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A numerical method for velocity determination in shallow refraction work

SURENDRA SINGH
School of Physics
University of Science of Malaysia, Penang, Malaysia

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ABSTRACT. Graphical method fails to give a reliable estimate of the velocity when the refracting boundary is uneven. A numerical routine is suggested in which the uneven boundary's arc lengths are estimated by straight line segments. This arc length method is tested for a hypothetical and some field data. Results indicate that there is a significant improvement in the estimate of the velocity when the refracting boundary is not flat.

1. Introduction

The velocity and depth are the two basic types of information that one seeks from a refraction survey. To derive the basic equations for the determination of velocity information, certain reasonable and, widely valid assumptions are made: each layer within a stratigraphic sepuence is isotropic with regard to its propagation velocity, ray paths are made up of straight line segments, and each layer has a higher velocity than the overlying one.

The simple case of two layers with plane and parallel boundaries is illustrated in Fig. 1. The velocity of the top layer in this case is the reciprocal slope of the first segment on the time-distance curve and the velocity of the second layer is the reciprocal slope of the second segment.

However, if the boundary between two layers is not as simple as shown in Fig. 1, there is an alternative numerical method (Redpath 1973) to determine the true velocity V_2, of the second layer. As shown in Fig. 2, the two points G_1 and G_2 define the length and position of the refraction traverse. The data for the direct profile will be taken as the source moves away from G_1 toward G_2 on G_1 G_2 line, where fixed geophones G_1 and G_2 are the end-of-line geophones. Similarly, the data for the reverse profile is obtained as the source starts from near the geophone G_2 and moves away toward the end-of-line geophone G_1. As can be seen from time-distance curves in Fig. 2, it is not always with certainty that one can determine the true velocity V_2 of the second layer by graphically fitting straight line segments. However, the velocity of the top layer V_1 can be, in all practical cases, determined reliably. The time taken for the refraction arrival through the second layer for the direct profile can be written as

$$T_d = Z_1/(V_1 . \cos\alpha) + [X - (Z_1 + Z_S) . \tan\alpha]/V_2 + Z_S/(V_1 . \cos\alpha). \quad (1)$$

where Z_1, Z_2 are the normal depths to the refracting horizon from G_1 and G_2, respectively; V_1, V_2 are the velocities in layer 1 and 2, respectively, X is the horizontal distance between the fixed geophone G_1 and the movable source S, and α is the angle of critical incidence and is equal to $\sin^{-1}(V_1/V_2)$. The refraction item for the reverse profile for the same source positon will be

$$T_r = Z_1/(V_2 . \cos\alpha) + (H - X) - (Z_2 + Z_S) \tan\alpha)/V_2 + Z_S/(V_1 . \cos\alpha) \quad (2)$$

where H is the total horizontal length of the traverse G_1G_2. The difference in the refraction times at the two fixed geophones G_1 and G_2 from the same position of the source S, will be

$$(T_d - T_r) = (Z_1 - Z_2)/(V_1 . \cos\alpha) + (Z_2 - Z_1) . \tan a/V_2 - H/V_2 + 2X/V_2 \text{ or}$$

$$(T_d - T_r) = \text{constant} + 2X/V_2. \quad (3)$$

Therefore, if the difference in arrival times is plotted against the horizontal distance X, the slope of such a line will be equal to $2/V_2$. The velocity of the second layer, V_2, can thus be computed numerically.

2. Theory

Strictly speaking, Eqns. (1), (2) and (3) would not be valid if refracted energy travels along an

