# The relation of raindrop size to intensity of rainfall in different types of tropical rain using a simple raindrop recorder

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ABSTRACT. The paper gives the results of studies of raindrop size characteristics in three other different<br>types of tropical rain to what Sivaramakrishnan had earlier studied (1961), using the simple raindrop recorder, The study of raindropsize characteristics refers to analysis of raindrop samples taken at Poona in (1) typical shower types<br>of rain without thunder (cumuliform clouds), (2) continuous type of rain with occasional thunder (cumu tions connecting the intensity of rainfall with the various rainfall parameters such as median drop size  $D_n$ , liquid water content W, radar reflectivity Z have been developed and compared with findings of other investigators all over the world. Best's formula is found to represent the size distribution of raindrops in the tropics also but with suitable changes in the constants. It is also shown in agreement with results obtained earlier by Sivaramakrishnan is the most suitable parameter for describing the average size of the drop size distribution in preference to other<br>average size parameters such as mean diameter  $D_m$ , mode diameter  $D_d$ , mean volume diameter  $D_p$  and pre nant diameter  $D_{\mathcal{D}}$ .

#### 1. Introduction

Considerable study has been made during the last few years of raindrop spectra and their relation to the average size of raindrop, liquid water content  $W$  and radar reflectivity  $Z$ . Basic references on the subject are the papers of Laws and Parsons (1943), Marshall and Palmer (1948), Spilhaus (1948), Best (1950, 1951), Blanchard (1953), Jones (1956), Mueller and Jones (1960), Fujiwara (1960), Atlas (1963). Sivaramakrishnan has given an up-to-date survey of raindrop distribution (1961) including data for tropical storms, showers and continuous type of rain.

The effects of evaporation, coalescence, wind sorting on drop size distribution have been considered by Rigby, Marshall and Hitschfeld (1954), Gunn and Marshall (1955), Imai et al. (1955), Imai (1956), Atlas and Chmela (1957), Sivaraman and Sivaramakrishnan (1962). The available evidence from all the above papers clearly shows that drop size spectra are highly variable in time, space and with type of rainfall.

The present study refers to the analysis of raindrop size characteristics of rain samples taken at Poona during (a) typical shower type of rain without thunder (from cumuliform clouds), (b) continuous type of rain with occasional thunder (from stratiform and cumuliform clouds mixed) and (c) non-freezing rain during typical monsoon rain from stratiform clouds.

Regression equations connecting the intensity of rainfall with the various rainfall parameters such as average size of raindrop, liquid water content W, radar reflectivity Z, have been developed and compared (Table 4) with the findings of other investigators - Boucher (1951), Higgs (1952), Kelkar (1961), Srivastava (1960). It is shown in agreement with the results obtained earlier by Spilhaus (1948), Best (1951), Blanchard (1953), Atlas and Chmela (1957), and Sivaramakrishnan (1961) that the median drop diameter is the most suitable parameter for describing the average size of the drop size distribution in preference to other average size parameters such as mean diameter  $D_m$ , mode diameter  $D_d$ , mean volume diameter  $D_v$  and the predominant diameter  $D_p$ .

Best's formula for the size distribution of rain $drop$  given  $by$  —

$$
1 - F = \exp\left[-(x/a)^n\right] \tag{1}
$$

where  $a = A/R'^p$ 

 $F =$  Fraction of liquid water in the air comprised by drops with diameter  $less than x$ 

## $R' =$ Rate of rainfall

#### $A, p$  and n are constants

is found to be suitable for representing the size distribution of raindrops in the tropics also but with suitable changes in Best's constants.

