Relationship between Vertical Currents and Intensity of Precipitation

J. M. SIL

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RATE of condensation in a rising column of saturated air can be easily found out when it is assumed that the column is thermally and physically isolated. For purpose of evaluating the rate of precipitation further assumption is made, *viz.*, all the products of condensation are immediately detached from the system and fall out as precipitation. On these assumptions formulas for the amounts of precipitation have been given by several writers. Fulks[1] recently derived a workable formula from which the rate of precipitation can be evaluated when the physical state of the upper layers and the velocity of the ascending layer are known. The subject has been briefly discussed here following Holmboe[2].

Take a sample saturated air of $(1+w_s)$ tons having one ton of dry air and w_s ton of water vapour. While it ascends under pseudo-adiabatic process, precipitation from this mass will be at the rate of $-\frac{dw_s}{dt}$.

$$\text{We write } -\frac{dw_s}{dt} = -\frac{dw_s}{dz} \cdot \frac{dz}{dt} = \frac{-dw_s}{dz} \cdot v_z$$

For simplifying calculation we take $v_z = 1$ m/sec to begin with. We may then say that when the mass of $(1+w_s)$ tons of the saturated air is lifted one metre the amount of precipitation will be $-\frac{dw_s}{dz}$ ton. Per ton of saturated air, the precipitation is therefore $-\frac{1}{1+w_s}\cdot\frac{dw_s}{dz}$ ton. If we consider in the above sample of air a column with cross-sectional area of one square metre and height of one dynamic—decimetre, the mass of the column will be $\frac{1}{\alpha\, g}$ ton. The precipitation from this column will therefore be

$$P_1 = -\frac{1}{\alpha\, g\ (1+w_s)}\cdot\frac{dw_s}{dz} \text{ or } = -\frac{1}{\alpha\,(1+w_s)}\cdot\frac{dw_s}{d\phi}$$

Here, α is the sp. vol. of the saturated air,

w_s the mixing ratio,

ϕ the dynamic height,

and g the acceleration of gravity.

During the pseudo-adiabatic process the heat released by condensation is used up in heating the entire mass of the air. Therefore $L dw_s = (1+w_s)\,(C_p\, dT - \alpha\, dp)$ or

dividing by $d\phi$, $-L\frac{dw_s}{d\phi} = (1+w_s)\,(C_p\frac{dT}{d\phi} - \alpha\frac{dp}{d\phi})$ $\cdots\cdots$ (2)

Eliminating $\frac{dw_s}{d\phi}$ between (1) and (2) and substituting

$d\phi = -\alpha\, dp$, we get, $P_1 = \dfrac{1+C_p\frac{dT}{d\phi}}{\alpha\, L}$

Here, C_p is the sp. heat of moist air at constant pressure, and L the heat of condensation or of sublimation according as the process takes place in the rain stage of snow stage.

If we assume that the condensed water will fall on ground over an area of one square metre the rain collected per second will be $P_1 \cdot 10^3$ millimetres, or at the same rate the rainfall is $3{\cdot}6\times10^6 P_1$ mm/hr.

For ease of calculation, a column of 100 dynamic metres in height, ascending with a velocity of v_z m/sec. may be taken. In that case the rate of precipitation becomes

$$P_{100} = 3{\cdot}6\times10^9\ \frac{1+C_p\frac{dT}{d\phi}}{\alpha\, L}\cdot v_z \qquad \cdots\cdots (4)$$

Or, putting $3{\cdot}6\times10^9\dfrac{1+C_p\frac{dT}{d\phi}}{\alpha\, L} = r$

We may write $P_{100} = r.v_z$

Precipitation from the entire column would be $P = \Sigma\, r.v_z$ $\cdots\cdots$ (5)
when the total height of the saturated layer is broken up into layers of 100 dy.m. thickness. The value of r may be obtained from RAOB data and that of P readily from rainfall intensity record; the value of v_z can thus be determined.

Take the rainfall at 2130 hrs. I.S.T. on 22nd June, 1945. From radiosonde ascent at 20.30 hrs. we find the conditions of the upper layers as follows :—

p	t	u	p	t	u
912	25	85	600	4	100
900	21	90	500	−4	100
850	19	95	400	−14	..
750	13	100	300	−28	..
700	11	100	200	−46	..

