Statistical characteristics of some Indian hydrologic time series

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ABSTRACT. Extensive literature exists on the analysis and construction of models of stream-flows in the temperate regions, whereas the hydrology of tropical monsoon affected rivers have not received much attention. An analysis of the monthly and annual rainfall and runoff data from the Krishna and Godavari river basins in India and the Wabash river basin in the United States are presented in this paper. The Krishna and Godavari rivers are in the tropical, monsoon region whereas the Wabash river is in the temperate zone. The basic statistics, and the spectral analysis of the data are presented and compared. The implications of the data structure to modeling these flow sequences are discussed.

1. Introduction

Since the early fifties considerable amount of water resources development has been undertaken in India. These projects are related to hydroelectric power generation, irrigation, flood control and so on. However, the characteristics of hydrologic time series such as those of Indian river flows and precipitation does not appear to have been systematically analyzed and the results reported in the literature. The ultimate use of such analyses is in the development of appropriate models which are useful both for simulation or synthetic generation of hydrologic data and for forecasting the processes one or more time steps ahead. The forecast results may be used in operation of water resource systems.

The use of simulated data in water resource systems design has been well known. The role of forecasting hydrologic data and using them in operating water resources systems is receiving greater attention only recently. In economies such as thatof India the delay or failure of even a single monsoon can lead to tremendous hardships. Therefore, it is important to have the capability of forecasting the important hydrologic variables and to use these forecasts to update the operating policies of water resources systems.

In view of these considerations a systematic study of the Indian precipitation and river flow data was undertaken and some of the results are presented herein. The characteristics of the monthly and annual river flows from the Krishna and Godavari rivers in south India, and the corresponding precipitation data from several stations in the Krishna and Godavari river basins are analyzed and the results are presented in this paper. These results give the details of the stochastic structure of the precipitation and river flow time series. Classes of stochastic models which are suitable to characterize the flows in these rivers may be selected by using the results of data analyses such as those presented in this paper.

The development and validation of stochastic models for the monthly and annual flows in the Krishna and Godavari rivers have been discussed in detail elsewhere (Rao and Kashyap 1973, Rao et al. 1975). The validation of the models included both the tests on residuals and the comparison of characteristics of the data generated by using these models with those of the observed data. Linear stochastic difference equation models with mixed distributions fitted to the histograms of residuals are adequate to characterize these river flows. Although it is possible to select a "best" model in a particular class of models, several equally "good" alternative models may be developed for a given time series. However, simulation and forecasting studies by using some selected models would normally reduce the choice and render the model selection much easier. These aspects are discussed in greater detail in Kashyap and Rao (1976). The type of analysis presented in this paper, however, is the

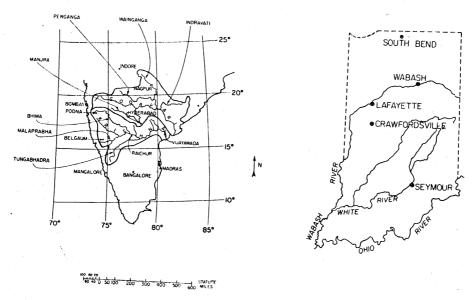


Fig. 1. Location of hydrologic stations

basic first step in the development of models for hydrologic time series.

Statistical characteristics of flows in rivers in different climatic zones significantly differ from each other. A desirable property in the family of models chosen for modeling river flow sequences is that it must be general enough so that flows in rivers which are in different climatic zones can be modeled by a member in the family. Furthermore, it is interesting to inquire whether the models of flows in rivers which are geographically close to each other and which are in the same general climatic zone are substantially different from each other. In order to answer these questions, the data analysis techniques of this paper were used to construct models of the monthly and annual flows of the Krishna, Godavari, and Wabash rivers. The Krishna and Godavari rivers are geographically close to each other, and are in the tropical zone. The Wabash river in the United States is in the temperate zone.

An important aspect which deserves attention is the similarity (or the lack of it) in the statistical characteristics of hydrologic time series from different parts of India. Such an analysis would greatly help in understanding the hydrology of the country much better. Methods such as those discussed in this paper may be used as a first step in such a study.

2. Physiographic and climatic information

The Krishna and Godavari rivers are two of the major rivers in central and south India. The watersheds of these rivers and some other details are shown in Fig. 1. Both of these rivers have their origins in the Western "Ghats" (mountains) near Bombay. Although these rivers originate near

Bombay, and run an approximately parallel course, they maintain their separate identities and discharge separately into the Bay of Bengal. The Krishna and Godavari rivers flow in a variety of terrain. Some of the major tributaries of Krishna are the Bhima river in the north, Malaprabha and Tungabhadra in the south. The Godavari has two major tributaries: Penganga river in the north and the Manjra river in the south. The Wabash river, in Indiana, USA, has a smaller watershed area than either Krishna or Godavari although it drains the surface runoff from a large part of the State of Indiana and from smaller parts of Illinois and Ohio. The most important tributary of the Wabash is the White river, and the Wabash itself drains into the Ohio river. Some details of these watersheds and the statistical characteristics of the data related to these revers are shown in Table 1.

