

Distribution of Saturation Potential Temperature(θ_s) with Height and Latitude

K. NAGABHUSHANA RAO

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DIAGRAMS showing the distribution with height and latitude of pressure (p), temperature (T) and potential temperature (θ) for Summer and Winter are given in many of the standard meteorological books and publications. But, as far as is known, there do not appear to be any similar diagrams showing the distribution of moisture with height.

Normand in his paper 'Wet-Bulb temperature and the thermodynamics of the air' discussed the vertical distribution of potential temperature and Equivalent potential temperature. The data used were only for Germany and limited to a height of 3 Km. He noticed that in all seasons of the year the potential temperature increases with height above the surface while on the other hand the E.P.T. in summer months actually decreases with height upto at least 2 Km. With the considerably increased volume of data that have now become available, Dr. (now Sir Charles) Normand suggested to me that it would be of interest to prepare a diagram showing the distribution of Saturation Potential Temperature (θ_s) with height.

Two diagrams showing the distribution of θ_s with height and latitude were prepared under his guidance during the closing part of 1943. The first of the two has a linear scale for latitude while in the second the sine of latitude has been used. A short note on 'Distribution of Wet Bulb Potential Temperature in Latitude and Altitude' including the first diagram by C.W.B. Normand and myself has appeared recently in *Nature*.

The publications from which the data have been taken for preparing the diagrams are given in the end. The values of θ_s have been calculated for the two periods Summer and Winter only. For Summer the mean is based on data of July and August while for Winter the mean is for data of December, January and February.

Relative Humidity data are not available beyond 10 to 12 g.km. As moisture content above 12 g.km. is generally very small, the mixing ratio (x) has been taken as zero for calculating θ_s above this height. It may be stated that this assumption does not materially affect the values of θ_s at these levels.

The times of ascent were not the same at all the stations but this has been taken into account for smoothing the lines.

The values of θ_s for the different stations are given in the following tables I to IV. Data in tables III and IV relate mostly to stations in the United States of America. The diagrams have been drawn mainly from data in tables I and II and of stations in high latitudes from tables III and IV.

TABLE III

Saturation Potential

gkm.	Barrow	Bethel	Anchorage	Nome	Fairbanks	Juneau	Ketchi Kan	Spokane	Seattle	Bismark	St. Paul	Boise	Buffalo	Medford	Omaha
17
16	306.1	306.0	305.0	05.2	03.7	305.1	05.5	303.5
15	07.2	..	06.3	06.5	06.5	04.8	04.1	01.7	03.7	302.2	02.6	02.8	302.1	03.1	300.5
14	..	03.6	03.7	03.7	04.0	03.5	02.9	301.1	301.1	99.7	99.7	300.6	99.5	300.8	98.3
13	302.5	300.9	300.9	300.9	301.1	00.8	301.0	97.7	93.1	97.1	97.2	97.3	97.0	97.7	96.4
12	99.7	97.2	97.1	97.2	97.5	93.1	96.1	94.5	95.1	94.8	95.4	94.7	94.9	94.9	95.3
11	95.8	93.6	93.3	93.6	93.1	92.5	92.6	92.8	92.8	93.3	91.0	93.0	91.6	93.2	94.2
10	91.0	91.1	90.6	91.2	90.3	90.4	90.6	92.0	91.6	92.6	91.2	91.2	92.6	92.3	93.6
9	88.3	89.5	89.1	81.5	88.1	88.8	89.7	91.1	90.4	92.0	92.5	91.4	92.0	91.5	93.2
8	86.9	88.7	87.8	83.4	87.6	88.2	88.6	90.4	89.8	91.4	91.7	91.7	91.8	91.0	92.4
7	85.8	88.1	87.3	87.6	83.8	87.7	88.1	89.6	89.0	90.9	91.2	90.2	90.3	90.3	92.0
6	84.9	87.3	86.8	86.8	85.7	86.9	87.4	89.5	88.7	90.7	90.6	90.1	89.7	90.0	91.4
5	84.1	86.1	85.9	83.0	83.3	83.0	86.6	88.9	87.9	91.6	90.6	90.0	89.4	91.0	91.5
4	83.3	85.8	85.4	85.4	85.0	85.7	83.5	89.1	87.1	91.1	90.5	90.2	88.9	90.2	91.8
3	82.0	85.2	84.9	85.4	84.6	84.6	86.1	89.5	88.4	92.0	90.6	90.9	88.7	90.9	92.3
2½	81.6	85.1	84.8	85.1	84.6	85.6	85.4	89.7	87.9	92.3	91.1	91.1	88.7	90.8	93.3
2	81.1	85.1	84.9	84.8	84.6	84.6	86.2	90.3	88.9	93.0	91.4	91.6	89.3	91.5	93.2
1½	80.3	84.5	85.0	84.5	84.8	84.8	86.3	91.4	83.9	93.3	91.7	91.8	89.9	90.8	93.2
1	79.3	84.1	85.0	84.0	84.8	85.5	86.4	61.6	88.8	93.2	91.2	92.1	89.8	91.0	293.9
½	277.7	83.7	85.4	83.3	85.2	85.8	86.7	..	88.6	..	92.1	..	90.4	90.9	293.5
Sur.	..	283.6	275.3	282.7	285.6	285.1	286.1	283.7	287.7	290.4	291.5	290.9	288.9	289.7	293.2

