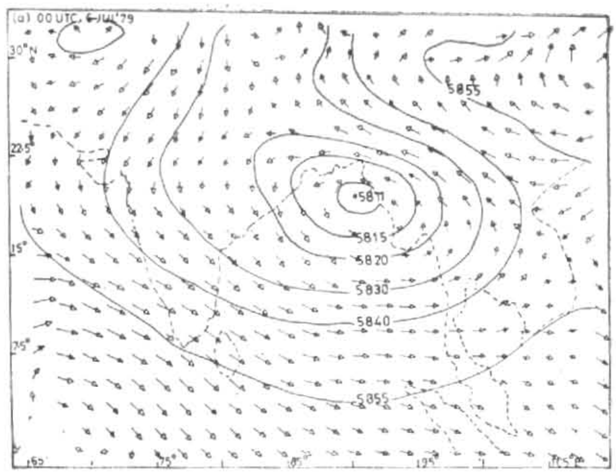


Fig. 6(a). Prediction with initialization at 500 hPa on 6 July 1979 (00 UTC)

Now the Eqn. (4.25) of Temperton (1988) in this case takes the form

$$\begin{bmatrix} 0 & f & 0 \\ -f & 0 & \nabla^2 \\ 0 & \Phi & 0 \end{bmatrix} \begin{bmatrix} \triangle\zeta \\ \triangle D \\ \triangle\phi \end{bmatrix} = \begin{bmatrix} (\dot{\zeta})_G \\ (\dot{D})_G \\ (\dot{\phi})_G \end{bmatrix} \qquad (11)$$

where, $\triangle\zeta$, $\triangle D$ and $\triangle\phi$ are the quantities to be estimated in each iteration of INMI and added to the initial fields. $\dot{\zeta}$, $\dot{D}$, $\dot{\phi}$ are the tendency obtained by running the model one time step forward. From the second equation of Eqn. (11) we have

$$-f \triangle\zeta + \nabla^2 (\triangle\phi) = \dot{D}_G \qquad (12)$$

Since the slow modes are stationary by property (*i*) we have

$$(\dot{D})_O = (\dot{D})_G, \qquad (\dot{D})_R = 0$$

and by the property (*ii*)

$$\triangle\zeta = \frac{f \triangle\phi}{\Phi} \qquad (13)$$

Therefore Eqn. (12) takes the form

$$\left( \nabla^2 - \frac{f^2}{\Phi} \right) \triangle\phi = (\dot{D})_O \qquad (14)$$

To find $\triangle D$ we have to consider the third equation of Eqn. (11)

$$i.e., \quad \Phi \triangle D = (\dot{\phi})_G \qquad (15)$$

Now by using the property (*ii*) and following the corresponding steps of Temperton (1988) we obtain the equation for $(\dot{\phi})_G$ as given below :

$$\left( \nabla^2 - \frac{f^2}{\Phi} \right) (\dot{\phi})_G = \nabla^2 (\dot{\phi})_O - f(\dot{\zeta})_O \qquad (16)$$

**6. Computation and convergence of INMI**

In the computational process we have to solve two Helmholtz equations [Eqns. (14) and (16)]. The boundary conditions should be selected in such a way that the unwanted gravity modes should be suppressed. Here the boundary of the domain is kept time fixed during initialization, in other words the increments or decrements of the dependent variables $\triangle u$, $\triangle v$ and $\triangle\phi$ are all zero along the boundary, *i.e.,* $\triangle x = \triangle\psi = \triangle\phi = 0$, where, $\triangle x$ and $\triangle\psi$ are the corresponding velocity potential and stream function respectively. This set can form a consistent set of boundary condition for initialization (Juvanon du Vachat 1988). During initialization the time filtering is switched off.

The convergence of iterative steps is monitored through the quantity "BAL" which measures the balance in the adjusted gravity modes. Here, BAL is defined (Briere 1982, Temperton 1988) as follows :

$$\text{BAL} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ (\dot{\phi}_{ij})^2_G + \Phi \left( (\dot{u}_{ij})^2_G + (\dot{v}_{ij})^2_G \right) \right] \cos\theta_j \qquad (17)$$

As INMI scheme converges BAL approaches zero. In this study we used 7 iterations for INMI but after 3 iterations the scheme seems to have converged within computational tolerance.

The computational procedure INMI can be summarized step by step (Temperton 1988) as follows :

*Step* (1) — Integrate the model for one time step forward and obtain the observed tendencies $(\dot{\zeta})_O$, $(\dot{D})_O$, $(\dot{\phi})_O$.

*Step* (2) — Solve the Helmholtz Eqns. (14) and (16) for $\triangle\phi$ and $(\dot{\phi})_G$ respectively with Dirichlet boundary condition.

