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# A creeping and surface breaking long strike-slip fault inclined to the vertical in a viscoelastic half space

# SANJAY SEN, SEEMA SARKAR and (Late) ARABINDA MUKHOPADHYAY Department of Applied Mathematics, University of Calcutta (Received 6 January 1992, Modified 23 February 1993)

सार - इस शोध-पत्न में श्यान अत्यास्थ अर्ध-समस्टि वाल लियो मंडल एवं एस्थेनो मंडल के सरल मॉडल में स्थित तथा स्वेच्छ कोण पर सीधे झुके हुए अभूकम्पी विसर्पण सतह विभंजन नतिलंब सर्पण अंश पर विचार किया गया है। मॉडल में क्सियापन, प्रतिबल तथा तनाव के सटीक समाधान निकाले गए हैं। अभिकलित परिणाम यह दर्शाते हैं कि विस्थापन, प्रतिबल तथा तनाव के मान पर अंश के झुकाव का महत्वपूर्ण एवं सार्थक प्रभाव है। ऊर्ध्वाधर नतिलंब सर्पण अंश के लिये नतिलंब सर्पण झुकाव बनाने को अग्रसर अपरूपण तनाव के संचयन की दर सबसे अधिक पाई गई है। जबकि क्षेतिज की ओर छोटे कोणों में झुके हुए अंशों में यह दर काफी कम है। इस कार के सैद्धान्तिक मॉडलों के उपयोग द्वारा भूकम्पी सक्रिय प्रदेशों में भूवाल की प्रक्रिया की गहन जानकारी प्राप्त करने तथा लियो मंडल एवं एस्थेनो मंडल प्रणाली की गतिकी के साथ इनके संबंधों का परीक्षण किया गया है।

ABSTRACT. An assistically creeping surface-breaking strike-slip fault inclined to the vertical at an arbitrary angle, situated in a simple model of the lithosphere-asthenosphere system consisting of a viscoelastic half space is considered. The exact solutions for displacements, stresses and strains in the model are obtained. Computed results show that the inclination of the fault has a significant influence on the values of the displacements, stresses and strains. The rate of accumulation of shear stress tending to cause strike-slip movement has been found to be greatest for vertical strike-slip fault, while for faults inclined at smaller angles to the horizontal, this rate is significantly smaller. The uses of such theoretical models in obtaining greater insight into the earthquake processes in seismically active regions and their relations to the dynamics of the lithosphere-asthenosphere system are examined.

Key words -- Earthquake, Tectonic force, Creep velocity, Surface shear strain, Strike-slip fault.

### 1. Introduction

The problem of earthquake prediction has attrated wide-spread attention among seismologists in recent years and the steady accumulation of relevant seismological data and improvements in the techniques of their analysis and interpretation, together with the development of relevant theoretical models and computer simulation techniques have made it possible to hope that effective programmes of earthquake prediction may become feasible in near future. In this connection it is realised that effective programmes of earthquake prediction would require a better understanding of the process of stress accumulation and release in seismically active regions and their relations to the dynamics of the litho sphere-asthenosphere system.

In this connection it may be mentioned that regular observations in seismically active regions in the recent years indicate that during apparently quiet aseismic periods, there are usually slow quasi-static aseismic surface movements of the order of a few cm per year or less, resulting in the accumulation of stress and strain. In some cases this may eventually lead to a sudden fault movement generating an earthquake, if the stress accumulation reaches sufficiently high levels. In some other cases there may be a continuous, slow, aseismic fault creep across the active faults, *e.g.*, the central part of the San Andreas fault in North America. The effect of this aseismic fault creep on the accumulation and release of stress in the region concerned is of great interest in the study of the dynamics of the lithosphereasthenosphere system in seismically active regions during asiesmic periods.

In recent years, some theoretical models of the lithosphere-asthenosphere system in seismically active regions during asiesmic periods have been developed starting with the theoretical model of Nur and Mavko (1974). The general features of the theoretical models of this type, developed till now, have been discussed by Cohen *et al.* (1984), Mukhopadhyay and Mukherji (1984 1986).

### 2. Formulation

We consider a simple theoretical model of the lithosphere-asthenosphere system with a creeping, surface



Fig. 1. The model and the co-ordinate axes

breaking, long, plane strike-slip fault F, inclined to the horizontal at an angle  $\theta$  and situated in a linearly viscoelastic half space with its material of Maxwell type. The upper and lower edges of the fault F are horizontal and D is the width of the fault. The case of a fault of finite length has been considered by Chinnery (1961), Pal *et al.* (1979). We, however, consider a long fault (keeping in view the San Andreas fault in north America whose length is about 1500 km) whose length is assumed to be >> D.

We introduce rectangular cartesian co-ordinates  $(y_1, y_2, y_3)$  with the plane free surface of the viscoelastic half space as the plane  $y_3=0$  and the  $y_3$ -axis pointing into the half space. The upper edge of the fault F is taken as the  $y_1$ -axis. For convenience of analysis, we also introduce another rectangular system of cartesian co-ordinates  $(y'_1, y'_2, y'_3)$  associated with the fault, with the same origin; the  $y'_1$ -axis coincident with the  $y_1$ -axis and the plane of the fault as the plane  $y'_2=0$  (Fig. 1). With this choice of axes, the half space occupies the region  $y_3 \ge 0$ , while the fault is given by  $F: (y'_2=0, 0 \le y'_3 \le D)$ . The relations between  $(y_1, y_2, y_3)$  and  $(y'_1, y'_2, y'_3)$  are given by :