TABLE IV

Saturation Potential

gkm.	Barrow	Nome	Fairbanks	Anchorage	Juneau	Ketchi Kan	Spokane	Bismark	St. Paul	Boise	Buffalo	Medford	Omaha	Denver
18
17
16	06.2	..
15	04.9	05.3	05.2	04.4	05.1	03.5	01.3	03.7
14	04.1	04.8	05.0	04.7	04.5	03.9	02.7	02.9	02.7	01.9	02.8	01.7	02.1	01.5
13	01.8	02.3	302.2	02.0	01.8	01.3	99.7	90.2	00.1	99.1	00.4	98.7	93.3	98.6
12	98.6	99.1	99.0	93.0	98.4	97.9	96.0	96.7	96.5	95.4	97.1	95.0	96.1	95.3
11	95.1	95.4	95.0	94.9	94.8	93.6	91.9	92.1	92.3	91.5	91.0	91.0	92.3	91.5
10	90.8	90.5	90.3	90.2	90.4	89.3	88.5	88.5	88.9	88.6	89.5	88.8	89.1	83.7
9	85.5	85.9	85.7	86.1	86.3	85.2	86.5	86.2	86.5	86.6	85.6	87.3	87.0	87.1
8	79.8	82.4	82.7	83.3	83.9	81.0	84.8	84.7	85.2	85.7	81.5	86.1	85.6	86.4
7	75.6	79.8	80.8	81.4	82.2	82.8	84.2	83.8	84.1	84.6	83.4	85.3	84.5	85.4
6	272.5	78.1	79.2	80.0	80.8	81.2	83.2	82.6	82.9	83.6	81.9	84.5	83.5	84.6
5	70.8	76.5	..	78.4	79.9	80.1	82.1	81.2	81.6	82.9	80.6	84.0	81.4	84.0
4	69.1	74.7	76.6	77.0	78.8	79.0	81.3	80.4	80.3	82.0	79.5	83.0	81.5	83.3
3	67.9	72.6	75.3	75.8	77.8	78.1	80.4	79.4	78.2	81.2	77.6	82.3	80.3	82.8
2½	68.8	72.2	75.2	75.4	77.1	77.6	79.9	78.4	77.4	81.2	76.0	82.2	79.6	82.6
2	..	70	74.0	74.9	76.2	77.2	79.4	77.2	76.2	81.6	74.9	82.1	78.9	281.4
1½	..	270	270.3	75.1	75.0	76.2	78.5	74.9	74.3	80.1	72.9	81.8	77.2	..
1	..	268	..	75.1	73.7	75.7	77.5	271.6	..	79.1	271.6	81.0	275.3	..
½	272.9	273.7	75.4	79.4
Sur.	274.5	275.3	277.6	..	279.1	..	277.0

