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Growth of random errors in temperature forecasts by numerical method using centred time-differences

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ABSTRACT. The growth of initial random errors in temperature forecasts by numerical method using centred time-differences is investigated. Horizontal advection in one dimension is considered. Assuming that there is no correlation between the initial random errors at the different grid points and neglecting any correlation that may develop in the course of computation, the random errors grow much more rapidly in this method than in forward time differencing. In both methods, correlations develop between the random errors at different grid points in the course of computation. When these are taken into account, the growth of random errors is further enhanced in the forward differences. In the centred time-differences method, these correlations keep the random error almost at the initial level.

1. Introduction

In an earlier paper, Rao and Ramamurti (1970) derived an expression for the growth of random errors in temperature forecasts by numerical method calculated by forward time differencing. In order to minimise truncation errors, it is more usual to prognosticate temperature using time-centred differences. Increase of random errors in temperature forecasts by the latter method is discussed in this paper.

The statistical method developed in the earlier paper is briefly recapitulated below, as it forms the basis of this paper as well. Let the functions, A, B, and C be related as —

$$C = A \times B$$

Assuming that the errors in A and B are uncorrelated, it was shown that —

$$\text{Var } C = \overline{A}^2 \text{ Var } B + \overline{B^2} \text{ Var } A + \text{Var } A . \text{ Var } B \tag{1}$$

$\overline{A}$ and $\overline{B}$ represent the correct values free of error. Var C is the variance of error in C, consequent on errors in A and B. The other relation used is that if,

$$M = P + Q \tag{2}$$

$$\text{Var } M = \text{Var } P + \text{Var } Q$$

provided P and Q are uncorrelated.

2. Error in forecasts by time-centred differences

The procedure of forecasts by time-centred differences is outlined below. From the initial temperature $T(0)$, the temperature $T(1)$ after the first time step is calculated by forward time-differencing as —

$$T(1) = T(0) + (\triangle T/\triangle t)_0 \triangle t \tag{3}$$

All subsequent steps are by time-centred differences, so that,

$$T(2) = T(0) + 2\,(\triangle T/\triangle t)_1 \triangle t \tag{4}$$

and

$$T(n) = T(n-2) + 2\,(\triangle T/\triangle t)_{n-1} \triangle t \tag{5}$$

where the suffix of $(\triangle T/\triangle t)$ indicates the time for which the ratio is calculated.

Therefore,

$$\text{Var}(1) = \text{Var}(0) + (\triangle t)^2 \text{ Var } (\triangle T/\triangle t)_0 \tag{6}$$

$$\text{Var}(2) = \text{Var}(0) + 4(\triangle t)^2 \text{ Var } (\triangle T/\triangle t)_1 \tag{7}$$

$$\text{Var}(n) = \text{Var}(n-2) + 4(\triangle t)^2 \text{ Var } (\triangle T/\triangle t)_{n-1} \tag{8}$$

In deriving these relationships it is assumed that there is no correlation between the T term and the corresponding $\triangle T/\triangle t$ term.

Neglecting diabatic heating and assuming that motion is dry adiabatic,

$$-\frac{\partial T}{\partial t} = u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} + w(\Gamma_d - \Gamma) \tag{9}$$

Assuming,

(*i*) $\text{Var } u = \text{Var } v = \text{Var } U$

(*ii*) $(\triangle T)_x = (\triangle T)_y = (\triangle T)$

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and that,

(*iii*) there is no correlation between the different terms in Eq. (9) and between the errors in temperatures at neighbouring grid points, it was derived in the earlier paper,

$$\text{Var}\left(\frac{\triangle T}{\triangle t}\right) = \frac{|\mathbf{V}|^2 \text{ Var } T}{2G^2} + \frac{(\triangle T^2) \text{ Var } U}{2G^2} + \frac{\text{Var } T \text{ Var } U}{G^2} + \frac{w^2 \text{ Var } T}{2G_Z^2} + (\Gamma_d - \Gamma)^2 \text{ Var } w + \frac{\text{Var } T \text{ Var } w}{2G_Z^2} \tag{10}$$

where, $|\mathbf{V}|^2 = u^2 + v^2$

In Eq. (10), terms involving vertical velocity may be of the same order as the other terms in certain situations, as $G_z << G$ though $w << |\mathbf{V}|$. For simplicity we neglect the terms involving vertical velocity, and write,

$$\text{Var}\left(\frac{\triangle T}{\triangle t}\right) = \frac{|\mathbf{V}|^2 \text{ Var } T}{2G^2} + \frac{(\triangle T)^2 \text{ Var } U}{2G^2} + \frac{\text{Var } T \text{ Var } U}{G^2} \tag{11}$$

noting that in convective situations Var $\triangle T/\triangle t$ will be more than the calculated value.