$$y_1 = y'_1, \ y_2 = y'_2 \sin \theta + y'_3 \cos \theta$$
  
 $y_3 = -y'_2 \cos \theta + y'_3 \sin \theta$ 

Let  $(u_1, u_2, u_3)$  be the components of the displacement u in the half space  $y_3 \ge 0$  in the directions  $(y_1, y_2, y_3)$ -axes respectively and let  $\tau_{12}$ ,  $\tau_{13}$ ,  $\tau_{23}$ ,  $\tau_{11}$ ,  $\tau_{22}$ ,  $\tau_{33}$  be the stress components while  $e_{ij}$  (i, j=1, 2, 3) are the components of strain. For long fault, all these quantities are taken to be independent of  $y_1$  and are functions of  $y_2$ ,  $y_3$  and t. These components separate out into two distinct and independent groups (Maruyana 1966) — one group consisting of  $u_1$ ,  $\tau_{12}$ ,  $\tau_{13}$  and  $e_{12}$ ,  $e_{13}$  is associated with strike slip movement, while the other group consisting of  $u_2$ ,  $u_3$ ;  $\tau_{22}$ ,  $\tau_{33}$ ,  $\tau_{23}$ ; and  $e_{22}$ ,  $e_{33}$ ,  $e_{23}$ 



Fig. 2. Rate of change of surface displacement per year due to fault creep (V=1.0 cm/year)

is associated with a possible dip-slip movement of the fault. Here we consider only the strike-slip movement of the fault.

### (i) Constitutive equations (stress-strain relations)

The stress-strain relations for the viscoelastic half space (Budiansky and Amazigo 1976) :

$$\begin{pmatrix} \frac{1}{\eta} + \frac{1}{\mu} & \frac{\partial}{\partial t} \end{pmatrix} \tau_{12} = \frac{\partial^2 & u_1}{\partial t & \partial y_2} \\ \begin{pmatrix} \frac{1}{\eta} + \frac{1}{\mu} & \frac{\partial}{\partial t} \end{pmatrix} \tau_{13} = \frac{\partial^2}{\partial t} \frac{u_1}{\partial y_3} \end{pmatrix} \begin{pmatrix} -\infty < y_2 < \infty, \\ y_3 \ge 0, t \ge 0 \end{pmatrix}$$
(1)

where,  $\eta$  is the effective viscosity and  $\mu$  is the effective rigidity of the material.

#### (ii) Stress equation of motion

For the slow aseismic quasi-static deformations of the system, inertial forces are very small and are neglected; the relevant stresses would satisfy the following relations:

$$\begin{aligned} \frac{\hat{c}}{\partial y_2} (\tau_{12}) &+ \frac{\partial}{\partial y_3} (\tau_{13}) = 0, \\ (-\infty < y_2 < \infty, \ y_3 \ge 0, t \ge 0) \end{aligned} \tag{2}$$

From Eqns. (1) and (2) :

$$\frac{\partial}{\partial t} \, \left( \bigtriangledown^2 u_1 \right) = 0$$

which is satisfied if

$$\nabla^2 u_1 = 0, \ (-\infty < y_2 < \infty, \ y_3 \ge 0, \ t \ge 0)$$
(3)

(iii) Boundary conditions

$$\tau_{13} = 0 \text{ on } y_3 = 0 \ (-\infty < y_2 < \infty, t \ge 0) \tag{4}$$

$$\tau_{13} \Rightarrow 0 \text{ as } y_3 \Rightarrow \infty \ (-\infty < y_2 < \infty, t \ge 0) \tag{5}$$

$$\tau_{12} \to \tau_{\infty}$$
 (t) as  $|y_2| \to \infty$   $(y_3 \ge 0, t \ge 0)$  (6)

where,  $\tau \infty$  (t) is the shear stress maintained by the tectonic forces far away from the fault which may or may not change with time but is independent of  $y_3$ .

### (iv) Initial conditions

Let  $(u_1)_0$ ,  $(\tau_{12})_6$ ,  $(\tau_{13})_0$ ,  $(e_{12})_0$ ,  $(e_{13})_0$  are the values of  $u_1$ ,  $\tau_{12}$ ,  $\tau_{13}$ ,  $e_{12}$ ,  $e_{13}$  respectively at time t=0. They are functions of  $(y_2, y_3)$  and satisfy the relations (1) to (6) where time t is measured from a suitable instant when there is no seismic activity in the system.

# 3. Displacements, stresses and strains in the absence of any fault movement

In this case the displacements, stresses and strains are all continuous throughout the system and the time t is measured from a suitable instant for which conditions (1) to (6) are satisfied for  $t \ge 0$ . The solutions can be obtained by taking Laplace transforms of the Eqns. (1) to (6) with respect to time t which give rise to a boundary value problem for  $\overline{u_1}, \overline{\tau_{12}}, \overline{\tau_{13}}$ , the Laplace transforms of  $u_1, \overline{\tau_{12}}, \overline{\tau_{13}}$  with respect to time t.