Ascent curve indicated that the surface air had a lapse rate just greater than the dry-adiabatic. It was, therefore, indifferent and would rise with small perturbations. Rising from the ground a parcel of air would get saturated at about 900 mb. level and thereafter continue to rise due to buoyancy until it came to equilibrium with its environment at about 200 mb. level.

Assuming v_z=1 m/sec. the rate of precipitation has been calculated from the entire saturated layer as follows :—

Layer. mbs.	H dyn. m.	r mm./hr.	$\frac{H r}{100}$
800—700	1188	0·75	8·9
700—600	1287	0·50	6·4
600—500	1428	0·44	6·2
500—400	1700	0·31	5·2
		Total ..	26.7 mm/hr.

Sample calculation :—Take the layer 800—700 mb.
Mean value in the layer p=75·0 cb., T=286 °·5 K
$w_s = 13{\cdot}3 \times 10^{-3}$
Thickness of the layer is given by

$$H_{700} - H_{800} = 28{\cdot}7\ (273 + t_m\ {}^{\circ}C)\ \text{in}\ \frac{p_{700}}{p_{800}}$$

$$= 28{\cdot}7\ (273 + 13{\cdot}5)\ \text{in}\ \frac{8}{7} = 1188 \text{ dyn. metres.}$$

$$\frac{dT}{d\phi} = \frac{-5}{11880} = -0{\cdot}42 \times 10^{-3} \text{ deg./dy. dm.}$$

$$C_p = C_{pd}\,(1 + 0.9w) = 1004\,(1 + 0{\cdot}9 \times {\cdot}013) = 1016 \text{ kj/ton.}$$

$$C_p \frac{dT}{d\phi} = -1016 \times 0{\cdot}42 \times 10^{-3} = -0{\cdot}426$$

$$\alpha = \frac{R_d T}{p} = \frac{287 + 286{\cdot}5\,(1 \times {\cdot}61 \times {\cdot}013)}{75{\cdot}0} = 1107 \text{ m}^3/\text{ton}$$

$$r = 3{\cdot}6 \times 10^9 \times \frac{1 + C_p \frac{dT}{d\phi}}{\alpha L} = 3{\cdot}6 \times 10^9 \times \frac{1 - 0{\cdot}426}{1107 \times 2{\cdot}5 \times 10^6}$$

$$= 0{\cdot}746 \text{ mm./hr.}$$

The rain commenced at 2130 hrs., *i.e.*, about one hour after the ascent was taken. The structure of the rainfall as shown by the intensity record is given below :—

Date	Rainfall: Total amt. in inches	Rainfall: Duration	Periods of intensity in minutes: 0·5"/hr	1"/hr.	2"/hr.	3"/hr.	4"/hr.	5"/hr.	6"/hr.	
22nd June, 1945.	1·2	52 mins.	9	7	5	3				1st shower.
			25	17	10	5	3	2	2	2nd shower.

The average rate of rainfall, as found from the record, is $\frac{1\cdot2 \times 60}{52} = 1\cdot39$ in./hr. or $= 35\cdot2$ mm./hr. Rate of precipitation, as calculated, is $26\cdot7$ mm./hr. The average rate of ascent of the saturated column is therefore v_z av. $= \frac{35\cdot2}{26\cdot7} = 1\cdot3$ m./sec.

During the fall the maximum intensity attained was 6 in./hr. or 152 mm./hr. for a very short time. It will not be incorrect to assume that only v_z changed during the period and that other factors contributing to the rate of precipitation remained more or less constant. Thus, v_z max. $= \frac{152}{26\cdot7} = 5\cdot7$ m./sec.

A few rainfall records have been studied together with the upper-air conditions prevailing shortly before the fall, and proceeding in the way mentioned above the speed values of the vertical currents of saturated air have been derived. Unfortunately the number of occasions selected for study so far has not been large. However, the results of study are given in Tables 1 and 2 below :

TABLE 1.

Structure of Rainfall.

