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A comparative study of plume rise formulae

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ABSTRACT. A comparative study of plume rise formulae by Briggs, Tennessee Valley Authority, Holland, Lucas ,Morton, *et al.* CONCAWE and whaley was made and it was found that slightly modified Lucas formulae appears to yield reasonable results under different stability and wind conditions.

1. Introduction

In any determination of concentrations downwind from a source it is essential to estimate the effective stack height—the height at which the plume becomes level. Seldom this height corresponds to the physical height of the stack. High emission velocity and higher temperature of the effluents than the ambient air at the stack top enchances the effective stack height above the physical height of stack. Aerodynamic downwash, eddies caused by the flow around buildings or the stack and also evaporative cooling of moisture droplets in the effluent may cause lowering of the plume to the extent that it may be lower than the physical stack height. Several investigators have proposed formulae for the estimation of effective stack height under given conditions. Moses and Storm (1961) have made a comparative study of some formulae and found that "there is no one formula which is outstanding in all respects".

Plume rise can be calculated as a function of source parameters, such as buoyancy, and meteorological conditions. Techniques for deriving this have been developed by several people and organisations but hardly any of them agree, either with each other or with new observations if they go outside the range of variables of observations the techniques were originally made to fit (Briggs 1975, Guldberg 1975). Guldberg (1975), after an extensive comparison of plume rise formulae, concluded that Briggs model (1969, 1971, 1972) predicts best the observed plume rise during periods of low wind speed and at higher wind speeds the Tennessee Valley Authority (TVA 1972) model suggested by Montgomery *et al.* (1972) performed best. Both these formulae need vertical temperature distributions which are not measured regularly at any station and hence cannot be used in general Holland (1953) developed an equation with experimental data from larger sources (stack diameters from 1.7 to 4.3 metres and stack temperatures from 82° to 204°C).

This equation, although physically sound, demands several parameters which also may not be available. Numerous empirical and semi-empirical formulae utilising heat emission and wind speed alone are developed both in Europe and North America (CONCAWE 1966; Whaley 1969, Lucas *et al.* (1963). Moore (1974) after a comparison of plume rises recommended the use of Lucas formula with certain modifications. From the above it is clear that the sophisticated formulae need additional parameters which are not conventionally measured at any meteorological observatory. Therefore, a necessity arose to seek simple formulae which require minimum data and the accuracy secured by using more refined formulae is not seriously affected. Therefore, in the present study a comparative sutdy of plume rise made by Holland, CONCAWE, Whaley, Lucas, Briggs, TVA and Morton *et al.* models was made to arrive, at an appropriate formula that can be adopted generally. In this comparison the conclusions of Guldberg (1975) referred to above were given due consideration.

2. Formulae used for comparison

(a) *Holland's equation*

Holland's equation is

$$\triangle h = \frac{V_s d}{u} \left[1.5 + 2.68 \times 10^{-3} \, p \left(\frac{T_s - T_a}{T_s} \right) d \right]$$

where,

$\triangle h$ =Plume rise (m)

V_s =Stack gas exit velocity (m/s)

d =Inside stack diameter (m)

u =Wind speed at stack level (m/s)

p =Atmospheric pressure (mb)

T_s =Stack gas temperature (°K)

T_a =Air temperature at stack level (°K) and 2.68×10^{-3} is a constant having units of $mb^{-1}\ m^{-1}$

Holland's formula frequently underestimates the effective stack height. A value between 1.1 and 1.2 times the $\triangle h$ from the equation should be used for unstable conditions; a value between 0.8 and 0.9 times the $\triangle h$ from the equation should be used for stable conditions.