In the case when  $\tau_{\infty}(t) = \text{constant} = \tau_{\infty}$ , say, the solutions are given by (Maji *et al.* 1979)

$$\begin{array}{c} u_{1}(y_{2}, y_{3}, t) = (u_{1})_{0} + \tau_{\infty} . t. y_{2}/\eta \\ \tau_{12}(y_{2}, y_{3}, t) = (\tau_{12})_{0} . \exp(-\mu t/\eta) + \tau_{\infty} \\ [1 - \exp(-\mu t/\eta)] \\ \tau_{13}(y_{2}, y_{3}, t) = (\tau_{13})_{0} . \exp(-\mu t/\eta) \end{array}$$

$$(7)$$

$$e_{12}(y_2, y_3, t) = \frac{\partial u_1}{\partial y_2} = (e_{12})_0 + \tau_{\infty} \cdot t/\eta$$

$$\tau_1'_2'(y_2, y_3, t) = \tau_{12} \sin \theta - \tau_{13} \cos \theta$$

$$(\pi'_1) = \tau_{12} \sin \theta - \tau_{13} \cos \theta$$

$$=(\tau_{1\,2})_0 \exp\left(-\mu t/\eta\right) + \tau_\infty \sin \theta \\ \left[1 - \exp\left(-\mu t/\eta\right)\right] \tag{8}$$

where,  $(\tau_{12}')_0$  is the value of  $\tau_{12}'$  at t=0 and is given by :

 $(\tau_{1'2'})_0 = (\tau_{12})_0 \sin \theta - (\tau_{13})_0 \cos \theta$ 

Thus, if the shear stress  $\tau_{12}'$  near the fault is  $< \tau_{\infty} \sin \theta$  at t=0, then there will be a continuous accumulation of shear stress  $\tau_{12}'$  near the fault for t>0 and ultimately as  $t \rightarrow \infty$ ,  $\tau_{12}' \rightarrow \tau_{\infty} \sin \theta$  in the neighbourhood of the fault.

It may further be noted that the rate of accumulation of the shear stress  $\tau_{12}'$  as well as the maximum limiting value of the shear stress  $\tau_{12}'$  (which is  $= \tau_{\infty} \sin \theta$ ) both increases as  $\theta$  increases and have the maximum values when the fault is vertical  $(\theta = \pi/2)$ . Thus the accumulation of shear stress  $r_{1'2'}$  tending to cause strike slip movement can reach comparatively greater values if the fault is vertical or nearly vertical, so that the possibility of a major strike slip movement is relatively greater for nearly vertical faults compared to those which are inclined at relatively smaller angles to the horizontal. This result is consistent with the general observations. If the characteristic of the fault be such that it starts creeping or a sudden seismic movement occurs across it when  $\tau_{12}'$  in the neighbourhood of the fault reaches some critical value, say  $\tau_c (< \tau_{\infty} \sin \theta)$ , then there will be a creeping or sudden seismic movement across F after a finite length of time and in that case the solutions (7) and (8) will no longer hold good and require some modifications. We assume here that the fault is such that it starts creeping after a time  $t=T_1$  (say) > 0, when  $(\tau_1'_2)$  reaches that critical value.

# 4. Displacements, stresses and strains after the commencement of the fault creep

We consider a slow, aseismic creep movement across the fault F commencing at time  $t=T_1$ . Here all the relations (1) to (6) are valid for  $t \ge T_1$ . In addition to these the following creep condition is also satisfied :

$$[u_1] = U(t_1) . (fy'_3) . H(t - T_1) \text{ across}$$
  

$$F : y'_2 = 0, \quad 0 \le y'_3 \le D \text{ where, } t_1 = t - T_1 \qquad (9)$$

 $f(v_3')$  gives the spatial dependence of the creep movement along the fault and  $[u_1]$  is the relative creep displacement across F defined by :

$$[u_1] = \lim_{y'_2 \to 0} (u_1) - \lim_{y'_2 \to 0} (u_1)$$

Here,  $U(t_1)$  and  $f(y_3')$  are assumed to be continuous functions of  $t_1$  and  $y_3'$  respectively and  $U(t_1) = 0$  for  $t_1 \leq 0$ . The creep velocity across F is given by :

$$\frac{\partial}{\partial t}\left\{ \left[ u_1 \right] \right\} = V(t_1) \cdot f(y'_3), \text{ where } V(t_1) = \frac{dU(t_1)}{dt_1}$$

which is assumed to be finite for all  $t_1 \ge 0$ .

To solve the initial and boundary value problem involving  $(u_1, \tau_{12}, \tau_{13})$  for  $t \ge T_1$  we try to obtain  $u_1, \tau_{12}, \tau_{13}$  in the following form :

$$\left.\begin{array}{l}u_{1}=(u_{1})_{1}+(u_{1})_{2}\\\tau_{12}=(\tau_{12})_{1}+(\tau_{12})_{2}\\\tau_{13}=(\tau_{13})_{1}+(\tau_{13})_{2}\end{array}\right\} \quad 9(a)$$

where,  $(u_1)_1$ ,  $(\tau_{12}, \tau_{13})_1$  are continuous everywhere in the model satisfying Eqns. (1) to (6) and assume the values  $(u_1)_0$ ,  $(\tau_{12})_0$ ,  $(\tau_{13})_0$  at t=0. The solutions for  $(u_1)_1$ ,  $(\tau_{12})_1$ ,  $(\tau_{13})_1$  will be obtained as in § 3 and when  $\tau_{\infty}$  (t) = constant =  $\tau_{\infty}$ , they are same as Eqns. (7) and (8) and are given by :

$$\begin{array}{c} (u_{1})_{1} = (u_{1})_{0} + \tau_{\infty} \cdot t \cdot y_{2}/\eta \\ (\tau_{12})_{1} = (\tau_{12})_{0} \cdot \exp(-\mu t/\eta) \\ + \tau_{\infty} \left[1 - \exp(-\mu t/\eta)\right] \\ (\tau_{13})_{1} = (\tau_{13})_{0} \cdot \exp(-\mu t/\eta) \end{array}$$

$$(10)$$

 $(u_1)_2$ ,  $(\tau_{12})_2$ ,  $(\tau_{13})_2$  are functions of  $y_2$ ,  $y_3$  and t and they satisfy Eqns. (1), (2), (3) boundary conditions (4) (5) and the following conditions :

$$(\tau_{12})_2 \to 0 \text{ as } \mid y_2 \mid \to \infty, (y_3 \ge 0, t_1 \ge 0)$$
 (11)

together with the following creeping condition :

$$[(u_1)_2] = U(t_1) \cdot f(y'_3) \text{ across } F, \ t \ge T_1 \text{ with}$$
$$U(t_1) = 0 \quad \text{for } t_1 \le 0 \tag{12}$$