$$\text{Var } \frac{\triangle T}{\triangle t} = \frac{\text{Var } T}{2G^2}\left[|\mathbf{V}|^2 + \frac{(\triangle T)^2 \text{ Var } U}{\text{Var } T} + 2 \text{ Var } U\right] = \frac{\text{Var } T}{(\triangle t)^2}\,\epsilon \tag{12}$$

where,

$$\epsilon = \frac{(\triangle t)^2}{2G^2}\left[|\mathbf{V}|^2 + \frac{(\triangle T)^2 \text{ Var } U}{\text{Var } T} + 2 \text{ Var } U\right] \tag{13}$$

We now substitute from equation (12) in equations (6), (7) and (8).

$$\text{Var (1)} = \text{Var (0)} + \epsilon_0 \text{ Var(0)} \tag{14}$$

$$\text{Var (2)} = \text{Var (0)} + 4\epsilon_1 \text{ Var (1)} \tag{15}$$

$$\text{Var } (n) = \text{Var}(n\text{-}2) + 4\epsilon_{n-1} \text{ Var}(n\text{-}1) \tag{16}$$

The three items in brackets in Eq. (13) are in decreasing order and under strong wind conditions it suffices to take into account the first term only. Using equations (13), (14), (15) and (16), it is possible to follow the growth of random errors in any actual computation, making some suitable assumption about Var U. $|\mathbf{V}|$, $(\triangle t)$ and Var T values in ϵ may vary at each step and appropriate values will have to be substituted.

In an attempt to estimate the growth of random errors under different wind and temperature regimes, we assume in the first instance that $|\mathbf{V}|$, $(\triangle t)$ and Var U are constant. Var T in the right hand side of equation (13) is taken as remaining equal to the initial value, *viz.*, Var (0). As the first term in the brackets dominates, this will not materially affect the computation of Var (n). With these assumptions ϵ becomes a constant and,

$$\text{Var (1)} = (1 + \epsilon) \text{ Var (0)} \tag{17}$$

$$\text{Var (2)} = (1 + 2\epsilon)^2 \text{ Var (0)} \tag{18}$$

and

$$\text{Var } (n) = \text{Var } (n-2) + 4\,\epsilon \text{ Var } (n-1) \tag{19}$$

As ϵ is positive and as variance is necessarily positive the solution of (19) can be written as

$$\text{Var } (n) = [A(q)^n + B(-q)^{-n}] \text{ Var (0)} \tag{20}$$

where,

$$q = 2\epsilon + \sqrt{1 + 4\epsilon^2} \tag{21}$$

Substituting for Var (0) and Var (1) in Eq.(20),

$$A + B = 1$$

and $Aq - B/q = 1 + \epsilon$

Hence,

$$A = \frac{1 + q(1+\epsilon)}{q^2 + 1}$$

and

$$B = \frac{q^2 - q(1+\epsilon)}{q^2 + 1} = \frac{q\,(q - 1 - \epsilon)}{q^2 + 1}$$

As ϵ is one order of magnitude smaller than 1,

$$q = 2\epsilon + \sqrt{1 + 4\epsilon^2} \approx 1 + 2\epsilon + 2\epsilon^2 \tag{22}$$

q is greater than but near about 1. A is nearly one and B about $\frac{1}{2}\epsilon$. Hence for larger values of n, the term with coefficient B in (20) can be neglected and

$$\text{Var } (n) = Aq^n \text{ Var (0)} \quad (23$$

TABLE 1

Values of $\sigma(n)/\sigma(0)$ by centred time-differences method (assuming that temperature errors at neighbouring grid points are uncorrelated at all time steps of computation)

ϵ	n				
	10	20	100	200	300
0·2	6·5				
0·1	2·6	7·1			
0·01		1·2	2·7		
0·001			1·1	1·2	1·3

Equation (19) and solution (20) appear similar to those for growth of truncation errors, but apply to random errors here.