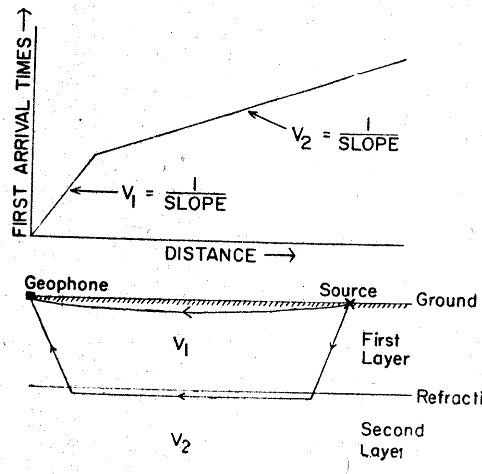

Fig. 1. Time-distance curve for two horizontal layers

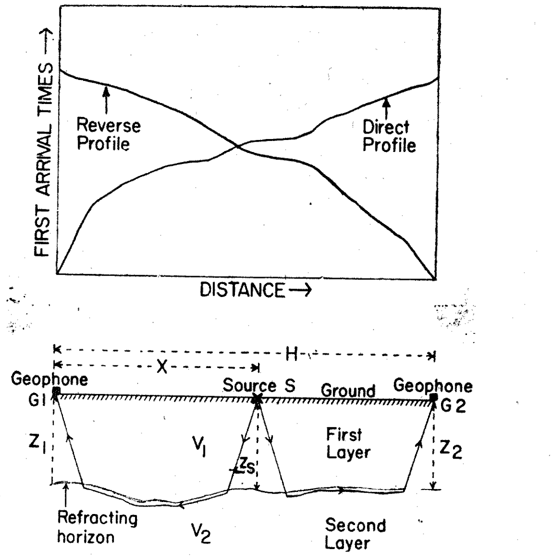

Fig. 2. Direct & reverse profiles for an uneven refracting boundary

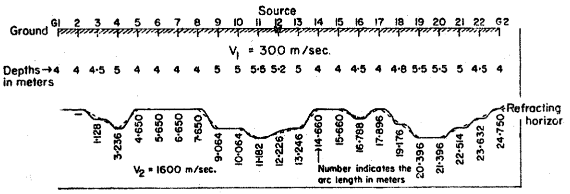

Fig. 3. A hypothetical configuration for a two-layer case

uneven surface, and the arc length distances should be used, therefore, rather than straight horizontal distances. Eqn. (3) is correct if the refraction horizon is reasonably flat, so that its length is about the same as the horizontal distance X.

Let $u_d(X_M)$ be the arc length along the smooth uneven refracting horizon, at a horizontal distance X_M, from a fixed geophone G_1 (Fig. 3) for the direct profile. Let Z_M be the normal depth at this distance. Then $X_{M+1}=X_M+I$, where I is the incremental horizontal distance between two consecutive positions of the movable source on the surface. It will be assumed here that the points which are vertically under the source are situated on practically horizontal parts of the uneven boundary. The implication is that the boundary should be horizontal for about $Z_M \cdot \tan\alpha$ length on each side under the source position. The assumption is similar to the one made in the conventional delay-time method. This would allow us to take the delay times at a particular position of the source to be equal for direct and reverse profiles, both. Let the incremental differences in depths be $\triangle Z_{M+1}=Z_{M+1}-Z_M$. Then, approximating the uneven boundary by straight line segments as done in Fig. 3.

$$u_d(X_{M+1}) \simeq u_d(X_M) + [I^2 + (\triangle Z_{M+1})_d{}^2]^{\frac{1}{2}},$$

subscript d denotes the direct profile, etc.

$$\vdots$$

$$u_d(X_N) \simeq u_d(X_M) + \sum_{i=1}^{N-M} [I^2 + (\triangle Z_{M+1})^2{}_d]^{\frac{1}{2}} \tag{4}$$

Using the delay time concept (Deobrin 1960), the refraction times can be expressed as

$$T_d = D_{G_1} + D_S + u_d(X_N)/V_2 \text{ for the direct profile} \tag{5}$$

$$T_r = D_{G_2} + D_S + u_r(H - X_N)/V_2 \text{ for the reverse profile} \tag{6}$$

where D_{G_1} is the delay time at geophone G_1, etc. Also the total time T_t taken from G_1 to G_2 is

$$T_t = D_{G_1} + D_{G_2} + \frac{u_d(H)}{V_2} \tag{7}$$

Substracting Eqn. (6) from Eqn. (5), we get

$$(T_d - T_r) = (D_{G_1} - D_{G_2}) + [u_d(X_N) - u_r(H - X_N)]/V_2 \tag{8}$$

The first term on the right hand side in Eqn. (8) is constant as geophones are fixed. The above equation shows that if there are no lateral variations in the velocity V_2, a plot of the difference in arc lengths *versus* the difference in refraction times for the direct and reverse profiles would be a straight line and the reciprocal slope of this line will be the velocity of the second layer.

3. Full equipment

The field results reported in this paper were obtained with a signal enhancement seismograph which is the modern version of the instrument described by Gough (1952). Various versions of this instrument are manufactured under different trade names. All of them have the common feature of enhancing the signal by electronically stacking the successively received signals from the same one location of the source. A ten pound hammer with a trigger switch mounted on it is the energy source. The operator may enhance the signal from the hammer by repeating the impact at the same hammer station and adding these two or more signals. In this fashion, successive measurements of the first arrival-times can be made as we increase the distance of the hammer from the geophone along the traverse line.