Lat. 71°20' 65° 65° 61° 57°30' 55°24' 47°45' 46°17' 45° 43°47' 42°52' 42°21' 41°12' 39°48'

SUMMER

Temperature (θ_s).

														gkm.	
306.5	306.2	..	305.5	305.7	305.7	305.4	305.1	305.2	305.3	304.7	04.8	303.8	305.3	303.9	17
303.3	303.0	304.5	302.3	302.4	302.1	301.8	302.2	302.5	301.5	301.4	01.5	301.9	302.1	301.2	16
300.6	300.4	302.1	97.6	99.9	99.6	99.3	99.4	300.1	98.9	99.0	99.0	99.2	99.7	98.9	15
97.9	98.4	99.6	97.6	97.9	97.9	97.3	97.8	98.1	97.2	97.7	97.8	97.3	97.9	96.8	14
96.2	96.6	97.1	86.4	96.7	96.6	96.1	96.7	96.8	96.3	91.7	93.1	95.9	95.6	95.2	13
94.6	95.6	94.8	95.5	95.7	95.7	95.4	95.8	95.9	95.6	96.0	95.2	95.0	95.6	94.6	12
93.6	94.6	94.3	94.8	94.9	94.7	94.9	94.9	94.6	95.0	95.4	94.5	94.2	94.7	94.1	11
93.0	93.9	92.1	94.3	94.0	94.1	94.3	94.2	93.5	94.4	94.6	93.7	93.5	94.1	93.7	10
92.6	93.4	92.1	93.7	93.7	93.6	93.8	93.7	92.6	93.8	94.1	93.5	93.4	93.9	93.3	9
92.0	91.5	90.6	93.0	93.1	92.9	93.3	91.2	92.1	93.4	93.2	92.7	92.8	93.2	92.6	8
91.9	92.0	89.9	92.5	92.7	92.5	92.9	91.5	91.7	92.9	92.7	92.0	92.3	92.3	92.1	7
91.8	91.7	89.6	92.5	92.5	92.4	92.9	92.6	91.2	92.7	92.6	92.0	92.0	91.5	91.9	6
91.8	91.7	89.3	92.1	92.5	92.9	93.1	92.8	91.2	93.1	92.9	91.8	92.0	91.1	92.0	5
91.9	91.9	88.9	92.1	92.5	91.5	93.0	93.5	90.8	93.8	92.5	92.3	92.1	89.9	91.5	4
93.7	92.3	88.7	92.4	93.5	91.3	93.1	91.5	91.4	94.8	93.2	91.9	92.7	90.3	91.9	3
94.3	93.0	88.8	94.2	94.0	94.8	93.6	94.9	91.4	95.3	94.1	92.2	94.0	90.7	92.7	2 1/2
93.9	93.6	83.5	94.2	95.2	94.7	93.7	95.2	92.9	95.6	95.2	93.0	93.4	91.9	93.5	2
..	94.1	90.0	95.2	96.0	..	95.0	95.4	92.5	95.8	96.1	94.2	94.2	92.4	94.6	1 1/2
..	94.2	90.1	95.5	95.9	..	95.4	95.8	92.9	95.8	96.1	95.0	95.2	92.6	95.7	1 1/2
..	94.2	88.3	96.2	93.1	..	95.6	96.0	91.1	..	97.3	96.9	293.7	93.1	93.9	1/2
293.8	293.1	286.4	294.6	295.5	294.6	295.6	294.1	290.1	291.6	296.5	297.5	..	293.8	296.1	Sur.
Denver	St. Louis	Oak Land	Nash Ville	Okla Homa City	Albu querque	Atlanta	Phoneix	Sandiego	Elpaso	San Antonio	Browns Ville	Miami	Pearl Har- bour	Sanjuan	