Date	Rainfall: Time I.S.T.	Rainfall: Duration in h. m.	Rainfall: Amt. in inches	Duration of Intensity of Rainfall in minutes: 0·5"/hr.	1"/hr.	2"/hr.	3"/hr.	4"/hr.	5"/hr.	6"/hr.	Average	Max.
22-6-1945	21·30	0—52	1·2	34	24	15 (5+5 +5)	8 (3+5)	3	2	2	1·4	6·0
6-7-1945	19·30	0—55	0·9	26	22	11 (3+2 +6)	3	2	..	..	0·88	4·0
17-7-1945	22·00	0—45	1·3	38	19	9 (2+3 +2 +2)	2	..	..	..	1·73	3·0
21-7-1945	22·11	3—48	4·87	134	92	31 (5+5 +2 +3+2 +2+3 +4 +5)	12 (2+3 2+7)	6 (2+4)	..	..	1·28	4·0
14-8-1945	19·45	1—17	1·3	57	23	8 (3+3 +2)	3	2	..	..	1·0	4·0

TABLE 2.

Speed of vertical currents in convective layer and their duration, as derived from analysis of rainfall data in table 1. V_z vertical velocity in metre per sec. and d the duration in minutes.

Date	V_z	d	V_z	d	V_z	d	V_z	d	Average speed m/sec	Max speed. m/sec.
22-6-1945	1·9 (3)	5	2·9 (2)	4	3·8 (1)	2	5·7 (1)	2	1·3	5.7
6-7-1945	1·8 (3)	4	2·7 (1)	3	3·6 (1)	2	..		0·78	3·6
17-7-1945	2·2 (4)	2·3	3·3 (1)	2	..	..	..	..	1·9	3·3
21-7-1945	2·5 (10)	3	3·8 (3)	4	5·1 (2)	3	..	..	1·6	5·1
14-8-1945	1·9 (3)	2·7	2·8 (1)	3	3·8 (1)	2	..	..	0·95	3·8

Table 1 shows among other things how long it rained at different intensities. For intensities greater than 2 in./hr. the number of showers together with their duration in minutes are shown by figures in bracket. The last two columns give respectively the average and the maximum values of intensity during the fall.

In respect of each rainfall the speed, which the ascending currents of saturated air might attain, has been derived and shown in Table 2 together with their duration in minutes. The figures in bracket indicate the number of occasions this value of the speed was found during the fall. It will be seen that the highest value of v_z is about 5·7 metres per sec., or about 12·7 miles per hour, which has been obtained during the rainfall on 22nd June, 1945. The average speed is found to be about 1·3 m./sec.

As the speed of ascending currents is closely related to intensity of rainfall the structure of convectional rain is of particular interest. Thunder showers occuring at Poona during 1939-45 were therefore grouped in Table 3 according to month, and from the records of Intensity Raingauge [3] information was collected about the intensities and their duration during each fall. During heavy rain the showers are quite distinct. Duration of each shower has therefore been indicated by figures in bracket. It will be seen that thunder-storms during September and October generally give more intense rain at Poona and the maximum intensity of rain recorded there is about 6 in./hr. During the 35 thunder-storms 211 showers occurred which had intensity of 2 in./hr. or greater. These high intensity showers have been grouped in Table 4 according to the maximum intensity attained by them. The duration of showers of a certain intensity has been averaged and its mean value also given.

TABLE 3.

Duration, in minutes, of intensity of thunder showers at Poona. Figures in bracket indicate duration of separate showers.

Date	Intensity of rain											Total amount inches
	Time	1"/ hr.	1·5"/ hr.	2"/ hr.	2·5"/ hr.	3"/ hr.	3·5"/ hr.	4"/ hr.	4·5"/ hr.	5"/ hr.	5·5"/ hr.	
25-4-1940	15·00	25	22	12 (5+5 +2)	10 (5+3 +2)	6 (4+2)	3	2	..	..	..	1·61
20-5-1940	16·41	10	6	1								0·11
21-5-1940	17·40	14	4	3								0·28
19-5-1943	17·50	24	10	7 (2+2 +3)								0·69