4. Procedure

(1) Choose a traverse length such that first arrivals from any third layer (if any there) do not reach the geophones. So this becomes a two layer problem.

(2) Next, take the data of refraction times for as many equally spaced source positions between two fixed geophones as possible.

(3) Plot the results on a graph paper and see if there are enough over lapping points where both the geohpones receive refraction arrivals from the second layer. If not, "phantom" the direct and reverse profiles (Redpath 1973) to get more of such overlapping points. The phantoming for the direct profile can be done by offsetting the geophone G_1 along the line by a distance approximately more than the critical distance and then hammering at the same old set of locations of the source, starting at the previous geophone location G_1. While plotting the results for the phantom curve, distances are measured from the old geophone location and not from the offset location. The reverse profile is phantomised in the same way. The phantom curve should be parallel to the previous curve when geophones were at G_1 and G_2 locations. The phantom data can thus be used to extrapolate the refraction times at distances smaller than critical distances.

(4) Determine the delay times $D_S = \frac{1}{2}(T_d + T_r - T_t)$, where T_t is the time taken from G_1 to G_2 or G_2 to G_1. Normal depths to the refracting boundary can then be determined by multiplying delay times with a factor which is $V_1/\cos[\sin^{-1}(V_1/V_2)]$. The value of V_2 used here is very rough estimate of V_2, and can be obtained graphically from the plot of the time-distance data. Error in the estimate of V_2 would not influence the computed values of depths appreciably. In most field cases, $V_1/V_2 < 0.3$. Suppose we take $V_1/V_2 = 0.3$, $V_1 = 300$ m/sec. and $V_2 = 1000$m/sec. The above factor, then, would have a value equal to 314. If the values are $V_1/V_2 = 0.2$, $V_1 = 300$ m/sec and $V_2 = 1500$ m/sec, the factor is 306. Thus a variation of 50 per cent in the value of V_2 would cause a variation of 2.54 per cent only in the value of the multiplying factor. Hence a rough estimate by the graphical method is good enough.

It should be pointed out here that the delay times are always given by $\frac{1}{2}(T_d + T_r - T_t)$ whether the boudary is flat or not, as can be verified from the basic Eqns. (5), (6) and (7).

(5) Compute depth increments $\triangle Z_M$. The value of the approximated arc length for the direct profile up to $X = X_N$ will be given by

$$u_d(X_N) = u_d(X_M) + \sum_{i=M+1}^{N} [I^2 + (\triangle Z_i)^2]^{\frac{1}{2}} = u_d(X_M) + F_d \tag{9}$$

and for the reverse profile, arc length from $X = X_N$ to G_2 will be given by

$$u_r(H - X^N) \simeq u_r(X_M) + \sum_{i=N+1}^{J-M+1} \left[I^2 + (\triangle Z_i)^2 \right]^{\frac{1}{2}} = u_r(X_M) + F_r \tag{10}$$

where $H = (J-1).I$,

and $u_r(X_M) = u(H) - u_d(X_{J-M+1})$

In other words, $u_d(X_5)$ would mean the arc length beginning at geophone G_1 and $u_r(X_5)$ would mean the arc length beginning at geophone G_2 and at the 5th source location from geophones G_1 and G_2 respectively. Further, incremental depths $\triangle Z_i$ always correspond to the direct profile. That is, $\triangle Z_5$ means the value at X_5 from geophone G_1. Then,

$$u_d(X_N) - u_r(H - X_N) \simeq u_d(X_M) - u_r(X_M) + (F_d - F_r) \tag{11}$$

The value of distance X_M will be decided by the nature of the data. For example, if the refraction phase starts only after 5th source location and the data before the 5th source location cannot be used for the computation of delay times, then $M = 5$. Delay time at a source location can be determined only if both the geophones receive the refracted signal through the second layer from the particular location of the source. Further, X_M does not have to be the same for the reverse profile as that for the direct profile. However, if it is different, then the summation index in Eqn. (10) should be adjusted accordingly.

(6) The first part of the right hand side of Eqn. (11) is a constant whose value need not be known. Substituting into Eqn. (8) from Eqn. (11),

$$(T_d - T_r) \simeq (D_{G_1} - D_{G_2}) + [u_d(X_M) - u_r(X_M)]/V_2 + [(F_d - F_r)]/V_2$$

$$\text{or } (T_d - T_r) = \text{constant} + (F_d - F_r)/V_2 \tag{12}$$

The slope of the curve between $(T_d - T_r)$ and $(F_d - F_r)$ will determine the value of the velocity V_2.