*Step* (3) — Calculate $\triangle D$ and $\triangle\zeta$ using the equations

$$\triangle D = \frac{(\dot{\phi})_G}{\Phi} \qquad (18)$$

$$\triangle\zeta = \frac{f \triangle\phi}{\Phi} \qquad (19)$$

*Step* (4) — Calculate the improvements of the initial fields $\triangle u$ and $\triangle v$ for any iteration by solving (*i.e.,* reconstruct the wind fields from known values of new vorticity and divergence fields)

$$\nabla^2 (\triangle\chi) = \triangle D \qquad (20)$$

$$\nabla^2 (\triangle\psi) = \triangle\zeta \qquad (21)$$

and using the equations

$$\triangle u = \frac{1}{\mu} \left( -\mu \frac{\partial(\triangle\psi)}{\partial\theta} + \frac{\partial(\triangle\chi)}{\partial\lambda} \right) \qquad (22)$$

$$\triangle v = \frac{1}{\mu} \left( \frac{\partial(\triangle\psi)}{\partial\lambda} + \mu \frac{\partial(\triangle\chi)}{\partial\theta} \right) \qquad (23)$$

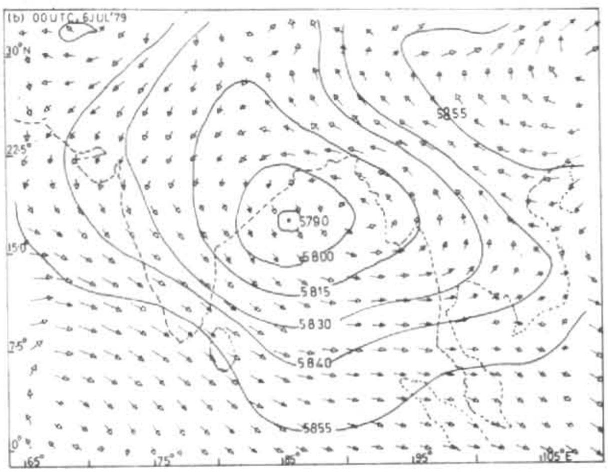

Fig. 6 (b). Prediction without initialization at 500 hPa on 6 July 1979 (00 UTC)

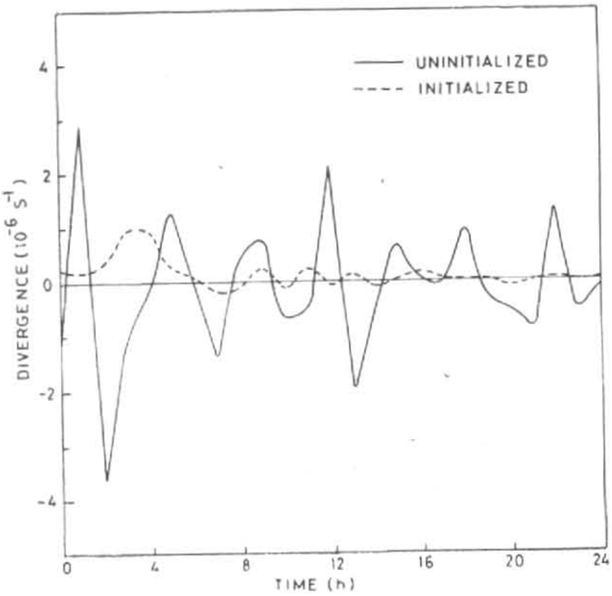


Fig. 8. Time-trace of divergence ($1 \times 10^{-6}$ $s^{-1}$) at the grid point (15°N, 86.25°E), solid line indicates prediction without INMI and dotted line indicates prediction with INMI

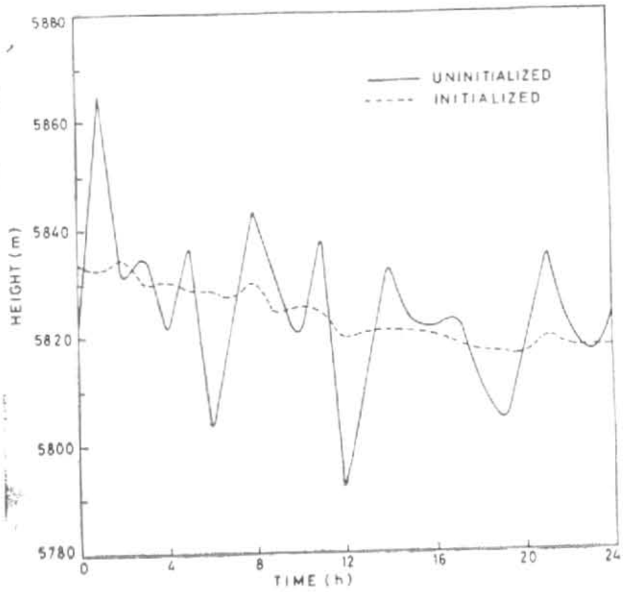


Fig. 7. Time-trace of height (m) fields at the grid point (15°N, 86.25°E), solid line indicates prediction without INMI and dotted line indicates prediction with INMI