Until about 1950, very few dams and other major retention structures existed in the Krishna, Godavari and their tributaries. During the midfities, several major dams on the Krishna and its tributaries were designed and constructed. The flow in the Wabash and its tributaries is controlled by several minor dams and other retention structures. Thus, there is some flow modification in these rivers, although it is very small.

The claimatic conditions of the Wabash river basin are different from the climatic conditions of the Krishna and Godavari river basins. Trevartha (1943) has classified the general climate of the Wabash river basin as belonging to "Humid Continental, Warm Summer" (Da) class whereas the climatic conditions in Krishna and Godavari river basins are classified by him as a combination of "Tropical Savannah (AW)

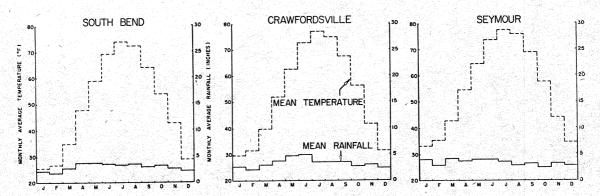


Fig. 2(a). Monthly mean rainfall and temperature variation in some stations in the Wabash river watershed

and Steppe (BS)". As the temperature and rainfall distributions are among the important factors which affect the climate and hence the hydrology and ecology of the river basins (MacArthurr 1972), it is instructive to compare the temporal distributions of average monthly temperature and rainfall in these two basins. The average monthly rainfall at several stations in or near the Wabash river basin indicates a rather uniform distribution throughout the year whereas the average temperature changes drastically (Fig. 2a). The average temperature at several stations in the Krishna and Godavari river basins and also at some neighbouring stations (Fig. 2b) does not fluctuate as much as the rainfall. Furthermore, drastic variations in average monthly rainfall in the Krishna and Godavari river basins are observed along the north-south direction (Nagpur, Hyderabad, Bangalore) and along the east-west direction (Madras, Bangalore, Mangalore). Such drastic variations in average monthly rainfall are absent in the Wabash river basin.

These rainfall and temperature variations at different locations have given rise to the development of different climatic regions within a rather small geographic area in central and south India. According to Mizukoshi (1971), who has used the classification by Koppen (1961) to delineate these climatic regions of south India, the following climatic regions can be found in Krishna and Godavari river basins. Most of the central Krishna river basin has the Steppe (or BS) climate, whereas the eastern portions and the western headwaters of the Krishna river basin are in regions of Tropical Savannah (AW) climate. The southern part of the Godavari river basin has the Steppe (BS) climate, the central portion is in the Tropical Savannah (AW) climate, whereas the northern parts of the Godavari basin are in the warm temperate rainy climate with dry winters (CW). The BS climates are dry, whereas the AW climates have lesser rainfall than the tropical rainforest climates (Af). The rainfall in the Af climate is unevenly distributed throughout the year so that there is a distinctly wet and a distinctly dry season. The CW climates are warm temperate and rainy.

The Wabash river basin, on the other hand, is almost entirely in the humid continental climate with warm summers (Da). Trevartha (1943) has a detailed description of the Da climate. A detailed analysis of the Krishna and Godavari river basin climates is found in Mizukoshi (1971).

3. Temporal and spatial variation in monthly rainfall

The average monthly rainfall at several points in the Krishna and Godavari river basins (Fig. 2b) indicate the effects of both the southwest (June-September) and the northeast (October-December) monsoons. The stations along the west coast of India (Bombay, Mangalore) receive very heavy rainfall during the southwest monsoon season with very small effects from the northeast monsoons. The stations along the east coast (Waltair, Madras) indicate the opposite effect in the sense that the rainfall contribution to the vearly total from the northeast monsoon is more than the contribution from the southwest monsoon. The stations in the central regions of the Peninsula (Bangalore, Hyderabad and Nagpur) exhibit a comparatively uniform rainfall distribution although there is a large variability in the annual rainfall among these stations. The average rainfall ranges from about 25 to 45 inches from east to west in the Krishna and Godavari river basins. The variations in the average annual rainfall in the Krishna and Godavari river basins are discussed in greater detail by Rao and Jagannathan (1963) and by Tsuchiya (1971).