3. Comparison of errors

Table 1 gives the ratio of $\sigma(n)/\sigma(0)$ for different values of ϵ and n.

Comparison may be made of the growth of random errors in this method with that by forward differences. For ϵ of the order of 0·1 or less, expressions for q and A can be further simplified by neglecting terms with 2nd and higher powers of ϵ.

$$q \approx 1 + 2\epsilon$$

$$A \approx \frac{2+3\epsilon}{2+4\epsilon} \approx \left(1 - \frac{\epsilon}{2}\right)$$

Eq. (23) becomes,

$$\text{Var}\,(n) = \left(1 - \frac{\epsilon}{2}\right)(1+2\epsilon)^n\,\text{Var}\,(0)\big] \quad (24)$$

Denoting the value by forward differences Varm (n)

$$\text{Varm}\,(n) = (1+\epsilon)^n\,\text{Var}\,(0) \quad (25)$$

as given in the earlier paper.

It is clear that growth of random errors is much more in time-centred differences than in the forward differences provided the assumption holds that no correlation exists or develops between error values at neighbouring grid points. Table 2 gives values of $\sigma'(n)/\sigma(0)$ for the same values of ϵ as in Table 1.

4. Numerical experiment to study the growth of errors

In the earlier paper a simple numerical experiment was conducted to test whether the growth of errors by forward differences was according to the theory presented. The result was encouraging. Nevertheless, it was considered that a more thorough test should be carried out.

TABLE 2

Values of $\sigma'(n)/\sigma(0)$ by forward differences (assuming that temperature errors at neighbouring grid points are correlated at all time steps of computation)

ϵ	n				
	10	20	100	200	300
0·2	2·5				
0·1	1·6	2·6			
0·01		1·1	1·6		
0·001			1·05	1·10	1·15

The propagation of temperature (in one dimension) along a closed circle is considered as in the earlier paper. The relevant differential equation is,

$$\partial T/\partial t = -U\,\partial T/\partial x \quad (26)$$

We shall again assume that U is a constant throughout the period of integration.

This may be written in a finite difference form in forward differences as,

$$T_j(n) = T_j(n-1) - \frac{U\triangle T}{2G}\Big[T_{j+1}(n-1) - T_{j-1}(n-1)\Big] \quad (27)$$

20 grid points were taken along the circle. Setting $U\triangle t/2G = p$, the equation may be written in the matrix form as,

$$\begin{vmatrix} 1 & -p & 0 & 0 & - & - & p \\ p & 1 & -p & 0 & - & - & 0 \\ 0 & p & 1 & -p & - & - & 0 \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ -p & 0 & 0 & - & - & p & 1 \end{vmatrix} \begin{vmatrix} T_1(n-1) \\ T_2(n-1) \\ - \\ - \\ - \\ T_{20}(n-1) \end{vmatrix} = \begin{vmatrix} T_1(n) \\ T_2(n) \\ - \\ - \\ - \\ T_{20}(n) \end{vmatrix} \quad (28)$$

Or

$$|\mathbf{P}|\;|\mathbf{T}(n-1)| = |\mathbf{T}(n)|$$

As by our assumptions p is a constant, Eq. (28) leads to,

$$\begin{vmatrix} 1 & -p & 0 & - & - & - & p \\ p & 1 & -p & - & - & - & 0 \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ -p & 0 & 0 & - & - & p & 1 \end{vmatrix}^n \begin{vmatrix} T_1(0) \\ T_2(0) \\ - \\ - \\ - \\ T_{20}(0) \end{vmatrix} = \begin{vmatrix} T_1(n) \\ T_2(n) \\ - \\ - \\ - \\ T_{20}(n) \end{vmatrix}$$

Or

$$|\mathbf{P}|^n \; |\mathbf{T}(0)| = |\mathbf{T}(n)| \qquad (29)$$

where the matrix in p is raised to the n^{th} power. The initial temperature $T(0)$ at any point may be regarded as composed of a true value $t(0)$ and an error $e(0)$.

$$T(0) = t(0) + e(0)$$

Equation (29) may be written as

$$|\mathbf{P}|^n \; |\mathbf{t}(0) + \mathbf{e}(0)| = |\mathbf{t}(n) + \mathbf{e}(n)|$$

where

$$|\mathbf{P}|^n \; |\mathbf{t}(0)| = |\mathbf{t}(n)|$$

and

$$|\mathbf{P}|^n \; |\mathbf{e}(0)| = |\mathbf{e}(n)|$$

The $t(n)$ series of temperatures would be subject to growth of truncation errors but not random errors while $e(n)$ series represent the growth of random errors. This shows the growth of random errors can be studied independent of the real temperature variation along the circle. Hereafter **T** in equation (29) will be regarded as referring to random error only. There is advantage in separating the treatment of truncation and random errors as the latter can be developed by statistical methods.