Also 
$$(u_1)_2, (\tau_{12})_2$$
 and  $(\tau_{13})_2 = 0$  for  $t_1 \leq 0$  (13)

To obtain the solutions for  $(u_1)_2$ ,  $(\tau_{12})_2$ ,  $(\tau_{13})_2$  for  $t_1 \ge 0$ , we take Laplace tranforms of Eqns. (1) to (5) and Eqns. (11), (12), (13) with respect to  $t_1$ , the resulting boundary value problem involving  $(\overline{u}_1)_2$ ,  $(\overline{\tau}_{12})_2$ ,  $(\overline{\tau}_{13})_2$  the Laplace tranforms of  $(u_1)_2$ ,  $(\tau_{12})_2$ ,  $(\tau_{13})_2$  respectively with respect to time  $t_1$ , can be solved by using a suitably modified form of Green's function technique developed by Maruyama (1966) as explained in the

Appendix. On inverting these Laplace transforms the solutions for  $(u_1)_2$ ,  $(\tau_{12})_2$ ,  $(\tau_{13})_2$  for  $t_1 \ge 0$  and  $y'_2 \ne 0$  can be obtained. For a constant creep velocity V, we have :

$$\begin{aligned} & (u_1)_2 (y_2, y_3, t) = H(t - T_1) , \frac{V_{t_1}}{2\pi} \cdot \Psi_1(y_2, y_3) \\ & (\tau_{12})_2 (y_2, y_3, t) = H(t - T_1) \cdot \frac{V \cdot \eta}{2\pi} \cdot [1 - \exp \\ & \{-\mu(t - t_1)/\eta\}] \cdot \Psi_2 (y_2, y_3) \\ & (\tau_{13})_2 (y_2, y_3, t) = H(t - T_1) \cdot \frac{V \cdot \eta}{2\pi} \cdot [1 - \exp \\ & \{-\mu(t - t_1)/\eta\}] \cdot \Psi_3 (y_2, y_3) \end{aligned}$$
(14)

where  $\Psi_1$ ,  $\Psi_2$ ,  $\Psi_3$  are given in the Appendix.

This shows that the displacements, stresses and strains due to the fault movement depends, besides the model parameters, on the inclinations of the fault to the vertical and  $U(t_1) = V \cdot t_1$  where V is a constant.

Thus the final solutions for displacements, stresses, and strains for  $t_1 \ge 0$  are given by :

$$u_{1}(y_{2}, y_{3}, t) = (u_{1})_{0} + \frac{\tau_{\infty} \cdot t \cdot y_{2}}{\eta} + H(t - T_{1}) \cdot \frac{Vt_{1}}{2\pi} \cdot \Psi_{1}(y_{2}, y_{3})$$

$$\tau_{1'2'}(y_2, y_3, t) = \tau_{12} \sin \theta - \tau_{13} \cos \theta$$
  
=  $(\tau_{1'2'})_0 \exp(-\mu t/\eta) - \tau_{\infty} \sin \theta \times$   
[1-exp $(-\mu t/\eta)$ ]- $H(t-T_1) \cdot \frac{V\eta}{2\pi}$  [1]  
- exp $\left\{\frac{-\mu(t-T_1)}{\eta}\right\}$ ]. $(\Psi_2 \sin \theta - \Psi_3 \cos \theta)$   
 $e_{12}(y_2, y_3, t) = \frac{\partial u_1}{\partial y_2} = (e_{12})_0 + \tau_{\infty} t/\eta$   
+  $H(t-T_1) \cdot Vt_1 \cdot \Psi_2/2\pi$ ]

It is found that the displacements, stresses and strains will be finite and single-valued everywhere in the model, including the points near the lower edge of the fault, if the following conditions are satisfied by  $f(y_3)$ :

(i)  $f(y'_3)$  and  $f'(y'_3)$  are continuous functions of  $y'_3$  for  $0 \le y'_3 \le D$ ,

(*ii*) 
$$f(D) = 0$$
 and also  $f'(0) = f'(D) = 0$ ,

(iii) 
$$f''(y'_3)$$
 is continuous in  $0 \le y_3' \le D$ ,  
except for a finite number of points   
of finite discontinuity in  $0 \le y'_3 \le D$ ,  
or  $f''(y'_3)$  is continuous in  $0 < y'_3 < D$   
and there exist real constants  $m$ ,  
 $n < 1$ , such that  $(y'_3)^m f''(y'_3) \to 0$  or  
to a finite limit as  $y'_3 \to 0+0$  and  
that  $(D-y'_3)^n f''(y'_3) \to 0$  cr to a  
finite limit as  $y'_3 \to D = 0$ 

These conditions imply that the displacements, stresses and strains will be bounded everywhere in the model,



Fig. 3. Changes in the surface shear strain near the fault  $(y_2 \approx 0, y_3 = 0)$ 

including the points near the lower edge of the fault, if the magnitude of the relative creep displacement across the fault varies smoothly over the fault and approaches zero with sufficient smoothness as  $y'_3 \rightarrow D - 0$  near the lower edge of the fault.