### 2. Verification of Best's formula

From Best's equation, it can be shown that- $\log \log_{10} [1/(1-F)] = -0.36 + n \left( \log_{10} x - \log_{10} x \right)$  (2)

If Eq. (1) represents the data and  $\log \log_{10}[1/(1-F)]$ is plotted against  $\log_{10} x$  the points should lie on a straight line with slope n. From the intercept of this line on the log  $log_{10}$  [1/(1-F)] axis, the value of  $(0.36 + n \log_{10} a)$  and hence of a can be obtained. Figs. 1 and 2 give sections of raindrop records with corresponding self-recording Siphon raingauge record during a heavy shower on 17 September 1960 without thunder and continuous type of rain with occasional thunder on 19 September 1960 (from stratiform and cumuliform clouds mixed). In the bottom of Figs. 1 and 2, log log<sub>10</sub>  $[1/(1-F)]$  on Y axis and on the X axis the corresponding dropsize  $\log x$  has been plotted. The points when joined is found to be a straight line showing that Best's formula for the drop size distribution is found to be suitable for the drop size distribution in tropical rains also.

#### 3. Results

A complete description of the raindrop recorder has been given in Sivaramakrishnan's earlier paper (1961), giving the method of obtaining a continuous record of the time of occurrence. duration of different types of rain and range of raindrop sizes using a dyed filter paper tape technique. Tables 1, 2, 3 and 5 give a summary of the following raindrop parameters taken with the raindrop recorder during  $(a)$  shower type of rain from cumuliform clouds,  $(b)$  continuous type of rain with thunder from stratiform and cumuliform clouds mixed and (c) continuous non-freezing rain from stratiform clouds at Poona, where-

1. 
$$
R' =
$$
 Rate of rainfall (mm/hr)  
=  $6 \pi 10^{-4} \Sigma N_D D^3 \delta D V$ , where

- V=Terminal velocity of dropsize  $D(m/sec)$ ,  $N_D =$  number of drops between diameter limits per unit volume of air (per cubic
	- metre) and

 $D =$  drop diameter (mm)

- 2.  $W =$ Liquid water content  $\rm (mgm/m^3)$  $\equiv_{\mathcal{B}}^1 \pi \Sigma N_D D^3 \delta D$
- 3.  $Z =$ Radar reflectivity factor (mm<sup>6</sup>/m<sup>3</sup>)  $= \Sigma N_D D^6 \delta D$
- 4.  $D_n = \text{Median}$ volume diameter (i.e., value of that drop diameter which divides the drop distribution into two parts such that each represents half of the liquid water content  $W$ )
- 5.  $D =$ Predominant diameter (the diameter corresponding to the maximum volume of water per unit interval of diameter range, *i.e.*, the diameter for which  $\frac{\partial F}{\partial x}$  of 'Best' formula is greatest)
- 6.  $D_m$  = Mean diameter (*i.e.*, the sum of all drop diameters divided by the number of drops)
- 7.  $D_d =$ Mode diameter (*i.e.*, the diameter corresponding to the maximum number of drops)
- 8.  $D_v =$ Mean volume diameter, *i.e.*, diameter of a drop whole volume is equal to the average drop volume
- 9.  $D_{\text{max}}$ , *i.e.*, the maximum size of the drop measured
- 10.  $D_{\min}$ , *i.e.*, the minimum size of the drop measured
- 11.  $n$ ,  $a$ , constants of Best's formula  $a = AR'^p$  (A, p constants)
- 12.  $G(n)$  = Radar reflectivity coefficient  $= D_n^{-3} \Sigma N_D D^6 / \Sigma N_D D^3,$ a quantity dependant on spectrum breadth, the median volume diameter  $D_n$  (mm)
- 13. Number of drops sampled
- 14.  $V/V_0$  = Weighted fall velocity given by Atlas (1957)

$$
R' = \cdot 0036 \ V_0 \left( \overline{V} / V_0 \right) W \tag{3}
$$

 $V_0$  Fall velocity of medium volume size in Eq.  $(3)$ .