Twenty normalised error series of twenty values each, with zero mean were constructed. These twenty series were associated with the twenty grid points. In the first computation, the first values of the twenty series were assigned to the twenty grid points and the temperatures after ten time steps were computed at all the grid points. Next the second set of values in the series were assigned to the grid points and computations were made up to ten time steps. Similarly computations were made with all the twenty sets in the series.

To make the point clear let us represent the twenty values of the twenty series as —

$e_{1,1}$	$e_{1,2}$	$e_{1,3}$	—	—	—	—	$e_{1,20}$
$e_{2,1}$	$e_{2,2}$	$e_{2,3}$	—	—	—	—	$e_{2,20}$
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
$e_{20,1}$	$e_{20,2}$	$e_{20,3}$	—	—	—	—	$e_{20,20}$

The series $e_{1,1}, e_{2,1} \ldots\ldots e_{20,1}$ is normalised with zero mean. Similarly the other columns. The column number shows association with the grid point. Values with the same row number are used as initial values for computation.

After the computation, twenty final temperature values are available for each grid point. The mean and standard deviation of the twenty values at each grid point were calculated. It was found that the mean of the final values was zero. The standard deviations at the twenty grid points ranged from 1·47 to 2·51 with a mean value (calculated from the mean variance) of 2·02. In the computation the following values were used: $U = 50$ kt, $G = 100$ n. miles, $p = 0{\cdot}25$ and $\epsilon = 0{\cdot}125$. By Eq. (25) the value of $\sigma'(n)/\sigma(0)$ should have been 1·8, while the average value is 2·0. Though the expected and computed values are reasonably close, the latter is definitely higher.

Similar computation was made for the centred time-differences method. Surprisingly the standard deviations at the twenty points ranged from 0·72 to 1·23 with a mean value of 1·05. As per Eq. (23) the value should have been 3·3.

To sum up, accumulation of error is more in the forward differences and less in the centred time-differences method than the expected values derived on the assumption that there is no correlation between the errors at various grid points at any time step. The discrepancies are explained in the next section as due to the correlation which develops between errors at neighbouring grid points in the course of computation.

5. Correlation between grid points — Forward difference

From Eq. (27),

$$\left.\begin{aligned} T_j(1) &= T_j(0) - pT_{j+1}(0) + pT_{j-1}(0) \\ T_{j+1}(1) &= T_{j+1}(0) - pT_{j+2}(0) + pT_j(0) \\ T_{j-1}(1) &= T_{j-1}(0) - pT_j(0) + pT_{j-2}(0) \end{aligned}\right\} \quad (30)$$

Similar equations for the next time step are:

$$\left.\begin{aligned} T_j(2) &= T_j(1) - pT_{j+1}(1) + pT_{j-1}(1) \\ T_{j+1}(2) &= T_{j+1}(1) - pT_{j+2}(1) + pT_j(1) \\ T_{j-1}(2) &= T_{j-1}(1) - pT_j(1) + pT_{j-2}(1) \end{aligned}\right\} \quad (31)$$

As in the previous section we shall consider that we are dealing with errors in the initial values $T(0)$ which are mutually uncorrelated.

From (30) we may write,

$$\text{Varm}\,(1, j) = \text{Varm}\,(0, j) + p^2\,\text{Varm}(0, j+1) + p^2\,\text{Varm}(0, j-1) \qquad (32)$$

In deriving (32) correlations between neighbouring grid points do not arise as we have assumed that the initial values are uncorrelated.

We had also set,

$$\text{Varm}(0,j) = \text{Varm}(0, j+1) = \text{Varm}(0, j-1) = \text{Varm}(0)$$

which is the same as Var (0) = 1.

Hence,

$$\text{Varm}(1) = (1+2p^2)\,\text{Var}(0) \qquad (33)$$

We have omitted the subscript j on the left hand side as the value on the right hand side has no j in it, that is, the variance is the same at all the grid points.