The rainfall in the Wabash river basin, in contrast to the rainfall in the Krishna and Godavari river basins is fairly uniform over the year. The average annual rainfall in the Wabash river basin is about 36 inches. Besides this uniform temporal distribution, the spatial distribution of rainfall over the Wabash basin is also relatively uniform (Fig. 2a). Consequently, the spatial and temporal charcteristics of rainfall in the Wabash and Krishna and Godavari river basins are very different. This is reflected in the runoff characteristics of these rivers,

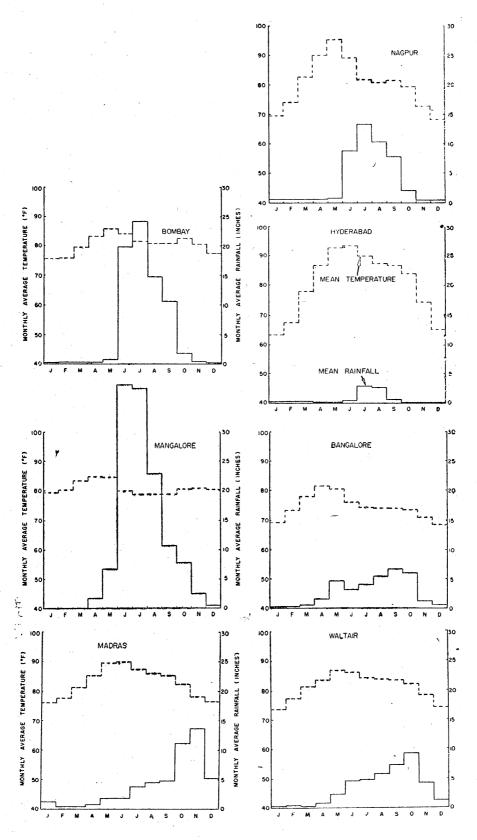


Fig 2(b). Monthly mean rainfall and temperature variation in several Indiana stations in or near Krishna and Godavari watersheds

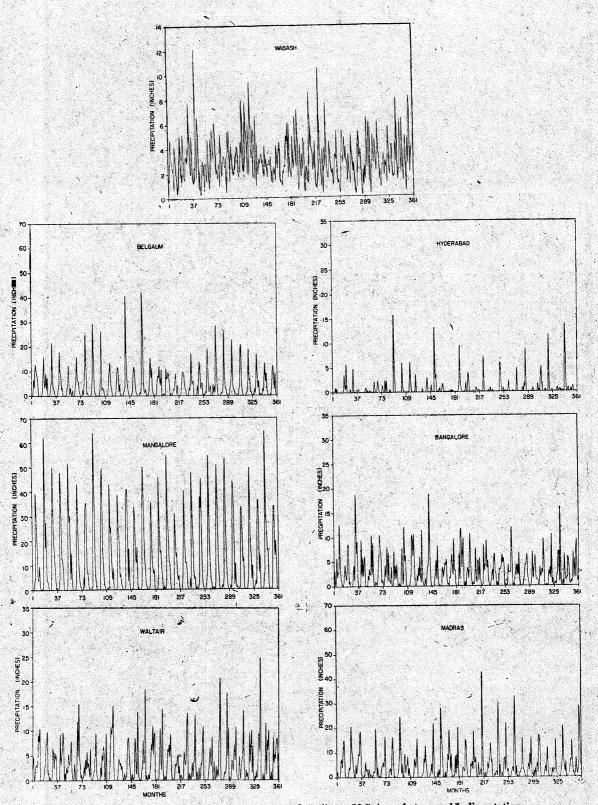


Fig. 3. Varaiation in monthly rainfall at Wabash, Indiana, U.S.A. and at several Indian stations

The variations in the monthly rainfall in several stations in the Krishna, Godavari and Wabash river basins are shown in Fig. 3. The data ex-

tends from January 1901 to December 1930 for the Indian stations and from October 1940 to November 1969 for the Wabash, Ind. station,