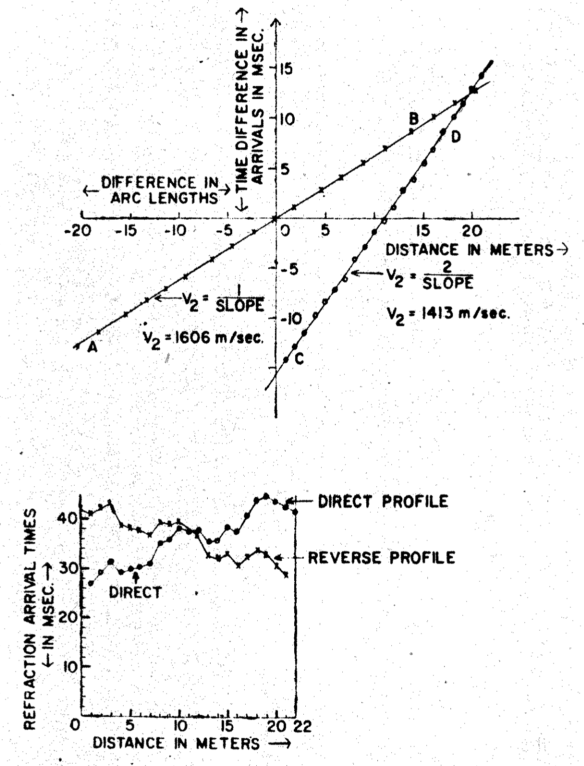

Fig. 4. velocity determination from refraction data for the hypothetical configuration

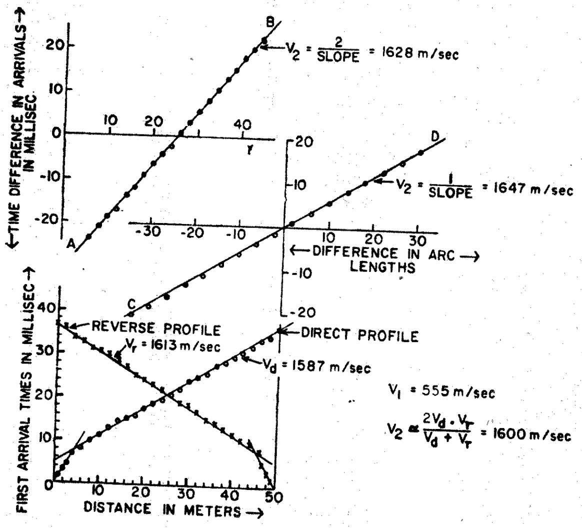

Fig. 5. velocity determination from refraction data for the Hockey field

5. Discussion of results

(1) A hypothetical case is shown in Fig. 3, with an uneven refracting boundary. Depths from the surface (around 4 metres) velocity of the top layer and second layer (300 and 1600 m/sec) are representative values found in field survey that will be discussed later in the following paragraphs. The actual boundary (solid line) is approximated, as required in the theory, by horizontal and sloping flat straight line segments (dashed line). If the source positions are closely spaced, this would give a very good approximation to the real length of the boundary. However, the sources should not be located closed to each other, otherwise the assumption of spot horizontality of the boundary may not be valid at all of these locations. A compromise has to be made. For example, in this case the boundary should be practically horizontal for about 0.76m on each side under the source location. Thus, the sources should be spaced from each other much more than 1.5 m apart. Refraction signal will start arriving in this case after 4. tan $(\text{Sin}^{-1}\ 300/1600) = 0.76$ metre. Refraction arrivals will not be always the first arrivals. But it is not any problem since computation can be done on a computer and direct arrivals through the top layer need not be considered.

Results of the hypothetical configuration in Fig. 3 are listed in Table 1 and plotted in Fig. 4. As can be seen, one cannot graphically determine the velocity accurately. Numerical methods have to be applied. A plot of horizontal distance in column 1 *versus* time difference in column 4 of Table 1, according to Eqn. (3), is shown by the line CD, the slope of it being $2/V_2$ and it gives a value of $V_2 = 1413$ m/sec. This value is in error by about 12 per cent. If we plot, (*see* Eqns. 10, 11 and 12) the difference in arc lengths in column 11 *versus* time difference in column 4, line AB is obtained. The reciprocal slop of AB is the velocity V_2 which is 1606 m/sec whereas the true velocity is 1600 m/sec. This demonstrates the significant improvement in the estimate of the velocity if arc lengths are used instead of the horizontal distances.