(*b*) *Briggs equation*

Briggs equation for unstable and neutral atmospheric conditions is

$$\triangle h = 1.6 \frac{F^{1/3}}{u} (3.5x^{*2/3})$$

$F = g\ Q_H/\pi\ C_p\ \rho T$ (Buoyancy flux parameter m^4/sec^3)

g =acceleration due to gravity (m/sec^2)

Q_H =emission of heat (cal/sec)

C_p =specific heat of air at constant pressure (cal/gm/°K)

ρ =density of air (gm/m^3)

T =air temperature (°K)

u =wind speed (m/s)

and x^* $= 14\ F^{5/8}$ when $F < 55\ m^4\ sec^{-3}$

$= 34\ F^{2/5}$ when $F \geqslant 55 m^4\ sec^{-3}$

For stable atmospheric conditions the equation is modified as

$$\triangle h = 2.4 \left(\frac{F}{us}\right)^{1/3}$$

where, $$s = \frac{g}{\theta}\frac{\partial \theta}{\partial z}$$

θ = Potential temperature (°K) at stack level

(*c*) *TVA* 1971 *model*

Tennesse Valley Authority (TVA) 1971 model is

$$\triangle h = \frac{114\ CF^{1/3}}{u}$$

where, $C = 1.58 - (41.4\ \frac{\partial \theta}{\partial z})$

C is a coefficient based on atmospheric stability (dimensionless).

and other symbols are as in the Brigg's equation.

(*d*) *TVA* 1972 *model*

Tennesse Valley Authority (TVA) 1972 model is

$$\triangle h = \frac{173 F^{1/3}}{uE}$$

where, $E = \exp\left(0.64 \frac{\partial \theta}{\partial z}\right)$

and the other symbols remain the same as above.

(*e*) *CONCAWE formula*

CONCAWE formula which is generally used in Europe is

$$\triangle h = \frac{0.047\ Q_H^{0.58}}{u^{0.7}}$$

$\triangle h$ =Plume rise (m)

u =meanwind speed at stack top (m/s)

Q_H =heat emission (cal/sec)

(*f*) *Whaley's formula*

Whaley's formula for plume rise is

$$\triangle h = \frac{262\ \ Q_H^{0.24}}{u}$$

Q_H =heat emmission in MW and the other symbols are the same as above.

(*g*) *Lucas formula*

Lucas has developed a formula for unstable and neutral conditions as

$$\triangle h = \left(\frac{60 + 5H}{u}\right) \times Q_H^{0.25}$$

For average meteorological conditions

Where, $$\triangle h = \left(\frac{275 + 2H}{u}\right) \times Q_H^{0.25}$$

H =physical stack height and

Q_H =Heat emission from stack (MW)

(*h*) *Morton et al. formula*

Morton, Taylor & Turner formula for very stable conditions with little or no wind is

$$\triangle h = \frac{5 \cdot 0 F^{1/4}}{s^{3/8}}$$

where,

F =buoyancy flux parameter

$= g\ \ Q_H/\pi\ c_p\ \rho T$

s =stability parameter and other symbols have the same notation as in the Briggs equation.

For computing plume rise by the above formulae, plant characteristics and meteorological parameters are required. Plant characteristics are collected from the industry, while meteoroloical parameters like wind speed, pressure, air temperature are culled out of the records of India Met. Dep. Values of potential temperature gradient for highly unstable, neutral and stable conditions are taken as -0.0017, 0 and $+\ 0.0173$°K/m according to Guldberg (1975).

TABLE 1

Comparison of Plume Rise

Stability	Wind speeds at stack level (kmh)**	Holland's eqn.	CON-CAWE formula	Whaley formula	Modified Lucas formula	Lucas formula	Brigg's model	TVA : 1971 model	TVA : 1972 model
Highly unstable and low wind speed	4.0	289.8	300.6	491.7	699.3	699.3	649.4	887.6	909.99
	11.2	102.3	145.0	173.05	246.8	246.8	229.2	313.3	321.2
Unstable and low wind speed	4.0	285.9	300.7	491.7	699.4	699.3	664.8	867.8	862.4
	11.2	100.9	145.0	173.5	246.8	246.8	231.7	308.8	307.0
Neutral and low wind speed	4.0	248.0	300.7	491.7	699.4	699.4	638.3	841.6	808.4
	11.2	87.5	145.0	173.5	246.8	246.8	225.2	297.0	285.3
Stable and low* wind speed	5.4	151.4	235.6	347.0	153.7	500.8	123.9	322.7	187.3
Highly unstable and high wind speed	20.2	56.9	95.2	95.1	135.3	135.3	126.3	172.2	176.7
	31.3	36.0	69.0	60.2	85.6	85.6	79.9	108.9	111.7
	43.9	26.3	55.5	44.0	62.6	62.6	58.4	79.7	81.7
Unstable and high wind speed	20.2	58.9	95.2	95.2	135.3	135.3	127.9	170.0	169.0
	31.3	37.2	69.1	60.2	85.6	85.6	80.9	107.5	106.9
	43.9	27.2	55.5	44.0	62.6	62.6	59.2	78.7	78.2
Neutral and high wind speed	20.2	54.1	95.2	95.2	135.3	135.3	128.5	166.5	159.9
	31.3	34.2	69.1	60.2	85.6	85.6	81.3	105.3	101.1
	43.9	25.0	55.5	44.0	62.6	62.6	59.4	77.0	74.0
Stable and high wind speed	15.5	61.7	113.6	122.5	74.8	172.6	87.9	117.9	68.7
	28.4	33.8	74.6	67.1	41.2	95.1	72.0	64.6	37.6
	44.3	21.4	54.1	42.5	25.9	61.1	61.9	40.9	23.8
	61.6	15.6	43.6	31.7	19.0	43.9	55.9	29.9	16.4