Considering the first of the equations (31)

$$\begin{aligned}\text{Varm}(2,j) = {} & \text{Varm}(1,j)+p^2\,\text{Varm}(1,j+1) + \\ & + p^2\,\text{Varm}(1,j-1)- \\ & -2pr(1_j, 1_{j+1})\,\sigma'(1,j)\,\sigma'(1,j+1)+ \\ & +2pr(1_j, 1_{j-1})\,\sigma'(1,j)\,\sigma'(1,j-1)- \\ & -2p^2r(1_{j+1}, 1_{j-1})\,\sigma'(1,j+1)\sigma'(1,j-1)\end{aligned} \qquad (34)$$

The correlation* $r(1_j; 1_{j+1})$ between $T_j(1)$ and T_{j+1} (1) can be obtained from the common terms between their expressions in Eq. (30). (The method of deriving this is given in Appendix B).

$$\left.\begin{aligned} r(1_j, \quad 1_{j+1}) &= 0, \\ r(1_j, \quad 1_{j-1}) &= 0, \\ r(1_{j+1}, 1_{j-1}) &= \frac{-p^2}{2+2p^2}\end{aligned}\right\} \qquad (35)$$

Using (35) and noting that $\sigma'(1)$ is the same at all the grid points, (34) becomes,

$$\begin{aligned}\text{Varm}(2) &= \text{Varm}(1)\left[1+2p^2+\frac{2p^4}{1+2p^2}\right] \\ &= (1+2p^2)\left[1+2p^2+\frac{2p^4}{1+2p^2}\right]\text{Var}(0)\end{aligned} \qquad (36)$$

Again the subscript j on the left hand side has been omitted. Neglecting the correlation that has developed between neighbouring points, the corresponding expression would have been $(1+2p^2)^2$

As $2p^2 = \epsilon$, (36) may also be written as,

$$\text{Varm}(2) = (1+\epsilon)\left[1+\epsilon+\frac{\epsilon^2}{2(1+\epsilon)}\right]\text{Var}(0) \qquad (37)$$

For the third time step,

$$T_j(3) = T_j(2) - pT_{j+1}(2) + pT_{j-1}(2) \qquad (38)$$

Values of $T(2)$ in terms of $T(0)$ will be given by

$$|\mathbf{P}|^2\,|\mathbf{T}(0)| = |\mathbf{T}(2)| \qquad (39)$$

The expression for variance from (38) is,

$$\begin{aligned}\text{Varm}(3,j) = {} & \text{Varm}(2,j)+p^2\,\text{Varm}(2,j+1)+ \\ & +p^2\,\text{Varm}(2,j-1)- \\ & -2pr(2_j, 2_{j+1})\,\sigma'(2,j)\,\sigma'(2,j+1)+ \\ & +2pr(2_j, 2_{j-1})\,\sigma'(2,j)\,\sigma'(2,j-1)- \\ & -2p^2r(2_{j+1}, 2_{j-1})\sigma'(2,j+1)\sigma'(2,j-1)\end{aligned} \qquad (40)$$

From common terms in (40)

$$\begin{aligned} r(2_j, 2_{j+1}) &= 0 \\ r(2_j, 2_{j-1}) &= 0 \\ r(2_{j+1}, 2_{j-1}) &= \frac{-2p^2(1+2p^2)}{(1+4p^2+6p^4)}\end{aligned}$$

In Eq. (35) similar correlation was only $-p^2/(1+2p^2)$ whereas at the next time step it is $(-2p^2-4p^4)/(1+4p^2+6p^4)$. Thus the numerical value of correlation entering into the error growth equation increases with the number of time steps.

Eq. (40) can be written with usual simplification as,

$$\begin{aligned}\text{Varm}(3) &= \left(1+2p^2+\frac{4p^4+8p^6}{1+4p^2+6p^4}\right)\text{Varm}(2) \\ &= (1+2p^2)\left(1+2p^2+\frac{2p^4}{1+2p^2}\right)\times \\ &\quad\times\left(1+2p^2+\frac{4p^4+8p^6}{1+4p^2+6p^4}\right)\text{Var}(0) \\ &= (1+\epsilon)\left[1+\epsilon+\frac{\epsilon^2}{2(1+\epsilon)}\right]\times \\ &\quad\times\left[1+\epsilon+\frac{\epsilon^2+\epsilon^3}{1+2\epsilon+\frac{3}{2}\epsilon^2}\right]\text{Var}(0)\end{aligned} \qquad (41)$$

Neglecting the correlations, the expression was $(1+\epsilon)^3$. In the actual calculation the correlation $r(n_{j+1}, n_{j-1})$ increased from 0·05 at $n=1$ to 0·45 at $n=9$. Taking these into consideration $\sigma'(10)/\sigma(0)=2\cdot05$ instead of 1·8, while the mean computed value is 2·02.