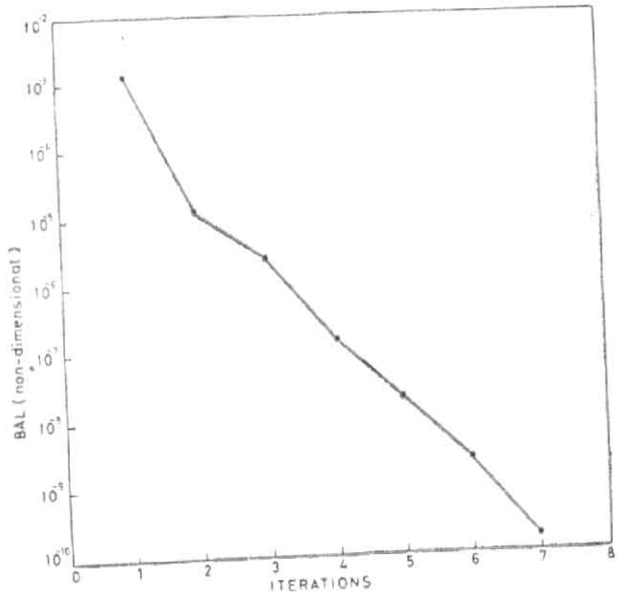


Fig. 9. The quantity BAL (non-dimensional) as a function of iterations in the convergence cycles of INMI

*Step* (5) — For any iterative step '$k$' the initialized fields are given by :

$$u^{(k)} = u^{(k-1)} + \triangle u \ ;$$

$$v^{(k)} = v^{(k-1)} + \triangle v \ ;$$

$$\phi^{(k)} = \phi^{(k-1)} + \triangle \phi .$$

In each iteration the improvements of wind fields have to be estimated accurately. The method as suggested by Lynch (1988) has been employed and the integration of the Eqns. (20) and (21) are performed using the staggered Arakawa-C grid. Since the model variables are computed on the same grid it is very desirable to adopt this method.

The BAL can be determined after step (3) from the Eqn. (17). The entire cycle is repeated till the BAL is sufficiently small. All the elliptic equations are solved by using SOR method.

## 7. Results and discussion

The main objective of this work is to implement a simple version of INMI to a tropical limited area model and study its effect. The INMI scheme has been tested to a barotropic LAM as described in sections 2 to 4. The mean depth of the fluid at 500 hPa is taken as 5700 m and the time step for integration as well as INMI is $\triangle t = 4$ minutes.

Figs. 1 and 2 show the observed fields at 12 UTC (5 July 1979) and 12 UTC (6 July 1979) respectively. The centre of depression over the Bay of Bengal as seen in Fig. 1 lies at (21.5°N, 91.0°E) while after 24 hours the depression has moved to (20.0°N, 90.0°E). Fig. 3 shows the initialized input of the model, in this case the position of the depression remained stationary while a little change observed in the height field (6 m). Fig. 4 shows the predicted field of 12 UTC (6 July) where the centre of depression has moved (20.0°N, 88.5°E), this shows that predicted field moved about 1.5° westwards more than the observed (Fig. 2).

Fig. 5 shows the difference fields, *i.e.*, initialized *minus* observed fields. The height fields are smooth which attain a maximum near by the centre of the domain. Prediction is made for 12 hours with and without INMI and the results are shown in Figs 6 (a & b) (00 UTC, 6 July 1979) respectively. In the initialized case the centre of the depression moved to (20.0°N, 90.5°E) after 12 hours but in the uninitialized case the centre of depression moved to (17.5°N, 86.0°E). The spurious prediction of the centre of the depression in the uninitialized case is mainly due to the influence of gravity modes (noise).

The Fig. 7 shows the plots of height field at the grid point (15°N, 86.25°E) nearer to the centre of the domain, at each hour of prediction with (dotted line) and without (solid line) INMI. A similar graph is given for the divergence field at the same grid point (Fig. 8). These graphs clearly show that the initialized data is free from spurious gravity oscillation to a great extent. The experiment is repeated for different sets initialized data obtained by iterating INMI schemes more than 3 times, all these cases the predictions are identical and initialized plots of Figs. 7 and 8 are indistinguishably same. This shows that the BAL is practically converged after 3 iterations. The convergence of INMI scheme is shown in Fig. 9 (logarithmic curve) for seven iterations. The BAL (non-dimensional) as shown in Fig. 9 is reduced by 3 orders of magnitude after 3 iterations.