15. Calculated value of  $D_n = 0.69^{1/n}a$ ; we obtain  $D_n$  as a solution of Best's equation  $(1)$ 

*i.e.*, 
$$
\frac{1}{2} = \exp [-(x/a)^n]
$$
  
\n
$$
D_n = a (\log 2)^{1/n}
$$
\n
$$
D_n = D_{50} = 0.69^{1/n} a
$$
\n(4)

16. Calculated value of 
$$
D_p = a \left(\frac{n-1}{n}\right)^{1/n}
$$

The volume of water comprised by drops with diameter between x and  $x + \Delta x$  is given by

$$
W \frac{\partial F}{\partial x} \triangle x = W \frac{n x^{n-1}}{\triangle n} \exp \left[-(x/a)^n\right] \triangle x
$$
\n(5)

The value of  $D_p$  is that value of x which gives maximum value to the right hand side of Eq. (5), i.e.,

$$
D_p = a \; (1-1/n)^{1/n} = a \; \left(\frac{n-1}{n}\right)^{1/n} \qquad (6)
$$



Samples of raindrop records at Poona on 17 Sep 1960 during a sharp shower



Poona-Heavy showers without thunder, 17 Sep 1960

F-Fraction of liquid water in the air comprised by drops of diameter less than x (mm) Fraction of volume of water as a function of drop size

Fig. 1

# **TABLE**

Summary of rain samples taken with the raindrop recorder during shower





 $\overline{V}/V_0$ : Weighted fall velocity given by equation (8)

 $\mathbf 1$ 

type of rain (cumuliform clouds) without thunder at Poona



Sample taken with dyed Whatman No. I filter paper tape

 $\bar{D}_n \;$ : Median volume diameter

- $\boldsymbol{D}_m\;$  : Mean diameter
- $D_d$ : Mode diameter

 $D_{\mathbf{v}}$ : Mean volume diameter

 $D_p$ : Predominant diameter

 $n, a$ : Constants of Best's formula  $a : AR'^p(A, p \text{ constants})$  $D_{\max}$ : Maximum diameter  $D_{\mbox{\footnotesize min}}~~:~~\mbox{Minimum diameter}$ 

# TABLE

Summary of rain samples taken with the raindrop recorder during continuous type of rain



## **TABLE**

Summary of rain samples taken with the raindrop recorder



 $\overline{2}$ 

	$D_p=\left. a\Big[\frac{n-1}{n}\Big]^{1/n} \right.$ $D_m$		$\mathcal{D}_d$	$\boldsymbol{D}_{\boldsymbol{v}}$	$D_{\max}$	$D_{\text{min}}$	$\boldsymbol{n}$	a	Reflectivity coefficient G(n)	Drops $\overline{V}/V_0$ sampled	
	1.55	1.21	0.09	1.37	2.40	1.21	4.00	1.67	1.55	20	$1 - 03$
	1.32	1.18	$1 - 13$	$1 - 28$	1.85	0.64	2.74	1.44	1.59	12	$1 - 08$
	0.89	0.74	0.64	0.89	2.18	0.50	3.20	$1-00$	1.92	40	$1 - 12$
	1.66	$1 - 14$	0.68	1.59	3.24	0.50	$3 - 32$	1.85	$2 - 86$	13	1.08
	2.60	1.29	0.67	1.94	3.92	0.48	2.52	$3 - 16$	1.98	25	$1 - 00$
	1.87	1.19	$1 - 12$	1.51	2.71	0.32	3.00	2.14	1.87	19	$1 - 05$
	2.43	1.48	0.87	1.97	3.30	0.37	2.68	2.89	1.62	13	1.01
	1.55	1.09	0.63	1.44	3.05	0.61	2.38	1.86	2.13	22	0.98
	2.64	1.28	1.20	2.05	4.08	0.24	2.54	3.24	1.97	28	0.99
	2.20	$1 - 33$	0.62	0.71	3.00	0.48	3.30	2.45	1.75	18	1.03
	1.18	0.89	0.62	$1 - 18$	2.81	0.27	3.34	$1 - 31$	2.17	32	0.98
	1.27	1.06	0.66	1.45	3.69	0.20	1.90	1.99	2.48	52	1.01
	1.53	0.83	0.62	$1 - 12$	2.67	0.20	2.70	1.82	2.48	35	1.01
	1.18	0.94	1.16	$1 - 16$	2.17	0.37	2.50	1:45	$1 - 62$	29	1.00
	$1 - 13$	0.65	0.40	$0 - 96$	2.02	0.09	2.00	1.60	1.46	29	0.95
	1.25	0.87	0.66	$1 - 13$	2.48	0.31	2.56	1.52	1.24	29	0.90
	1.25	1.16	0.82	1.45	2.66	0.43	$3 - 12$	1.93	1.39	24	0.97
	0.71	0.91	1.14	$1 - 12$	2.41	0.33	2.82	1.50	1.55	23	0.96
	1.13	0.83	0.46	1.04	$2 - 06$	0.28	2.34	1.48	$1 - 18$	22	0.90