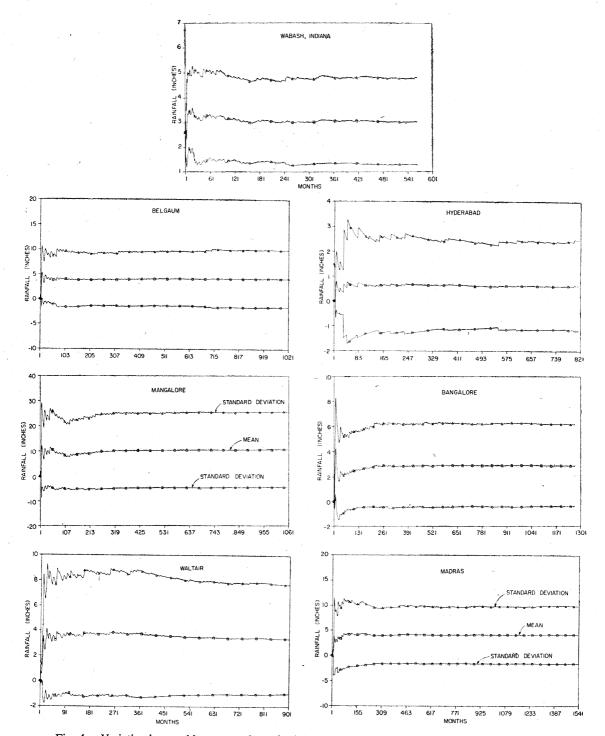


Fig. 4. Variation in monthly mean and standard deviation of rainfall with sample size

For the stations in the Krishna and Godavari watersheds, apart from the variation in the rainfall in the same month, at the same station, from one year to the next, there is considerable variation in rainfall among different stations. For example, during the month of July in 1908, the rainfall in Bangalore was 64 inches while in Bangalore it was only 4.12 inches. Because of

these large variations, the estimation of rainfall over the entire Krishna and Godavari watersheds is difficult.

4. Some statistical characteristics of monthly rainfall

4.1. Continuous means and standard deviations The continuous mean values of monthly rainfall were computed by Eqn. (1) where P(i) is

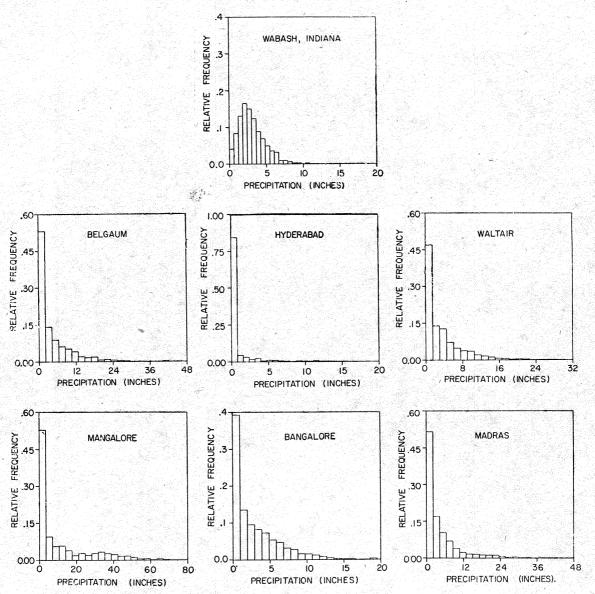


Fig. 5. Histograms of monthly rainfall from some Indian stations

the monthly rainfall

$$\overline{P}(j) = \frac{1}{j} \sum_{i=1}^{j} P(i) \tag{1}$$

The continuous standard deviation of monthly rainfall values were computed by Eqn. (2).

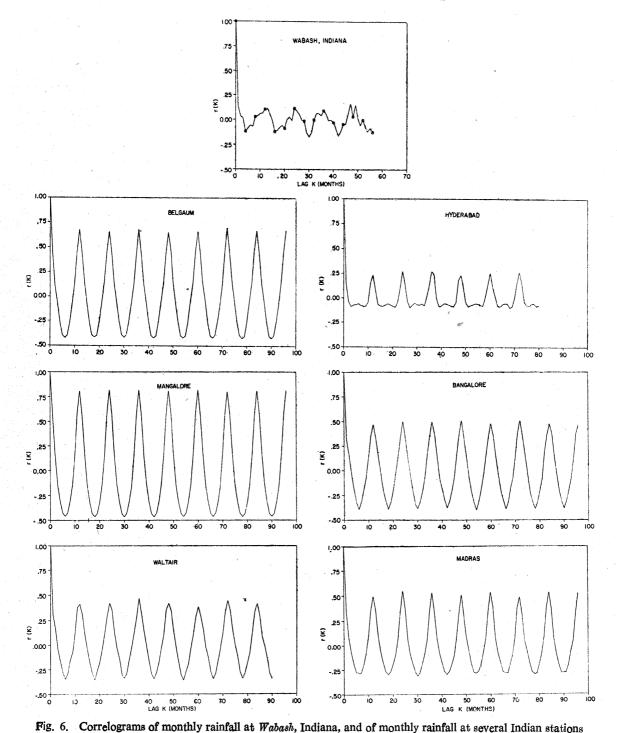
$$S(j) = \sqrt{\sum_{i=1}^{j} [P(i) - \overline{P}(j)]^{2}}$$
 (2)

The variation in the continuous means and standard deviations are shown in Fig. 4 for rainfall from several stations in the Krishna and Godavari river basins and for the Wabash, Ind. station in the Wabash river basin. The details of data used in the computation of the continuous means and standard deviations are given in Table 1. The continuous means and standard

devitations of monthly rainfall data show large fluctuations up to about 180 samples (15 years) after which they tend to a constant value. However, both the continuous mean and standard deviation show a slight "drift" even after about 500 samples.