The initialization was performed using the basic geopotential data extracted from FGGE-IIIb as input. In this case also the results are the same as the case of geopotential data obtained through nonlinear balance equation (Eqn. 4). This confirms the ability of the INMI scheme to adjust the mass and wind fields of the initial (observed) data.

## 8. Conclusions

A simple version of implicit nonlinear normal mode initialization has been implemented to a tropical one-level primitive equation model, for predicting the movement of monsoon depression. This initialization procedure was found to be very suitable to suppress unwanted gravity wave oscillation for a one-level tropical LAM. INMI is computationally economical as it does not require explicit knowledge of normal mode coefficients and can be incorporated to a multi-level baroclinic model. Inclusion of all the $\beta$-terms in the linearized system could have improved the results at the cost of solving more complicated Helmholtz equations (Temperton 1989).

In spite of the limitations of the one-level barotropic primitive equation model and the fact that the monsoon depression is a baroclinic system, this study has been successful in predicting the depression movement.

*Acknowledgements*

The authors wish to thank Dr. A. Chandrasekar (Dept. of Physics and Meteorology) and Mr. V.V. Vinod (Dept. of Computer Science), Indian Institute of Technology, Kharagpur for their helpful assistance in the preparation of the manuscript. The authors are thankful to the anonymous reviewer for suggestions that substantially improved the manuscript.

## References

Baer, F. and Tribbia, J., 1977, "On complete filtering of gravity modes through nonlinear initialization", *Mon. Weath. Rev.*, **105**, pp. 1536-1539.

Bourke, W and McGregor, J.L., 1983, "A nonlinear vertical mode initialization scheme for a limited area prediction model", *Mon. Weath. Rev.*, **111**, pp. 2285-2297.

Briere, S., 1982, "Nonlinear normal mode initialization of a limited area model", *Mon. Weath. Rev.*, **110**, pp. 1166-1186.

Juvanon du Vachat, R., 1986, " A general formulation of normal modes for limited area models", *Mon. Weath. Rev.*, **114**, pp. 2478-2487.

Juvanon du Vachat, R., 1988, "Non-normal mode initialization : Formulation and application to inclusion of $\beta$-terms in the linearization", *Mon. Weath. Rev.*, **116**, pp. 2013-2024.

Kasahara, A., 1982, "Nonlinear normal mode initialization and the bounded derivative method", *Rev. Geophys. Space Phys.*, **20**, pp. 385-397.

Krishnamurti, T.N., 1969, "An experiment in numerical prediction in the equatorial latitudes, "*Quart. J. R. Met. Soc.*, **95**, pp. 596-620.

Krishnamurti, T.N., Arun Kumar, Yap, K.S., Dastoor, A.P., Noel Davidson and Jian Sheng, 1990, "Performance of a high resolution mesoscale tropical prediction model," *Advances in Geophysics*, **32**, Academic Press, pp. 133-284.

Lynch, P., 1987, The slow equations, Part I and II, Tech. Note No. 50, Irish Meteorological Service, Dublin.

Lynch, P., 1988, "Deducing the wind from vorticity and divergence", *Mon. Weath. Rev.*, **116**, pp. 86-93.

Machenhauer, B., 1977, "On the dynamics of gravity oscillations in a shallow water model, with application to normal mode initialization", *Beitr. Phys. Atmos.*, **50**, pp. 253-273.

Sadourny, R., 1975, "The dynamics of finite difference models of shallow water equations", *J. Atmos. Sci.*, **32**, pp. 680-689.

Saha, S., 1983, "Behaviour of monsoon depression in a primitive equation barotropic model", *Mausam*, **34**, pp. 27-32.

Singh, S.S. and Sugi, Mastino, 1986, "Prediction of monsoon depression with a regional primitive equation model", *Mausam*, **37**, pp. 17-26.

Temperton, C., 1985, "Application of new principle of NMI : Seventh conf. on NWP", Montreal, Amer. Met. Soc., pp. 105-107

Temperton, C., 1988, "Implicit normal mode initialization", *Mon. Weath. Rev.*, **116**, pp. 1013 - 1031.

Temperton, C., 1989, "Implicit normal mode initialization for spectral models", *Mon. Weath. Rev.*, **117**, pp. 436-451.

Williamson, D.L., 1976, "Normal mode initialization procedure applied to forecast with global shallow water equations", *Mon. Weath. Rev.*, **104**, pp. 195-206.