523.2

AN INTERESTING RELATION IN PLANETARY MOTION

1. From a plot of log L^3 *versus* log S^2 of 17 solar system planets (where L is the orbital angular momentum and S is the spin angular momentum of the planet), it is seen that L^3/S^2 is very nearly constant for all the planets and is equal to be about 10^{60} gm cm²/sec. This value is again found to be numerically and dimensionally equal to the total orbital angular momentum (L_g) of the solar system planets as a whole, taken as a single equivalent planet, revolving round the centre of the Milky Way galaxy.

2. We know that all the planets in the solar system have two important types of motion : (i) they spin about their own axes of rotation and (ii) they revolve in great orbits around the Sun which controls them in its gravitational field. In addition, the Sun is situated in one of the spiral arms of our galaxy, the Milky Way, at a distance of about 30,000 light years from the galactic centre. The Sun with its planetary system revolves around the galactic centre with a speed of about 250 km/sec and takes about 240 million years to complete one revolution around the galactic nucleus. We may call this long period as the galactic year. The planetary system as a whole has, therefore, this third type of motion also around the galactic centre.

Kepler and Newton have clearly explained the nature of the orbital motion of the planets around the Sun. In this note an attempt is made to show that there exists a relationship connecting all the above three types of planetary motion.

As a first approximation, we may consider all the planets as spherical and their motions circular. In all the problems of circular motions, the angular momentum which is a product of moment of inertia (I) and angular velocity (ω) is an important conservative parameter and as such it is chosen for the present analysis.

3.1. The spin angluar momentum (S) and the orbital angular momentum (L) of 17 solar system planets (from the biggest planet Jupiter with a mass of 2×10^{30} gm to a small minor planet, Eros whose mass is only 10^{18} gm (*i.e.*, million million times smaller than Jupiter) have been computed and given in Table 1 along with a final column giving the value of the ratio L^3/S^2 for each of the planets. In spite of the fact that the physical and orbital parameters of the planets being widely different, the final ratio L^3/S^2 works out to be nearly constant, *viz.*, about 10^{60} gm cm²/sec for almost all the planets. This feature is better appreciated when we plot log L^3 *versus* log S^2 for all the planets (Fig. 1). All the points except for Mercury and Venus lie very nearly on a straight line, cutting the log L^3 axis at a value of about 60 with a slope of nearly unity (Fig. 1). Thus, we arrive at an interesting empirical relationship showing that L^3/S^2 for the solar system planets is nearly constant and is equal to about 10^{60} gm cm²/sec.

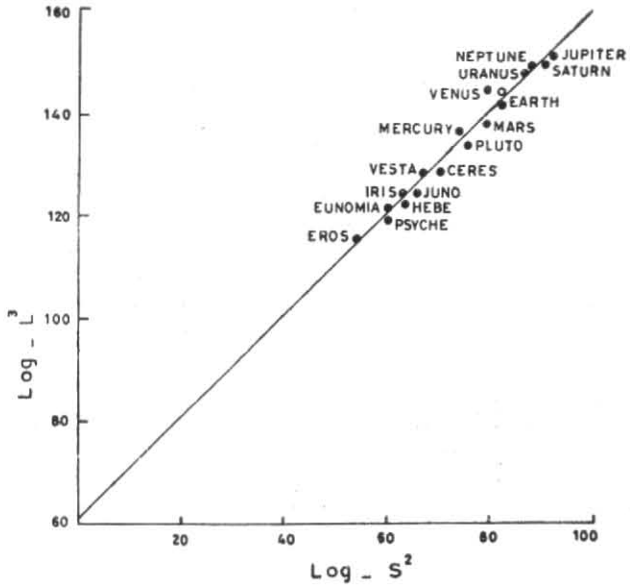

Fig. 1. Log L^3 *versus* log S^2 for all planets

TABLE 1

Planet	Mass (gm)	Orbital (L) (gm cm²/sec)	Spin (S) (gm cm²/sec)	L^3/S^2 (gm cm²/sec)
Mercury	3.3×10^{26}	8.9×10^{45}	9.6×10^{36}	8×10^{63}
Venus	4.9×10^{27}	1.9×10^{47}	1.3×10^{40} 2.6×10^{38}	4×10^{61} 4×10^{65}
Earth	6.0×10^{27}	2.7×10^{47}	5.9×10^{40}	5×10^{60}
Mars	6.4×10^{26}	3.6×10^{45}	2.0×10^{39}	1×10^{61}
Jupiter	1.9×10^{30}	1.9×10^{50}	4.3×10^{45}	4×10^{59}
Saturn	5.7×10^{29}	7.8×10^{49}	7.8×10^{44}	8×10^{59}
Uranus	8.7×10^{28}	1.7×10^{49}	1.8×10^{43}	3×10^{61}
Neptune	1.0×10^{29}	2.5×10^{49}	1.7×10^{43}	5×10^{61}
Pluto	1.5×10^{25}	4.2×10^{45}	2.5×10^{37}	1×10^{62}
Ceres	6.0×10^{23}	4.5×10^{43}	5.7×10^{34}	3×10^{61}
Juno	2.0×10^{22}	1.5×10^{42}	2.3×10^{32}	5×10^{61}
Vesta	1.0×10^{23}	6.9×10^{42}	4.7×10^{33}	2×10^{61}
Hebe	2.0×10^{22}	1.4×10^{42}	2.3×10^{32}	5×10^{61}
Iris	1.5×10^{22}	1.0×10^{42}	1.5×10^{32}	5×10^{61}
Eunomia	4.0×10^{22}	3.0×10^{42}	9.0×10^{32}	3×10^{61}
Psyche	4.0×10^{22}	3.0×10^{42}	1.3×10^{33}	2×10^{61}
Eros	5.0×10^{18}	2.7×10^{38}	6.4×10^{26}	5×10^{61}

Mass of the solar system planets as a whole$=2.7\times10^{30}$gm.
Its distance from the galactic centre=30,000 light years.
Its speed around the galactic centre=250 km/sec.
Its orbital angular momentum (L_g)$=10^{60}$ gm cm²/sec.

3.2. When we consider that the solar system planets, as a whole, as one single equivalent planet with a mass of about 2.7×10^{30} gm revolving round the centre of the Milky Way, its total orbital angular momentum (L_g) works to be nearly 10^{60} gm cm²/sec which is again numerically and dimensionally equal to the constant.

4.1. In the above analysis we find that the two innermost planets Mercury and Venus with their rotational periods of 58 days and 243 days respectively do not fall exactly in line with the other 15 planets on L^3/S^2 straight line graph. It is very likely that the original shorter rotational periods of these two planets had been greatly affected by the combined tidal effects due to the close proximity of the Sun and the neighbour planets and, therefore, became larger later. However, in the case of Venus it has been observed that cloud tops over Venus move very fast with a rotation period of four days. Using this value of 4 days rotation period the spin angular momentum for Venus is calculated and the point L^3/S^2 is marked in the graph as a small open circle which falls almost on the line.

4.2. Pluto, the outermost planet in the solar system is very small and it was difficult to determine its physical parameters accurately. Sometimes in the past, it was also suspected to be orginally a satellite of Neptune. But using recent observational data of Pluto with a satellite of its own (*Astronomical Almanac*, H. M. S. O. 1984) the value of L^3/S^2 for Pluto has been computed and this point also falls well on the straight line graph (Fig. 1). This fact confirms that Pluto also is a original planet in the Solar system.

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