4.2. Histograms

The seasonal nature of Indian rainfall is clearly brought about by the histograms of monthly data shown in Fig. 5. These histograms are highly skewed. Monthly rainfall values are very small over 40 to 70 per cent of the time. The histogram of rainfall data from the Wabash, Ind. stations is not as skewed as the histograms of rainfall data from Indian stations. The probability distributions of monthly rainfall data from all the Indian stations are clearly non-Gaussian Logarithmic, square root or cube root transformation of these data do not render them to be appriximated by a Gaussian distribution. The



monthly rainfall data from the stations in the Wabash river basin, however, can be made to fit Gaussian distribution by using a square root transformation.

4.3. Correlograms and power spectra of monthly rainfall data

The correlograms of moothly rainfall data are shown in Fig. 6. The correlation coefficeints were computed by calculating the auto-

covariances $\gamma(k)$ of the rainfall data at lags k by using Eqn. (3).

$$\gamma(k) = \frac{1}{N-k} \sum_{i=1}^{N-k} \left[P(i) - \overline{P}(N) \right]$$

$$\left[P(i+k) - \overline{P}(N) \right]; k = 0, 1, \dots, (3)$$

In Eqn. (3), N is the total number of data points, $\overline{P}(N)$ is the overall average monthly rainfall. The autocovariances $\gamma(k)$ were divided by the variance $\gamma(0)$ to obtain the correlation coefficients r_k . The correlograms are shown in Fig. 6.

$$r_k = \frac{\gamma(k)}{\gamma(0)} \tag{4}$$

Two observations are apparent from an inspection of the correlograms of monthly rainfall data shown in Fig. 6. First, the strong, consistently seasonal effect of the southwest monsoon in comparison with the rather weak seasonal effect of the northeast monsoon is revealed, for example, by a comparison of the correlograms of rainfall data from Mangalore and Belgaum with those from Madras and Banglore and Waltair. The correlogram of rainfall data from Hyderabad reflects the effects of long periods of scanty rainfall. Secondly, in all these correlograms of monthly rainfall from Indian stations, a yearly cycle is predominant.

The correlogram of monthly rainfall at Wabash, Ind. does not reflect the effects of strong periodicities. Even the annual cycle is not as clearly apparent in the correlogram of monthly rainfall data from the Wabash, Ind. station as it is in the correlograms of rainfall from Indian stations. The correlation coefficients are much smaller for the rainfall from Indian stations. The rainfall in the Wabash basin in comparison with the rainfall in the Krishna and Godavari basins. The correlogram of the Wabash, Ind. rainfall data has a decaying tendency which is not apparent in the correlograms of Indian rainfall data.

The power spectra of the monthly rainfall data were also computed and are shown in Fig. 7. The raw estimates of the power spectrum were obtained by using Eqn. (5).

$$PS(\omega_h) = \frac{2}{\pi} \sum_{K=0}^{M} E_h \gamma(k) \cos \frac{hk\pi}{M}$$
 (5)

where, $\omega_h=\frac{\hbar\pi}{M}=$

frequency in radians per time unit.

$$E_{k} = \begin{cases} 1 & 0 < k < \overline{M} \\ \frac{1}{2} & k = 0, M \end{cases}$$

 $h = 0, 1, 2, \ldots, M$

M = Integer nearest to 0. IN, N being the number of observations

 $\gamma(k)$ = Autocovariance at lag k in months

 $PS(\omega_h)$ = Raw estimate of the power spectral density at frequency w_h .

These raw estimates were smoothed by using the Hamming window to obtain the final estimates of the power spectrum which are given by:

for
$$k = 0$$
: $S(\omega_0) = 0.54 PS(\omega_0) + 0.45 PS(\omega_1)$
 $0 < k < M$: $S(\omega_k) = 0.23 PS(\omega_{k-1})$
 $+ 0.54 PS(\omega_k) + 0.23 PS(\omega_{k+1})$
(6)

$$k = M$$
 : $S(\omega_M) = 0.54 PS(\omega_M) + 0.46 (\omega_{M-1})$

The accuracy of computations was checked by computing the quantity

$$(\pi/M) \left[\frac{1}{2} [S(\omega_0) + S(\omega_M)] + \sum_{K=1}^{M-1} S(\omega_k) \right]$$

which must be equal to γ (0).