# 5. Discussion of the results and conclusions

We study the nature of the rate of change of surface displacement D and surface shear strain R and the shear stress  $\tau_1'_2$  near the mid-point (at  $y'_3 = D/2$ ) of the fault with the following choice of the polynomial  $f(y_3')$  and the model parameters :

$$f(y'_3) = (y'_3{}^2 - D^2)^2/D^4$$
, so that  $f(0) = 1$ , with  
 $U(t_1) = V \cdot t_1$ , V being a constant,  
 $\mu = 3.78 \times 10^{11}$  dyne/sq cm  
 $\eta = 3.0 \times 10^{21}$  poise  
 $D = 10$  km  
 $V = 0.0 - 5.0$  cm/yr  
 $T_1 = 60$  and 180 years  
 $\tau_{\infty} = 200$  bars  
 $(\tau_1'_2')_0 = 0.5 \quad \tau_{\infty} \sin \theta$ ,  
 $\theta = 30^\circ, 45^\circ, 60^\circ, 90^\circ$ .

We now consider :

(i) The rate of change of surface displacement per year due to fault creep and compute

$$\frac{\partial}{\partial t} \left[ u_1 - (u_1)_0 - \tau_{\infty} \ t y_2 / \eta \right] y_3 = 0$$



Fig. 4. Rate of release (R) (per year) of surface shear strain due to fault creep

(*ii*) The rate of release (per year) of the surface shear strain due to creep and compute

$$R = -\frac{\partial}{\partial t} \left[ e_{12} - (e_{12})_0 - \tau_\infty t/\eta \right] y_3 = 0$$

(*iii*) The shear stress  $(\tau_1'_2)_{\text{MD}}$  near the mid-point of the fault (*i.e.*, at the point  $y_2'=0$ ,  $y_3'=D/2$ ).

We consider first in greater detail the rate of change of surface displacement with distance  $y_2$  from the fault trace. Fig. 2 shows that the nature of this change depends significantly on the inclination of the fault to the horizontal. However, the following features are common for faults with different inclinations :

- (i) The maximum magnitude of the rate of change of surface displacement due to fault creep is attained near the fault for both  $y_2 > 0$  and  $y_2 < 0$ .
- (ii) The rate of change of surface displacement due to fault creep decreases rapidly as we move away from the fault on the free surface, and for  $y_2 >> D$ , it becomes very small.
- (iii) For  $y_2 > 0$  and  $y_2 < 0$ , the rate of change of surface displacement due to fault creep has opposite signs, the surface displacement due to fault creep being in opposite direction for  $y_2 > 0$  and  $y_2 < 0$ .
- (*iv*) The rate of relative displacement across the fault on the surface is equal to U in all cases, as expected.

Apart from these similarities there are considerable differences between the rate of change of surface displacement due to fault creep for faults with different inclinations. We consider here only four different values of  $\theta$ , viz.,  $\theta = 30^{\circ}$ ,  $45^{\circ}$ ,  $60^{\circ}$ ,  $90^{\circ}$ . It may be mentioned in this connection that smaller values of  $\theta$  are not considered, because of the fact that such situations are not likely to occur in reality.

The differences in the rate of change of surface displacement due to fault creep for different values of  $\theta$  as revealed from numerical computations, are stated in the following :

For  $y_2 > 0$ , this rate is found to increase as  $\theta$  decreases and the rate for  $\theta = 30^\circ$  is more than about 1.6 times the rate for  $\theta = 90^\circ$  as  $y_2 \rightarrow 0 + 0$ . However, for  $y_2 < 0$ , this rate has a smaller magnitude for smaller values of  $\theta$ and the magnitude of this rate for  $\theta = 90^\circ$  is nearly 3 times the magnitude for  $\theta = 30^\circ$  as  $y_2 \rightarrow 0 - 0$ . For  $\theta = 90^\circ$ , the rate is anti-symmetrical with respect to  $y_2 = 0$ . However, for  $\theta \neq 90^\circ$ , there is no such anti-symmetry.

Fig. 3 shows the changes in the total surface shear strain near the fault and near free surface ( $y_2 \approx 0$ ,  $y_3 =$ 0) for a vertical fault. In the case V=0, there is a steady accumulation of surface shear strain near the fault with time. This rate is of the order of  $10^{-6}$  per year which is of the same order of magnitude as the observed rate of accumulation of surface shear strain near the locked parts of the San Andreas fault in California as reported by Savage and Burford (1970) and others. Fig. 3 shows that fault creep commencing at  $t=T_1$ , leads to reduction in the rate of surface shear strain accumulation near the fault, this rate being greater for greater creep velocities. For faults with different inclinations  $\theta$  to the vertical, the general nature of the changes in the surface shear strain near the fault with time is found to be similar, the rate being the same for V=0 for all inclinations. For V > 0, the effect of creep is qualitatively similar for different inclinations of the fault, corresponding to different values of  $\theta$ . However, the effect of fault creep on the surface shear strain near the fault trace is found to decrease significantly as  $\theta$  decreases.

Fig. 4 shows the rate of release (per year) of the surface shear strain due to fault creep near the fault  $(y_2 \approx 0, y_3 = 0)$  and away from the fault for different inclinations.