Date	Time	Intensity of rain 1"/hr.	1·5"/hr.	2"/hr.	2·5"/hr.	3"/hr.	3·5"/hr.	4"/hr.	4·5"/hr.	5"/hr.	5·5"/hr.	Total amount inches.
21-5-1943	18·00	17	9	6 (3+3)	6 (3+3)	5 (3+2)	5 (3+2)	5 (3+2)	3	2		} 1·15
"	22·00	10	8	6 (3+3)	4 (2+2)	2						
18-6-1939	13·45	8	4	2								0·20
19-6-1939	14·42	18	10	6	5	3	3	2	2			0·50
22-6-1940	15·15	6										0·23
27-6-1941	14·30	7	6	3	2	2						1·27
23-6-1942	16·15	18	12	10 (6+1)	9 (6+3)	8 (6+2)	8 (6+2)	6	4	3	2	0·88
28-6-1943	14·45	6										0·50
10-6-1944	19·30	7	5	3	2	2	2					0·78
3-7-1939	19·30	10	7	2								1·18
29-8-1943	15·15	7	7	3	2	2						0·31
17-9-1940	13·45	7	5	3								} 3·54
"	14·30	15	9	8	4	4	3	2				
"	16·30	16	9	2								
"	17·40	8	6	6 (3+3)	(3+3)	5 (2+3)	5 (2+3)	5 (2+3)	5 (2+3)	2		
19-9-1940	17·47	12	3									0·30
29-9-1943	22·00	36	31	29 (6+7 +8 +3+3 +2)	23 (5+7 +8 (+3)	19 (4+6 +7 +2)	19 (4+6 +7 +2)	16 (6+7 +3)	15 (6+7 +2)	12 (5+7)	7 (4+3)	} 3·04
16-9-1944	13·32	8	4	2	2	2						0·65
25-9-1944	16·17	3	2	2								0·14
1-10-1939	16·14	16	14	12 (5+4 +3)	8 (3+3 +2)	4	3	2	2	2		0·58
2-10-1939	14·15	34	23	9 (3+6)	8 (3+5)	3	2					0·60
19-10-1939	17·45	15	11	10 (4+6)	7 (4+3)	5 (3+2)						0·90
13-10-1940	17·17	24	18	15	12	10 (4+3 +3)	7 (4+3)	6 (3+3)	2	2		1·30
14-10-1940	21·45	19	14	7 (4+3)	6 (4+2)	4	3	2	2	2		1·70
10-10-1941	15·08	11	10	10 (4+3 +3)	8 (3+3 +2)	4 (2+2)	4 (2+2)	2				0·47
17-10-1943	14·10	22	20	20 (5+5 +4 3+3)	16 (4+4 +3 3+2)	9 (3+3 +3)	9 (3+3 +3)	6 (3+3)	3			1·83
10-10-1944	18·30	5	4	4	2	2						0·41
11-10-1944	17·16	23	22	11	10	9 (4+5)	7 (3+2 +2)	7 (3+2 +2)	5 (3+2)	4 (2+2)	4 (2+2)	1·25
13-10-1944	22·13	10	4	4	3	2	2					0·27
20-10-1944	15·14	9	7									0·17
21-10-1944	18·45	5	4									0·14

TABLE 4.

Thundershowers having intensity of 2 in./hr. or greater.

Intensity attained		5·5"/ hr.	5"/ hr.	4·5"/ hr.	4"/ hr.	3·5"/ hr.	3"/ hr.	2·5"/ hr.	2"/ hr.
Number of showers	..	5	10	14	21	28	36	41	56
Per cent	..	2·4	4·7	6·6	10·0	13·3	17·0	19·5	26·6
Average duration in mins. during a shower	..	2·6	2·9	3·07	3·0	3·04	3·1	3·8	3·94

It will be seen that out of 211 showers only 5 reached an intensity of 5·5 in./hr. while 56 of them reached a maximum intensity of 2 in./hr. only. Generally a shower with an intensity of 4 in./hr., or greater, lasts for 2 to 3 mins. and that with an intensity between 2 to 4 in./hr. lasts for about 3 minutes only.

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