WINTER

Temperature (θ_s).

														gkm	
..	06.3	04.8	04.2	05.3	18
..	17
..	05.6	05.3	04.2	04.7	04.9	04.4	04.5	04.0	04.0	04.0	02.5	301.6	98.8	98.8	16
..	03.2	03.0	02.7	02.7	02.7	02.3	02.3	301.9	01.7	01.7	00.5	99.4	97.6	97.6	15
04.1	301.0	00.8	300.7	300.5	00.4	300.2	00.1	99.7	99.5	99.5	98.6	97.7	96.7	96.7	14
01.8	98.1	98.0	98.0	97.9	97.6	97.5	97.5	97.6	96.9	96.9	93.8	95.9	95.9	95.9	13
99.2	94.1	95.0	94.8	94.8	94.8	95.3	94.8	94.7	95.0	95.0	95.0	94.3	95.3	95.3	12
95.9	91.6	92.1	91.8	91.8	92.1	93.1	92.3	92.0	92.5	93.3	93.0	94.3	94.3	94.3	11
91.9	89.5	90.0	89.4	89.7	90.0	90.6	90.4	90.2	90.9	91.9	92.0	93.7	93.7	93.7	10
88.3	83.3	68.6	88.2	88.7	88.5	89.4	88.8	89.0	89.5	91.0	91.3	93.1	93.1	93.1	9
87.2	87.4	88.1	87.2	87.7	87.8	88.5	88.5	88.1	88.6	89.9	90.6	92.3	92.3	92.3	8
86.0	86.6	86.7	86.6	87.6	86.9	87.7	87.5	87.3	83.1	89.4	89.9	91.7	91.7	91.7	7
84.0	85.8	85.7	85.8	86.3	86.1	87.0	86.9	86.6	87.6	88.8	89.1	91.3	91.3	91.3	6
82.9	84.9	84.8	84.9	85.8	85.2	86.2	85.9	86.1	86.9	88.1	88.4	91.0	91.0	91.0	5
81.8	84.3	83.8	83.7	85.2	84.4	85.5	85.1	85.3	86.5	88.2	88.3	90.7	90.7	90.7	4
80.6	83.8	82.3	83.4	84.8	82.8	84.8	84.8	84.7	86.0	83.7	87.7	81.4	81.4	81.4	3
79.8	83.5	81.1	83.0	84.8	81.8	84.3	84.2	84.7	85.7	88.2	87.5	91.8	91.8	91.8	2 1/2
78.7	83.2	78.4	82.5	84.5	81.4	83.8	84.5	84.3	85.1	88.1	87.6	92.0	92.0	92.0	2 1/2
77.0	83.0	79.5	..	84.6	80.0	84.2	84.3	84.5	84.0	87.8	87.9	92.9	92.9	92.9	1 1/2
274.5	84.2	77.9	..	84.8	78.4	84.5	84.6	..	83.2	87.8	87.9	93.4	93.4	93.4	1 1/2
..	82.2	276.4	..	84.5	76.1	84.7	78.9	..	81.7	237.8	287.8	93.2	93.2	93.2	1/2
..	280.3	..	280.0	281.5	274.7	84.7	74.9	281.8	279.3	292.8	292.8	292.8	Sur.
St. Louis	Oakland calif	Nashville	Albuquerque	Phoneix	Atlanta	Sandiego	Charleston	Elpaso	San-Antonio	Browns Ville	Miami	Swan Island			
38°36'	37°48'	36°	35°	33°20'	33°45'	32°47'	32°54'	31°50'	29°30'	25°55'	25°46'				

Wet-Bulb potential temperature (θ') is a very close approximation to θ_s when θ' is derived from adiabatic diagrams (*vide* 4.2.2 of my paper on "Saturation and Wet-Bulb Temperatures") Both the θ_s diagrams may, therefore, be regarded as diagrams showing the distribution of θ' with height. Again, as Equivalent potential temperature (θ_E) is a single-valued function of θ_s , the diagrams also represent the vertical distribution of θ_E . It may also be mentioned here that as $\log \theta_E$ is proportional to the total entropy of air (*i.e.* the entropy of mixture of dry air and water-vapour), the θ_s diagrams give the vertical distribution of total entropy of air with height and latitude.