Figs. 5 (a&b). Shear stress  $\tau_{1,2}^{++}$  near the mid point of the fault for : (a)  $\theta = 30^{\circ}$ , 45°, 135°, 150°, and (b)  $\theta = 60^{\circ}$ , 90°, 120°

It is found that the fault creep results in release of the surface shear strain. But this effect falls off rapidly as we move far away from the fault trace on the free surface for all inclinations  $\theta$  of the fault to the horizontal. For  $\theta$ =90°, the surface shear strain release due to creep is greatest near the fault trace ( $y_2 \approx 0$ ,  $y_3 = 0$ ) and is symmetrical about the fault trace  $(r_2=0, r_3=0)$ . For  $\theta \neq 90^{\circ}$ , the effect is not symmetrical about the fault trace, and the maximum rate of release of surface shear strain occurs a little away from the fault trace. The distance from the fault trace of the point of the maximum rate of release of surface shear strain increases as  $\theta$ decreases. Near the fault trace  $(y_2 \approx 0, y_3 = 0)$  the surface shear strain release effect of fault creep decreases as  $\theta$  decreases. It is also noted that as  $\theta$  decreases from 90°, the maximum magnitude first decreases till  $\theta \approx 60^{\circ}$ and then increases as  $\theta$  decreases further. Thus, the effect of fault creep on the surface shear strain depends significantly on the inclination of the fault to the horizontal. The differences in the effects of fault creep on the surface shear strain for different inclinations of the fault may be useful in estimating the inclinations of creeping faults, using observational data on aseismic changes in the surface shear strain near the fault.

Figs. 5(a&b) show the variations with time of  $(\zeta_{1'2'})_{MD}$ the shear stress  $\zeta_{1'2'}$  near the mid-point of the fault for different inclinations of the fault to the horizontal  $(\theta=30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 135^\circ, 150^\circ)$  and for different values of the creep velocities from V=0 to V=5 cm/ year. It is found that in all cases, in the absence of fault creep, there is a steady accumulation of shear stress  $\zeta_{1'2'}$  near the fault with gradually decreasing rate of accumulation. If fault creep commences at  $t=T_1$ , there is a reduction in the rate of accumulation of shear stress near the fault due to creep and this effect is greater for larger values of creep velocities V. For sufficiently large creep velocities, there is a gradual release of the shear stress near the fault after  $t=T_1$  instead of accumulation and if V=5 cm/year, there is more or less complete release of the accumulated shear stress  $\zeta_{12}'$  near *F* after a sufficient time. The gradual release of shear stress would be expected to reduce progressively the possibility of the sudden seismic fault movement, thus reducing the possibility of a major tectonic earthquake due to sudden seismic movement across the fault.

The rate of accumulation of the shear stress as well as the maximum value attained by  $\zeta_{1'2'}$  near the fault

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depend significantly on the inclination of the fault. In the absence of fault creep, the rate of accumulation of  $\zeta_{1'2'}$  near the fault is greatest for  $\theta=90^{\circ}$  and decreases as the inclination  $\theta$  decreases. For  $0 \leq t \leq T_1$ , the average rate of accumulation of  $\zeta_{1'2'}$  near F for  $\theta=90^{\circ}$ has a value nearly twice the average rate for  $\theta=30^{\circ}$ . Also it is found that the value of creep velocity V for which there is no accumulation or release of  $\zeta_{1'2'}$  near F for  $t>T_1$  is nearly 1 cm/year for  $\theta=30^{\circ}$ , nearly 1.37 cm/year for  $\theta=45^{\circ}$ , nearly 1.67 cm/year for  $\theta=60^{\circ}$  and nearly 2 cm/year for  $\theta=90^{\circ}$ .

For relatively large values of V (say, V=5 cm/year) the rate of release of the shear stress  $\zeta_{1'2'}$  near 'the fault for  $t > T_1$  is found to depend to some extent on the inclination of the fault and is found to be smallest for  $\theta=90^{\circ}$ . As  $\theta$  decreases from  $\theta=90^{\circ}$ , the average rate of release of shear stress for large values of V (say, V=5 cm/year) increases slowly upto  $\theta=30^{\circ}$  as shown in Figs. 5 and 6. The total release of  $(\zeta_{1'2'})_{\text{MD}}$  over the period t=180 years to t=500 years, for V=5 cm/year, is nearly 56 bars for  $\theta=90^{\circ}$ , 61 bars for  $\theta=60^{\circ}$ , 65 bars for  $\theta=45^{\circ}$  and 72 bars for  $\theta=30^{\circ}$ .

Finally, we conclude that the inclination of the fault to the horizontal has an important influence on the effect of fault creep on the surface displacement, surface shear strain and the shear stress near the fault tending to cause strike-slip movement.

#### APPENDIX

## A 1. Displacements, stresses and strains for $t > T_1$ after the commencement of the fault creep—the method of solution

The displacements and stresses after the commencement of the fault creep have been found in the form given in Eqn. (9A). Taking Laplace transforms of Eqns. (1) to (5), (11), (12) and (13) with respect to time  $t_1$ , a boundary value problem involving  $(\overline{u_1})_2$ ,  $(\overline{\tau_{12}})_2$ ,  $(\overline{\tau_{13}})_2$ , the Laplace transforms of  $(u_1)_2$ ,  $(\tau_{12})_2$  and  $(\tau_{13})_2$  respectively with respect to  $t_1$  defined by :

$$\overline{(u_1)}_2, (\overline{\tau_{12}})_2, (\overline{\tau_{13}})_2\} = \int_0^\infty \{(u_1)_2, (\tau_{12})_2, (\tau_{13})_2\} e^{-p_1} dt_1$$