The power spectral estimates which were computed by the procedure outlined above were normalized by dividing them by γ (0) to obtain the power spectral density functions which are shown in Fig. 7. The following dominant frequencies are apparent from these plots.

	12 Month	6 Month	4 Month	3 Month
Wabash, Ind.	X	x		
Belgaum	X	- x	x	x
Hyderabad	x	. х	x	
Waltair	X	x ·	x	
Mangalore	x	x	X	x
Bangalore	x	x	x	
Madras	, x	x	x	X
in the second second			1.5	

The power at frequencies other than those shown is very small for the rainfall data in the Krishna and Godavari river basins. On the other hand, for the monthly rainfall at Wabash Ind., although the power in the frequencies corresponding to 6 to 12 months is predominant than at other frequencies, it is far from negligible at other frequencies.

In view of these considerations, the monthly runoff data from the *Krishna* and *Godavari* rivers may be expected to exhibit strongly periodic behaviour and also to have highly skewed non-normal distributions. The *Wabash* river runoff data, on the other hand, may be expected to have comparatively weaker periodic behaviour.

5. Some statistical characteristics of monthly river flows

5.1. Monthly means and standard deviations

The monthly mean flows in rivers are obtained by avearging the daily mean flows and hence these are called monthly mean flows or monthly

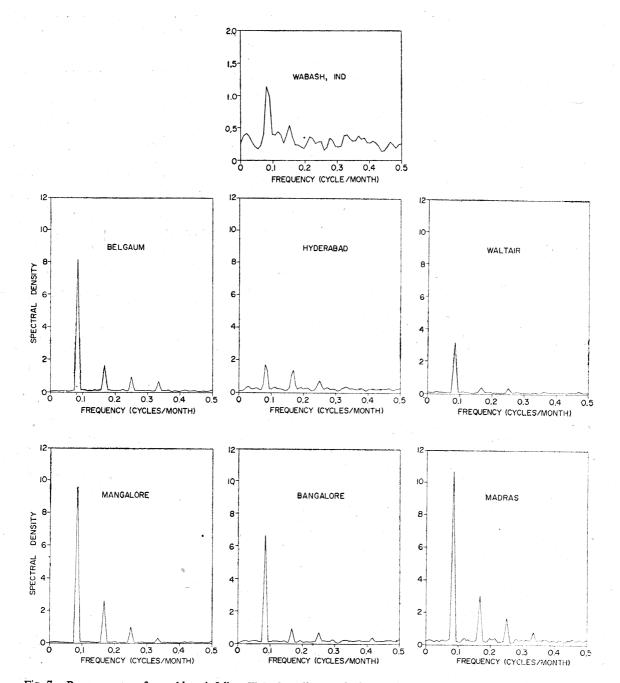


Fig. 7. Power spectra of monthly rainfall at Wabash, Indiana and of monthly rainfall at some Indian stations

average discharges. In the following discussion we will refer to the monthly mean flows as monthly flows.

The flows in the same month over the years were used to compute the mean and standard deviations of flows in that month. The resulting values are shown in Fig. 8 for the Krishna, Godavari and Wabash rivers. The high flows in the Wabash occur during the months of April to August whereas they occur during July to September in the Krishna and Godavari rivers. Al-

though the combination of snowmelt and summer thunderstorms cause the high flows in the Wabash river in the April-August period, the differences in the monthly mean flows in the Wabash river are not so drastic as they are for the Krishna and Godavari rivers. For example, the greatest change in the monthly flow in the Wabash river is in the mean flows between March and April when the flows increase from 590 to 1106m³/sec. For the Krishna and Godavari the greatest change occurs between June and July when the flows increases from 597

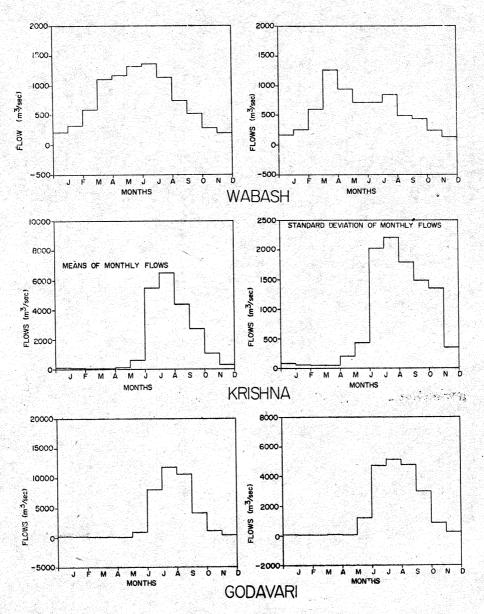


Fig. 8. Monthly means and standard deviations of (A) Wabash (B) Krishna and (C) Godavari river flows

to 5486 m³/sec for Krishna and from 930 to 8029 m³/sec for the Godavari. Furthermore, the flows in the non-monsoon seasons are extremely low in comparison with the monsoon season flows, indicating the absence of sustained, large scale ground water contribution to the flow. Secondly, the southwest monsoon affects the flows much more significantly than the northeast monsoon. The means and standard deviations of the entire sequences are given in Table 1.