# with thunder (cumuliform and stratiform clouds mixed) at Poona

 $\bf{3}$ 

during continuous non-freezing rain at Poona





\*Median volume diameter  $Dn$ , liquid water content W, radar echo intensity Z, as a function of rain intensity computed from various sources with additional tropical data from India



\*Sivaramakrishnan's earlier paper (1961) may also be seen in this connection

#### TABLE 5

# Regression equations and rainfall parameter for different types of rain at Poona (Sivaramakrishan and Miss Selvam)



# RELATION OF RAINDROP SIZE TO INTENSITY OF RAINFALL



Samples of raindrop records taken at Poona on 19 Sep 1960 during a severe thunderstorm



F-Fraction of liquid water in the air comprised by drops of diameter less than x (mm) Fraction of volume of water as a function of drop size Fig. 2

Table 4 gives a summary of the regression equation and rainfall parameters for different types of rain as a function of rain intensity computed from various sources for comparison with the data obtained at Poona.

## 4. Ayalysis of results

# Relation of intensity of rainfall R' with distribution of raindrops N<sub>D</sub> for different types of rain at Poona

Fig. 3 shows the raindrop distribution  $a$  during a convective shower type of rain on 17 September 1960 without thunder (cumuliform clouds at the time of shower). As can be seen from the weather diary the rain has occurred from cumuliform clouds and that -

(1) For very low intensity, viz., 7 mm per hour the curve is more or less a straight line without peaks and troughs and in general agreement with that given by the empirical relationship of Marshall and Palmer (1948).

(2) As the rainfall rate increases, the curve instead of being a straight line consists of a number of peaks and troughs as noticed by previous investigators (Mason 1953, Blanchard 1953, Ramana Murthy and Gupta 1959, Mason and Andrews 1960, Sivaramakrishnan 1961). One explanation for the peaks and troughs observed during thunderstorm rain at Poona given by Sivaramakrishnan (1961) is due to a marked discontinuity in the updraft rate with the maximum occurring at a certain level which acting as a sort of barrier to falling raindrops below a certain size will cause maximum concentration of raindrops belonging to a particular size group. Two other explanations are also possible for the peaks and troughs noticed. The first is due to wind shear in the atmosphere. With wind shear, and the fact that the drops that are collected at a given spot and time on the ground may have various positions of origin within the cloud, one might conceive of ways in which the drop distribution can vary with drop size. The other explanation may be in the fact that the drop distribution may not contain enough drops in a given sample to be representative of the entire population of drops.

(3) Marshall and Palmer have given the following expression connecting drop concentration and drop diameter for stratiform type of rainfall originating as snow -

$$
N_D = N_0 e^{-\Delta D} = N_0 e^{-3.67D/D_n}
$$
 (7)

where D is drop diameter,  $N_D \delta_D$  is the number of drops of diameter between D and  $D + \delta D$  per unit volume of space and  $N_0$  is the value of  $N_D$ for  $D = 0$ ,  $N_0 = 0.08$  cm<sup>-4</sup> for any intensity of

rainfall and  $\Delta = 3.67/D_n = 41 R'^{-0.21} cm^{-3}$ where  $R'$  is the rainfall rate in mm/hr and  $D_n$  is the median value diameter. The relation of  $\triangle$  to  $D_n$  is due to Altas (1953).