*Plume rise by Morton *et al.* formula came out to be 279.4 m

**When wind speed >14.4 km/hr (High wind speed)

<14.4 km/hr (Low wind speed)

3. Results

A comparative study of plume rise by the above formulae under different stability and wind conditions is shown in Table 1. It is seen that plume rise as determined by slightly modified Lucas formula is close to Briggs model at low wind speed and TVA :1972 model at higher wind speeds. It, therefore, appears that slightly modified Lucas formula could be utilised for determining plume rise under the conditions obtainable at Delhi-Mathura and Agra region. Lucas formula for plum rise as modified by the authors is

$$\triangle h = \frac{60 + 5H}{u} Q_H^{0.25} \quad \text{(unstable and neutral conditions)}$$

$$\triangle h = \frac{116}{u} Q_H^{0.25} \quad \text{(stable and low wind speed)}$$

$$\triangle h = \frac{160}{u} Q_H^{0.25} \quad \text{(stable and highwind speed)}$$

where,

$\triangle h =$ Plume rise (m)

$H =$ Physical stack height (m)

$u =$ Wind speed at stack level (m/s)

and $Q_H =$ heat emission from stack (MW)

Winds at stack level are obtained from a power law derived from continuous records of two levels sensitive electrical anemometers and temperature sensors.

$$u = u_1 \left(\frac{z}{z_1} \right)^p$$

where u_1 and u are wind speeds at z_1 (lower) and z (higher) levels respectively and

$p = 1/9$ (unstable, very unstable)

$p = 1/7$ (neutral)

$p = 1/3$ (stable)

Five different winds speeds, *viz.*, 3.0, 8.5, 15.5, 24.0 and 33.5 km/hr are considered. Wind speeds at stack level when exceeded 14.4 km/hr are considered as high otherwise low.

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References

Briggs, G.A., 1969, Plume rise, AEC critical Review series TID-25075, 81pp.

Briggs, G.A., 1971, *Proc. Second Int. Clean Air Congress*, Academic Press, pp 1029-1032.

Briggs, G.A., 1972, *Atm. Env.*, **6**, pp. 507-510.

Briggs, G.A., 1975, Plume rise predictions, ATDL cont. Nov. 75/15.

CONCAWE, 1966, The calculation of dispersion from a stack, stichting CONCAWE, The Hague, The Netherlands.

Guldberg, P.H., 1975, *J. app. Met.*, **14**, pp. 1402-1405.

Holland, J.Z., 1953, AEC Rep. CRC-99, Wash. D.C 584 pp.

Lucas D.H., Moore, D.J. and Spur, G., 1963, *Int. J. Water Poll.*, **7**, pp. 473-500.

Lucas, D.H., 1967, *Atm. Env.*, **1**, pp. 421-424.

Moses, H. and Storm, G.H.,1961, *J. Air. Poll, Cont. Assoc.*, **11**, 10, pp. 455-466.

Montgomery, T.L., Carpenter, S.B., Colbaugh, W.C. and Thomas F.W., 1972, *J. Air Poll. Cont. Assoc.*, **22**, pp. 779-784.

Moore, D.J., 1974, *Atm. Env.*, **8**, **5**, pp. 441-458.

Whaley, H., 1969, *Atm. Env.*, **3**, p. 177.