(*p* being the Laplace tranform variable) is obtained. The resulting boundary value problem is characterised by the following relations :

$$\overline{(\tau_{12})_2} = \left\{ p \left/ \left( \frac{1}{\eta} + \frac{p}{\mu} \right) \right\} \left| \frac{\partial}{\partial y_2} \overline{(u_1)_2} \right|$$

$$\overline{(\tau_{13})_2} = \left\{ p \left/ \left( \frac{1}{\eta} + \frac{p}{\mu} \right) \right\} \left| \frac{\partial}{\partial y_2} \overline{(u_1)_2} \right|$$
(A1)

$$\frac{\partial}{\partial \nu_2} (\overline{\tau}_{12})_2 + \frac{\partial}{\partial \nu_3} (\overline{\tau}_{13})_2 = 0 \tag{A2}$$

$$\nabla^2 (\overline{u_1})_2 = 0 \tag{A3}$$

$$(-\infty < v_2 < \infty, v_3 \ge 0)$$

Also,

$$(\overline{\tau}_{13})_2 = 0 \text{ on } y_3 = 0, \quad (-\infty < y_2 < \infty)$$
 (A4)

$$\overline{\tau_{13}}_2 \to 0 \text{ as } y_3 \to \infty, \quad -\infty < y_2 < \infty$$
 (A5)

$$(\overline{\tau}_{12})_2 \to 0 \text{ as } |y_2| \to \infty, \quad y_3 \ge 0$$
 (A6)

and 
$$[(\overline{u_1})_2] = \overline{U}(p) f(y'_3)$$
 across  $F: (y'_2 = 0, 0 \le y'_3 \le D)$  (A7)

 $(\tau_{12})_2$  and  $(\tau_{13})_2$  are continuous across F,  $\overline{U}(p)$  being the Laplace transform of  $U(t_1)$  with respect to  $t_1$ , so that

$$\overline{U(p)} = \int_{0}^{\infty} U(t_1) . \exp\left(-pt_1\right) dt_1$$

To solve this boundary value problem, a suitably modified form of Green's function technique, developed by Maruyama (1966) and Rybicki (1971) is used. Following Maruyama (1966) :

$$\begin{aligned} (\tilde{u}_1) (Q) &= \int_F \left[ (\tilde{u}_1)_2 (P) \right] \{ G^1{}_{13} (Q, P) \ d\xi_2 \\ &- G^1{}_{12} (Q, P) d\xi_3 \} \end{aligned}$$
 (A8)

where,  $Q(y_1, y_2, y_3)$  is the field point in the half space and  $P(\xi_1, \xi_2, \xi_3)$  is any point on the fault F and  $[(\tilde{u}_1)_2(P)]$ is the discontinuity in  $(\tilde{u}_1)_2$  across F at the point P, while  $G_{13}^{-1}(Q, P)$  and  $G_{12}^{-1}(Q, P)$  are two Green's functions given by :

$$G^{1}_{13}(Q, P) = \frac{1}{2\pi} \left[ \frac{(y_{3} - \xi_{3})}{L^{2}} - \frac{(y_{3} + \xi_{3})}{M^{2}} \right]$$
$$G^{1}_{12}(Q, P) = \frac{1}{2\pi} \left[ \frac{(y_{2} - \xi_{2})}{L^{2}} - \frac{(y_{2} - \xi_{2})}{M^{2}} \right]$$

where,  $L^2 = (y_2 - \xi_2)^2 + (y_3 - \xi_3)^2$ ,  $M^2 = (y_2 - \xi_2)^2 + (y_3 + \xi_3)^2$ .

Now,  $P(\xi_1, \xi_2, \xi_3)$  being a point on the fault F,  $0 \leq \xi_2 \leq D \cos \theta$ ,  $0 \leq \xi_3 \leq D \sin \theta$  and  $\xi_2 = \xi_3 \cot \theta$ . A change in co-ordinate axes from  $(\xi_1, \xi_2, \xi_3)$  to  $(\xi_1', \xi_2', \xi_3')$  connected by the relations :

$$\xi_1 = \xi_1', \ \xi_2 = \xi_2 \sin \theta + \xi_3 \cos \theta$$
  
$$\xi_3 = -\xi'_2 \cos \theta + \xi'_3 \sin \theta$$

is introduced so that  $\xi'_2=0$  and  $0 \leq \xi'_3 \leq D$  on F. Then from Eqn. (A8) using Eqn. (A7).

$$\begin{split} (\overline{u_{1}})_{2} (Q) &= \frac{\overline{U}(p)}{2\pi} \int_{0}^{D} f(\xi'_{3}) \\ & \left[ \frac{(y_{2} \sin \theta - y_{3} \cos \theta)}{\xi'_{3}^{2} - 2\xi'_{3} (y_{2} \cos \theta + y_{3} \sin \theta) (y_{2}^{2} + y_{3}^{2})} + \frac{(y_{2} \sin \theta + y_{3} \cos \theta)}{\xi'_{3}^{2} - 2\xi'_{3} (y_{2} \cos \theta - y_{3} \sin \theta) + (y_{2}^{2} + y_{3}^{2})} \right] d\xi'_{3} \\ & (\overline{u_{1}})_{2} (Q) &= \frac{\overline{U}(p)}{2\pi} \cdot \Psi_{1} (y_{2}, y_{3}) \end{split}$$
(A9)

where, 
$$\Psi_1(y_2, y_3) = \int_{0}^{D} f(\xi'_3)$$

or

$$\frac{\left[\frac{(y_{2}\sin\theta - y_{3}\cos\theta)}{\xi_{3}^{\prime 2} - 2\xi_{3}^{\prime}(y_{2}\cos\theta + y_{3}\sin\theta) + (y_{2}^{2} + y_{3}^{2})\right]}{(y_{2}\sin\theta + y_{3}\cos\theta)} + \frac{(y_{2}\sin\theta + y_{3}\cos\theta)}{\xi_{3}^{\prime 2} - 2\xi_{3}^{\prime}(y_{2}\cos\theta - y_{3}\sin\theta) + (y_{2}^{2} + y_{3}^{2})}\right]d\xi_{3}^{\prime}$$
(A9A)