Isopleths of θ_s in both the diagrams have been drawn at intervals of 2°C. The Tropopause also has been indicated.

The following points regarding the distribution of θ_s are of interest.

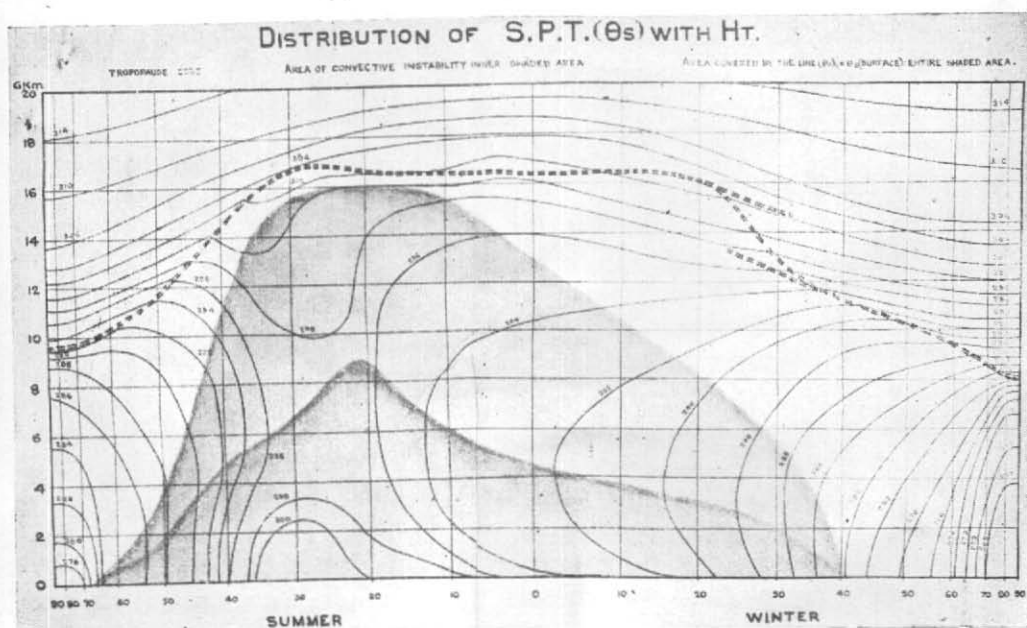
(1) Two shadings are shown in each of the diagrams. The lower shaded area represents the area where θ_s and hence also θ' , θ_E and total entropy (S) decrease with height (*i.e.* $\frac{\delta \theta_s}{\delta z} < 0$, $\frac{\delta \theta}{\delta z} < 0$, $\frac{\delta \theta_E}{\delta z} < 0$, $\frac{\delta S}{\delta z} < 0$). As $\frac{\delta \theta_s}{\delta z} < 0$, is the condition for convective instability, the lower shaded area represents the area of convective instability. This, as may be seen from the diagrams extends from 0° to 60° in Summer and from 0° to 40° in Winter. The maximum height upto which convective instability extends is about 7 to 8 g.km. in Summer near lat. 25°.