(2) Fig. 5 shows the results of a refraction traverse in the Hokey field of the University of Science of Malaysia. This location had been extensively surveyed previously since students run their geophysical experiments in seismics and resistivity here every year. The refracting horizon is a pretty flat horizontal boundary over lengths of hundreds of feet. The velocity estimate by the usual graphical method is 1600m/sec, the numerical method using horizontal distances (line AB) gives a value of 1628m/sec and the numerical method using are lengths (line CD) gives a value of 1647m/sec. As expected, all three different estimates are close to each other.

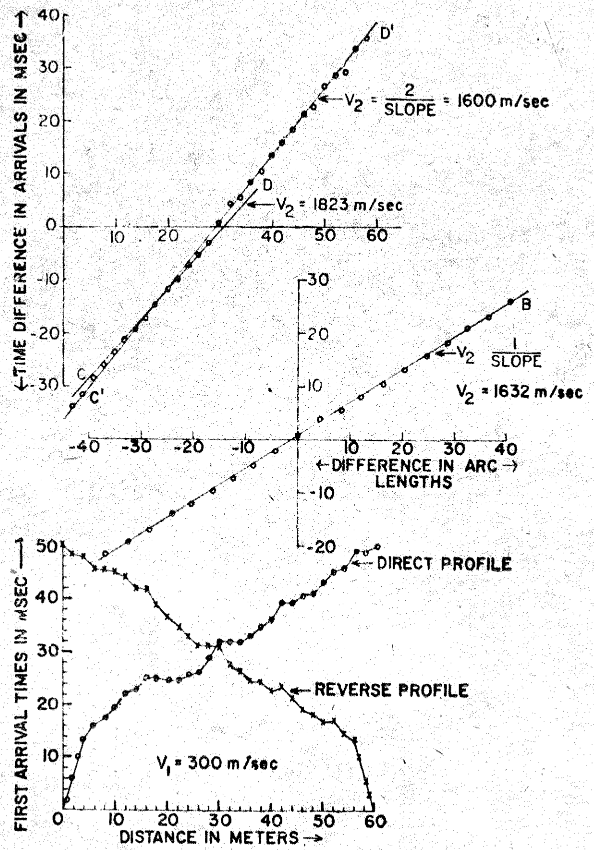

Fig. 6. Velocity determination from refraction data for the Balik Pulau area

(3) Need for a numerical routine as presented in this paper was felt while running a refraction survey in the coastal area of Balik Pulau on the Penang Island where the University of Science of Malaysia is situated. Coastal area of Balik Pulau is flat Holocene and Pleistocene sediments between the abruptly rising hills and the Indian Ocean, about one and a half mile away. Streams must have flown over the granite bedrock to make it an uneven boundary for the depositing sediments. In such a case, it was felt that if an accurate determination of velocity could be made, then we would not need to worry about the velocity values at every location of the traverse in the area since the area is pretty much homogeneous, geologically.

The results of a refraction traverse in the Balik Pulau area are plotted in Fig. 6. Many such traverses were taken in this area, but only a typical traverse is presented here. Again, graphical method cannot reliably estimate the velocity since there is too much uncertainty involved in fitting a straight line to the field data. Plots of direct and reverse profiles shows clearly that the refracting horizon is not a flat one. When time differences are plotted against the horizontal distances, there is still some uncertainty as shown by two possible lines (CD and C'D'). velocity $V_2 = 1823$ m/sec using line CD and 1600m/sec using line C'D'. However, if arc lengths are used (line AB), the velocity can be determined more reliably and it is 1632m/sec.

6. Conclusion

The velocity of the second layer in a two layer case can be determined more accurately by using arc lengths of an uneven refracting horizon instead of horizontal distances. The added accuracy of the method suggested herein is due to the fact that it uses a better approximation to the real length of the refracting boundary. A good approximation of arc lengths can be obtained by straight line segments. Results presented indicate that arc length method of the velocity determination can estimate the velociy more reliably for an uneven refracting boundary between two layers. In a geologically homogeneous area, an accurate estimate of the velocity can be representatively used in the whole area. One situation where considerably accurate velocities must be known is the shallow-reflection problem. In the absence of any drill-hole data, large errors can accur in the computations of depths if rough values of velocities are used. Arc length method involves more computation and relatively more time. However, considering that it has be done only once, the extra effort is worth it. The velocity of the top layer can, in all practical cases, be determined reliably. Once the velocity of the second layer also is estimated reliably, the velocity of the third layer, depths to the second and third layers etc can be estimated better than using a rough estimate of the second layer velocity at each and every location of the traverse in the area.