Mueller and Jones (1960) show that Eq. (8) is not applicable to convective showers in Florida. They show that instead of decreasing exponentially with size throughout, the number of drops increases exponentially to a maximum and then drops exponentially from there. The intersection point of the two exponentials occurs close to  $D=$ 1.5 mm. Rainfall rate in these storms is a more sensitive function of the total number of drops than of the slope. Atlas (1963) has fitted a simple linear relation to the  $(N_t - R')$  curve presented by Mueller and Jones with the result  $N_t = 100 + 7k$ , where  $N_t$  is the total number of drops per m<sup>3</sup> and  $R'$  is the rainfall rate in mm/hr. This compares to  $N_t = 1.95 \times 10^3 R'^{0.21}$  for the Marshall and Palmer (M.P.) distribution.

(4) A similar characteristics of peaks and troughs is also found in the drop spectra in the other two types of rain discussed (Fig. 3).

Relation of intensity of rainfall R' with liquid water content W in different types of rain

By the method of least squares the following relations between  $W$  and  $R'$  have been obtained for the different types of rain-

1.  $W = 88.3R^{0.81}$ 

shower type of rain for from cumuliform clouds

2.  $W = 81 \cdot 1R'^{0.81}$ 

for continuous type of rain with occasional thunder from stratiform and cumuliform clouds (mixed)

3.  $W = 87.3R^{0.80}$ 

for non-freezig rain from stratiform clouds.

Relation of intensity of rainfall R' with average size diameter  $(D_d, D_m, D_v, D_n, D_p)$  in different types of rain

By the method of least squares the following relationship between the average size of raindrop and  $R'$  has been obtained for the different types of rain. The average size of raindrop can be represented by-

 $D_d$  (mode diameter),  $D_m$  (mean diameter),  $D_u$ (mean volume diameter),  $D_n$  (median diameter),  $D_p$  (predominant diameter)



RAINDROP DISTRIBUTION OF CONTINUOUS TYPE RAIN WITH THUNDER (STRATIFORM AND CUMULIFORM CLOUDS) AND NON-FREEZING RAIN (STRATIFORM CLOUDS) ON 19-9-1960 AND 26-6-1960 RESP. AT POONA

Fig. 3

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Average size of raindrop	Type of rain						
	Shower	Continuous	Non-freezing				
$D_d$	0.45R'0.24	$0.57R$ <sup>10</sup> .10	$0.65R'$ 0.09				
${\cal D}_m$	$0.52R'$ 0.26	$0.54R^{6.23}$	$0 - 0.5R'0 - 19$				
$D_{\vartheta}$	$0.65R^{6.24}$	$0.56R^{0.31}$	0.71R'0.23				
D	$0.80R^{r}0.23$	0.71R'0.32	$0.59K^{0.33}$				
D $\mathcal{P}$	$0.76R'$ <sup>0.27</sup>	$0.41R^{0.50}$	0.42R'0.52				

The above values have been plotted in Fig. 4(a). It will be seen that for the same rate of rainfall, mode diameter is lowest for all types of rain in the following order  $D_d$ ,  $D_m$ ,  $D_v$ ,  $D_n$ ,  $D_p$ .