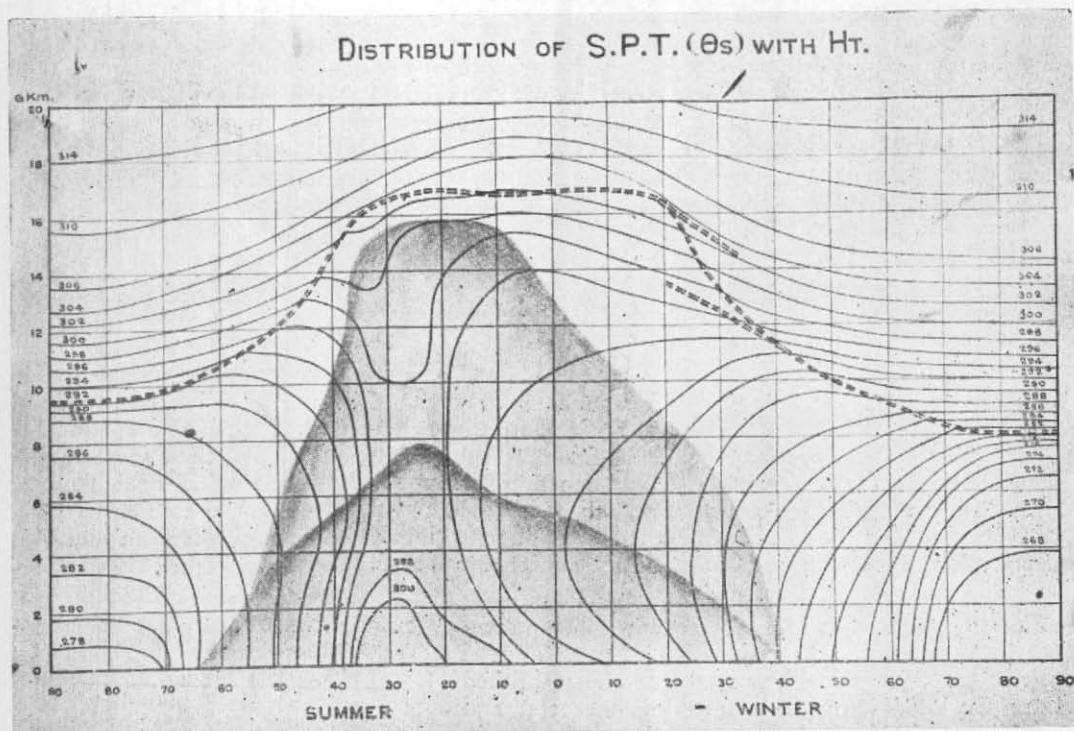
(2) Above the boundary of the lower shaded area θ_s increases with height. The boundary of the upper shaded area gives the heights at which θ_s equals the surface value.

$$(\theta_s)_{\text{surface}} = (\theta_s)_h$$

(3) The height of the upper boundary surface does not anywhere extend into the stratosphere. It has a maximum height of about 16 g.km. between lat. 10° and 30° during Summer.

(4) The difference between θ_s isopleths and those of potential temperature (θ), in the troposphere except near the poles is very striking. This is due mainly to the influence of Water-vapour.





In conclusion I wish to express my grateful thanks to Sir Charles Normand (former Director General of Observatories) for suggesting the problem and for valuable guidance in the course of the work.

Source of data :—

- | | | |
|-------------------------|-----|---|
| (1) Indian Stations : | ... | Data collected by the India Meteorological Department. |
| (2) England : | ... | Observatories year book of the London Meteorological Office (upto 1937). |
| (3) U. S. A. Stations : | ... | These form the largest in number. The data of these stations are based on Radio-sonde ascents during 1939-41. These have been taken from the 'Monthly Weather Review' Washington for the years 1939-41. |
| (4) Munich etc. : | ... | 'Deutsche Seewarte. 1934, 35, 36. |
| (5) Wagner : | ... | Climatologic der Freien Atmosphere Handbuch der Klimatologic Band 1, Teil F—Berlin 1931. |
| (6) Batavia : | ... | W. V. Bemmlen, Konig. Magn. en. Met. Obs. Batavia, Verhandhugen No. 4, 1916. |

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