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TABLE 1

Determination of the velocity of the second layer for the hypothetical configuration in Fig. 3

X (m) (1)	T_d (msec.) (2)	T_r (msec.) (3)	ΔT (msec.) (4)	D_S (msec.) (5)	Z_M (m) (6)	ΔZ_M (m) (7)	W_M (m) (8)	A_K $(=F_d)$ (9)	A_J $(=F_r)$ (10)	$(A_k—A_J)$ (m) (11)
1	26.8	28.5	—14.2	13.1	4.0					
2	29.1	30.8	—12.8	14.7	4.49	0.49	1.114	1.114	21.512	—20.4
3	31.5	33.2	—11.4	16.4	5.0	0.51	1.122	2.236	20.389	—18.1
4	29.1	33.8	—9.6	13.1	4.0	—1.0	1.414	3.650	18.975	—15.3
5	29.7	32.2	—8.4	13.1	4.0	0.0	1.0	4.650	17.975	—13.3
6	30.3	30.4	—7.2	13.1	4.0	0.0	1.0	5.650	16.975	—11.3
7	30.9	32.8	—6.0	13.1	4.0	0.0	1.0	6.650	15.975	—9.3
8	35.1	31.9	—4.2	16.4	5.0	1.0	1.414	8.064	14.561	—6.5
9	35.7	32.5	—2.9	16.35	4.99	—0.01	1.0	9.064	13.561	—4.5
10	38.1	36.6	—1.4	18.0	5.49	0.5	1.118	10.182	12.443	—2.3
11	37.7	37.9	—0.2	17.0	5.19	—0.3	1.044	11.226	11.399	—0.2
12	37.7	39.5	1.1	16.35	4.99	—0.2	1.019	12.246	10.379	1.9
13	35.3	38.6	2.8	13.1	4.0	—0.99	1.407	13.653	8.972	4.7
14	35.9	39.3	4.0	13.1	4.0	0.0	1.0	14.653	7.972	6.7
15	38.3	36.9	5.5	14.75	4.5	0.5	1.118	15.771	6.854	8.9
16	37.4	37.5	7.0	13.1	4.0	—0.5	1.118	16.889	5.736	11.1
17	40.8	38.1	8.6	15.7	4.79	0.79	1.274	18.164	4.461	13.7
18	43.8	38.7	10.0	18.0	5.49	0.70	1.221	19.384	3.241	16.1
19	44.5	42.9	11.3	18.05	5.51	0.02	1.0	20.385	2.241	18.1
20	43.5	41.9	12.7	16.35	4.99	—0.52	1.127	21.512	1.114	20.4
21	42.6	41.0	14.1	14.75	4.5	—0.49	1.114	22.625		
22	41.6	41.6$=T_t$								

Explanation of Table 1

X is the horizontal distance in metres of the hammer from geophone G_1,

T_d is the time of arrival of head waves for the direct profile,

T_r is the time of arrival of head waves for the reverse profile, where T_d and T_r both were computed using arc lengths,

ΔT is the time difference $(T_d - T_r)$ in arrivals at geophones G_1 for the direct profile, and G_2 for the reverse profile from the same location of the hammer,

D_s is the delay time at the location of the hammer and is equal to $\frac{1}{2}(T_d + T_r - T_t)$, where T_t is the total time taken by the head wave from G_1 to G_2,

Z_M is the normal depth at the location of the hammer and is euqal to $D_S . V_1 / \cos(\sin^{-1} V_1/V_2)$. Values of V_1 and V_2 are known in this hypothetical case,

ΔZ_M is the incremental depth, *i.e.*, $\Delta Z_M = Z_M - Z_{M-1}$.

W_M is the incremental arc length which is equal to $[(\Delta Z_M) + I^2]$ where I is the incremental horizontal distance between two successive locations of the hammer,

A_K is the arc length u p to K^{th} hammer location and is equal to $\sum_{i=3}^{k} W_i$

$K=3, 4, 5, \ldots\ldots, 22$ in Table 1, and is equal to F_d in Eqn. (12).

A_J is the arc length from K^{th} to the last hammer location, and is equal to $\sum_{i=K+1}^{J} W_i$, and $J=22$ in Table 1.

A_J is equal to F_r in Eqn. (12).

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