Atlas (1957) has shown that

$$
R' = 0.036 \times V_0 \left( \frac{V}{V_0} \right) \times W \tag{8}
$$

where,  $V_0 = \text{fall velocity of median volume size}$ ,  $\overline{V}/V_0$  = weighted fall velocity defined by Eq. (8) itself. He as well as the author (Sivaramakrishnan 1961) have found that for most raindrop spectra  $\overline{V}/V_0$  falls very close to 1. From Tables 1 to 4 the ratio  $\overline{V}/V_0$  is found to be very close to 1 which clearly shows that a drop distribution in general can be better represented by a uniform collection of drops with size equal to the median volume diameter  $D_n$ . Best (1951) has also shown from his study of drop size distribution in clouds that if *n* is not known,  $D_m$  and  $D_v$  are quite unsuitable for specifying the mean drop size not only because they vary with the minimum measurable diameter but also, they vary considerably with In this respect he has concluded that  $D_n$  is  $\overline{n}$ . better than any other representation of average size.

## Radar reflectivity factor and intensity of rainfall

The power received at a radar from a rain target is proportional to the radar reflectivity factor  $Z = \mathcal{Z} N_D D^6 \delta D$ , where N is the number of drops per cubic metre of diameter  $D$  on the size interval  $\delta D$ .

Values of  $Z = \Sigma N_D D^6 \delta D$  have been tabulated in Tables 1 to 3 for different types of rain and plotted in Fig. 4(b). By the method of least squares, the following relations between radar reflectivity factor  $Z$  and  $R'$  have been determined for the different types of rain and plotted in Fig.  $4(b)$ .



Bartnoff and Atlas (1951) have given a basic equation for Z (The radar Reflectivity Factor) as follows-

$$
Z = -\frac{6}{\pi} G(n) . D_n^3 \frac{W}{\rho} \text{ mm}^6/\text{m}^3 \tag{9}
$$

 $D_n =$  Median volume diameter in mm where.  $W =$  Liquid water content in mgm/m<sup>3</sup>  $\rho =$  particle density in gm/cc

 $G(n)$  = Reflectivity coefficient =  $D_n$  -3  $\sum N_D D^6$  $\sum N_D D^3$ , a quantity dependant on spectrum breadth.  $G$  is typically found between  $1.5$  and 3.0 for the different types of rain.

The mean values of  $G(n)$  for the different types of rain discussed here are given below-



Equations (a), (b), (c) are useful in estimating liquid water content (LWC) from radar measurements of Z provided there are negligible updrafts to alter the size distribution from that observed at the ground. This assumption is generally valid in stratiform type of rain. It is seen from the above equations that  $Z$  is almost directly proportional to  $W^2$  for Poona rains also (cf. Battan 1959).

#### 5. Conclusions

1. From the study of the size distribution of raindrops it is shown that the relation of intensity of rainfall with (1) number of drops per cubic metre  $N_D$ , (2) average size of raindrops ( $D_d$ ,  $D_m$ ,  $D_v$ ,  $D_n$ ,  $D_p$ ), (3) liquid water content W and (4) radar reflectivity is found to be different for the different types of rain taken for study. But it is seen that the regression equation connecting  $W$ and  $R'$  for the different types of rain studied are not very much different. As regards the relation between  $Z$  and  $R'$  the regression equation between shower type of rain (without thunder) and nonfreezing rain is not very much different.

2. The average size of raindrop is better represented by the median size diameter  $D_n$  than by the other raindrop size parameters  $(D_d, D_m, D_v)$ ,  $D_p$ ). It is interesting to see that the regression equation connecting median size diameter and intensity of rainfall (*i.e.*,  $D_n = 0.80R^{r_0.23}$ ) is

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found to be practically same as for thunderstorm. rain observed earlier by Sivaramakrishnan (1961) at Poona (viz.,  $D_n = 0.82R'^{0.29}$ ), also Marshall and Palmer distribution (1948) (mostly stratiform type is as reported by Best) gives an expression<br>for  $D_n$  as  $D_n = 0.82 R' 0.24$ . Thus while individual samples deviate sharply from the M. P. distribution, it appears that the latter is a fairly good average representation even in the tropics. This has been supported by Atlas also (1963).

3. Best's formula, *viz.*,  $1-F = \exp \{- (x/a)^n \}$ 

is found to be suitable for representing the size distribution of raindrops in the tropics also but with suitable changes in the constants.

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