5.2. Histograms of monthly flows

The predominance of very low monthly flows during several months of a year in *Krishna* and *Godavari* rivers results in extremely skewed his-

tograms (Fig. 9). A comparison of histograms of monthly rainfall in the basin and monthly runoff in the Krishna and Godavari rivers indicate considerable qualitative similarity. Approximately 60 per cent of the monthly flows in Krishna are less than about 600 m³/sec whereas 65 per cent of the flows are less than about 1700fl m³/sec in Godavari. The histogram of Wabash river flows is not as skewed as that of the Krishna and Godavari flows. However, there is a substantial difference in the histograms of monthly rainfall in the Wabash river basin and of the monthly runoff in the Wabash river. The skewness coefficient and the non centrality parameters corresponding to the histograms shown in Fig. 9 are given in Table 1.

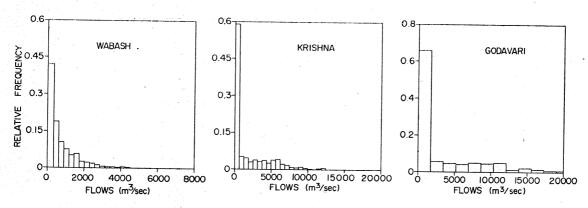


Fig. 9. Histograms of monthly flows

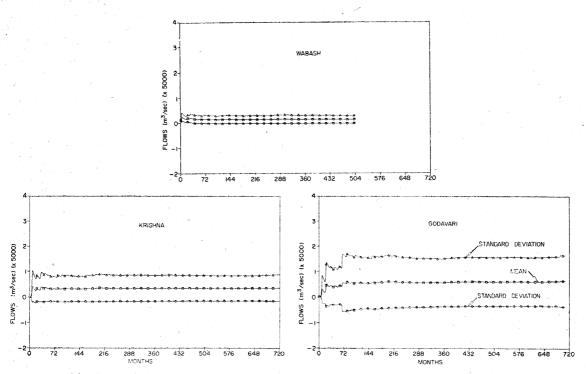


Fig. 10. Variation in mean and standard deviation of monthly flows with sample size

5.3. Continuous means and standard deviations of monthly flows

The continuous means and standard deviations of monthly flows which were computed by using Eqns. (1) and (2) are shown in Fig. 10. The monthly mean flows tend towards a "constant" value for about 140 to 150 observations (months) in all three rivers. The variability in the continuous means and standard deviations of monthly flows is not as large as the variability observed in the continuous means and standard deviations of rainfall (Fig. 4). However, the continuous means and standard deviations exhibit a slow drift, especially in the *Godavari* river data.

5.4. Correlograms and power spectra of monthly flows

The correlograms and power spectra of monthly flows are shown in Fig. 11. The correlograms of the *Krishna* and *Godavari* rivers exhibit strong periodic behaviour. A concentration of covariance at frequencies corresponding to 12, 6, 4 and 3 months is clearly discernable for the data from *Krishna*, while only the periodicities corresponding to 12, 6 and 4 months are prominent for the data for *Godavari*. The variance concentrated in other frequencies is much smaller. A slight decaying effect is present in the correlogram of *Godavari* river flows, which is not obvious in the correlogram of the *Krishna* river

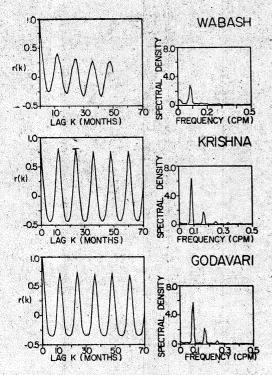


Fig. 11. Correlograms and power spectra of monthly flows

flows. The correlogram of Wabash river flows does not show the strongly periodic behaviour evident in the correlograms of Krishna and Godavari data, although the annual cycle is clearly evident. Also, the correlogram of the Wabash river flows is decaying with increasing lags, while this is not the case for the Krishna and Godavari river flows. The decaying effect in the correlogram of Wabash river flows is reflected in the increase in the low frequency power in the power spectrum.