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On taking inverse Laplace transforms of (A9) with respect to  $t_1$  and noting that  $(u_1)_2=0$  for  $t \le 0$ :

$$(u_1)_2(y_2, y_3, t) = H(t - T_1) \frac{U(t_1)}{2\pi} \cdot \Psi_1(y_2, y_3)$$

where,  $\Psi_1(y_2, y_3)$  is given in Eqn. (A9A)

Again from Eqn. (A1)

$$(\overline{\tau}_{12})_2(y_2, y_3) = \left\{ p \middle/ \left( \frac{1}{\eta} - \frac{p}{\mu} \right) \right\} \frac{\widehat{c}}{\widehat{c}y_2} \left\{ (\overline{u}_1)_2(y_2, y_3) \right\}$$

This gives

$$(\tau_{12})_2(y_2, y_3) = \left\{ p \left/ \left( \frac{1}{\eta} - \frac{p}{\mu} \right) \right\}$$
  
 $\frac{U(p)}{2\pi} \cdot \Psi_2(y_2, y_3)$  (A10)

where,  $\Psi_2(y_2, y_3) = \frac{\partial \Psi_1}{\partial y_2} = \int_0^D f(\xi'_3)$ 

$$\begin{bmatrix} \frac{\xi'_{3}^{2} \sin \theta - 2\xi'_{3}y_{3} - (y_{2}^{2} - y_{3}^{2}) \sin \theta + 2y_{2}y_{3}\cos \theta}{\{\xi'_{3}^{2} - 2\xi'_{3}(y_{2}\cos \theta + y_{3}\sin \theta) + (y_{2}^{2} + y_{3}^{2})\}^{2}} \\ + \frac{\xi'_{3}^{2} \sin \theta + 2\xi'_{3}y_{3} - (y_{2}^{2} - y_{3}^{2}) \sin \theta - 2y_{2}y_{3}\cos \theta}{\{\xi'_{3}^{2} - 2\xi'_{3}(y_{2}\cos \theta - y_{3}\sin \theta) + (y_{2}^{2} + y_{3}^{2})\}^{2}} \end{bmatrix} \\ \frac{d\xi'_{3}^{2} - 2\xi'_{3}(y_{2}\cos \theta - y_{3}\sin \theta)}{d\xi'_{3}} + (y_{2}^{2} - y_{3}^{2})^{2}} \end{bmatrix}$$

On taking inverse Laplace tranform of Eqn. (A10) with respect to  $t_1$  and noting that  $(\overline{\tau_{12}})_2 = 0$  for  $t_1 \leq 0$ 

$$(\tau_{12})_2(y_2, y_3, t) = H(t - T_1) \frac{\mu}{2\pi} \cdot \left[ U(t_1) - \frac{\mu}{\eta} \right]$$
$$\int_0^t U(\tau) \exp\left\{-\frac{\mu}{\eta}(t_1 - \tau)\right] d\tau = \left[ \cdot \Psi_2(y_2, y_3) \right]$$

Noting that  $U(t_1)=0$  for  $t_1 \leq 0$ , this can be written as :

$$(\boldsymbol{\tau}_{12})_2 (y_2, y_3, t) = H(t - T_1) \frac{\mu}{2\pi} \left[ \int_0^{t_1} V(\tau) \right].$$

$$\exp\left(-\frac{\mu}{\eta}\left(t_1-\tau_{-}\right)\right)d\tau\right],\Psi_2(y_2,y_3)$$

where,  $\Psi_2$  ( $y_2$ ,  $y_3$ ) is given in (A10A). Similarly

$$(\tau_{13})_2(y_2, y_3, t) = H(t - \mathcal{T}_1) \frac{\mu}{2\pi} \int_0^{t_1} V(\tau).$$

$$\left[\exp\left(-\frac{\mu}{\eta}\left(t_{1}-\tau\right)\right)d\tau\right]$$
,  $\Psi_{3}(y_{2}, y_{3})$ 

where,

$$P_{3}(y_{2}, y_{3}) = -\int_{0}^{D} f \xi'_{3}.$$

$$\begin{bmatrix} \xi'_{3}^{2}\cos\theta - 2\xi'_{3}y_{2} + (y_{2}^{2} - y_{3}^{2})\cos\theta + 2y_{2}y_{3}\sin\theta \\ (\xi'_{3}^{2} - 2\xi'_{3}(y_{2}\cos\theta + y_{3}\sin\theta) + (y_{2}^{2} + y_{3}^{2})\}^{2} \end{bmatrix} \\ -\frac{\xi'_{3}^{2}\cos\theta - 2\xi'_{3}y_{2} - (y_{2}^{2} - y_{3}^{2})\cos\theta - 2y_{2}y_{3}\sin\theta \\ (\xi'_{3}^{2} - 2\xi'_{3}(y_{2}\cos\theta - y_{3}\sin\theta) + (y_{2}^{2} + y_{3}^{2})\}^{2} \end{bmatrix} d\xi_{3}'$$

From Eqns. (A1) and (A2) the expressions for  $c_{12} = \frac{\partial u_1}{\partial y_2}$  and  $\tau_1' _2' = \tau_{12} \sin \theta - \tau_{13} \cos \theta$  can easily be obtained. The solutions thus obtained are unique.

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