The strong periodicities observed in the correlograms and spectra of monthly flows must be incorporated into models of monthly flows in these rivers. Secondly, the characteristics of the correlograms and spectra of the Wabash river data, which is in the temperate climate, are quite different from the coresponding characteristics of the monsoon-affected Krishna and Godavari, which have strong periodic behaviour.

6. Statistical characteristics of annual flows

6.1. Histograms of annual flows

The hitograms of annual flows are shown in Fig. 12. The histograms of anual flows are considerably more symmetrical than those of monthly flows which are given in Fig. 9. The statistics of annual flows are given in Table 1. The variances of annual flows are, as expected, smaller than those of the monthly flows.

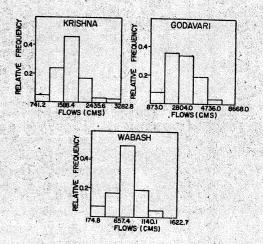


Fig. 12. Histograms of annual flows

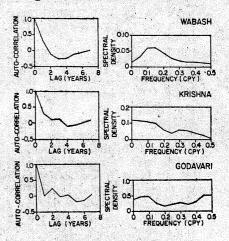


Fig. 13. Correlograms and power spectra of annual flows.

Two obvious inferences can be derived from the histograms of annual flows. The first of these is that the annual flows may be approximated by simple statistical distributions. The second is that the variability in the annual flows in *Krishna* and *Godavari* rivers is much larger than that in the *Wabash* river flows.

6.2. Correlograms and powerspectra of annual flows

The correlograms and power spectra of annual flows are shown in Fig. 13. The correlation coefficients decrease, and even assume negative values, up to lags of 4 to 5 years and then indicate a gradual increasing trend, although oscillating around zero. The negative values of the correlation coefficients at higher lags are as large or larger than the lag-one correlation coefficient for all the three rivers. Such an occurrence indicates that a model of orders higher than one or two would be necessary to model these flows. This aspect is discussed in detail in the second paper of the series. The power spec-

tral density functions also reflect these characteristics. There is a small concentration of low frequency effects apparent in the power spectral density plots.

7. Conclusions

The following conclusions may be presented as a result of the analysis of the statistical characteristics of the monthly and annual flow data:

- (i) The monthly flows of the Krishna and Godavari rivers are highly seasonal and cannot be considered as weakly stationary. The annual flows may be considered as weakly stationary processes.
- (ii) There is a much larger variability in the monthly and annual flows of Krishna and Godavari rivers compared to the Wabash river flows.
- (iii) The histograms of the monthly flows of all the three rivers are highly skewed and cannot be considered to be Gaussian. The annual flow histograms are much more symmetrical.
- (iv) The second order properties of Krishna and Godavari river flows are very similar to those of rainfall in the Krishna and Godavari river basins. This similarity is less evident for the data from the Wabash river basin.

References

- Köpen—Geiger, 1971, Klima Der Erde (Map) Hermann-Haak.
- Kashyap, R.L. and Rao, A.R., 1976, Dynamic Stochastic Models from Empirical Data, Academic Press, New York, NY, 330 pp.
- MacArthur, R.H., 1972, Geographical Ecology, Harper and Row, New York.
- Mizukoshi, M., 1971, Regional Division of Monsoon Asia by Köppen's Classification of Climate in *Water Balance* of Monsoon Asia, ed. by M.M. Yoshino, University of Hawaii Press, Honolulu, pp. 259-274.
- Rao, K.N. and Jagannathan, P., 1963, Climatic Changes in India, in *Chagnes in Climate*, Proceedings of the Rome Symposium organized by UNESCO and WMO, UNESCO, Paris, pp. 53-66.
- Rao, A.R. and Kashyap, R.L., 1973, Analysis, Construction and Validation of Stochastic Models for Monthly River Flows, Tech. Rept. CE-HYD-73-1, School of Civil Engineering, Purdue University, W. Lafayette, IN, 135 pp.
- Rao, R.G.S., Rao, A.R. and Kashyap, R.L., 1975, Aggregation of Empirical Data, Tech. Rep. CE-HID-75-6
 School of Civil Engineering, Purdue University, W
 Lafayette, IN, 71 pp.
- Trevartha, G.T., 1968, An Introduction to Climate, McGraw Hill Book Co., New York (4th Edn.).
- Tsuchiya, I., 1971, Fluctuation of Rainfall in Southeast Asia—Equatorial Pacific and Low and Middle Latitude Circulations in the Southern Hemisphere, in *Water Balance of Monsoon Asia*, ed. by M.M. Yoshino, University of Hawaii Press, Honolulu, pp. 217-240.