6. Correlation between grid points—centred time differences method

The effect of correlations in the centred time differences method will now be considered. The

*As T's represent errors, $Tj(0)$ can be taken to represent a series of error values. Correspondingly $Tj(n)$ has a series of values. Correlations are between these series.

first step is by forward differences and no correlations enter on account of the initial assumption. Hence,

$$\text{Var}(1) = (1 + \epsilon)\ \text{Var}(0)$$

The expression for the second time step in finite difference form is —

$$T_j(2) = T_j(0) - 2pT_{j+1}(1) + 2pT_{j-1}(1) \qquad (42)$$

$T_{j+1}(1)$ and $T_{j-1}(1)$ are to be taken from (30).

$$\begin{aligned}\text{Var}(2,j) = {} & \text{Var}(0,j) + 4p^2\,\text{Var}(1,j+1) + \\ & + 4p^2\,\text{Var}(1,j-1) - \\ & - 4pr(0_j, 1_{j+1})\,\sigma(0,j)\,\sigma(1,j+1) + \\ & + 4pr(0_j, 1_{j-1})\,\sigma(0,j)\,\sigma(1,j-1) - \\ & - 8p^2 r(1_{j+1}, 1_{j-1})\,\sigma(1,j+1)\sigma(1,j-1)\end{aligned} \qquad (43)$$

$$\begin{aligned} r(0_j, 1_{j+1}) &= p/(1+2p^2)^{\frac{1}{2}} \\ r(0_j, 1_{j-1}) &= -p/(1+2p^2)^{\frac{1}{2}} \\ r(1_{j+1}, 1_{j-1}) &= -p^2/(1+2p^2)\end{aligned}$$

Substituting σ (1) at all grid points,

$$\text{Var}(2) = \text{Var}(0)\ (1 + 6\epsilon^2) \qquad (44)$$

Neglecting the correlation Var (2)/Var (0) is $(1+2\epsilon)^2$, whereas it is now only $(1+6\epsilon^2)$. This is even less than the ratio after the first time step which is $(1+\epsilon)$.

Similarly,

$$\text{Var}(3) = \text{Var}(0).\,[1 + \epsilon - 12\epsilon^2 + 40\epsilon^3] \qquad (45)$$

Without taking into account the correlation Var(3)/Var(0) would have been $1+5\epsilon+16\epsilon^2+16\epsilon^3$ whereas it is now only $1+\epsilon-12\epsilon^2+40\epsilon^3$ which is much smaller. This process repeats itself and the value of Var(n) is kept low, almost at the initial value. The correlations between different terms in the formula of the centred time differences method is such as to keep the growth of random errors very low. Taking into account the correlations, $\sigma(10)/\sigma(0)$ should according to theory have been $1 \cdot 05$ while the actual in the computation was $1 \cdot 05$.

One important point in the numerical experiments is that the value of p has been kept constant at all the grid-points. This arises, from assuming the same wind speed at all points which remains constant throughout the period of integration. Though this is not likely to be realised in actuality, the wind speeds may not vary much at neighbouring points. Hence the error growth in this method may be restricted and the error may not change radically from its original value. The above result of the effect of correlation has been derived for motion in one dimension. The effect in two and three dimensional grids and for more generalised motion in the atmosphere requires further study.

In the earlier paper it was made out that due to the increase of random errors with each step in the numerical prognostication by forward differences, some time limit is set for the period of a useful temperature forecast. In the centred time differences method due to correlation between computed error values, the period for which useful forecasts can be made appears to be considerably extended.

7. Conclusions

(1) In temperature forecasts by centred time differences method growth of random errors is much more rapid than in the forward differencing if correlations that develop between computed errors at different grid points are neglected.

(2) If these correlations are taken into account, the increase in the errors in the centred time differences method seems negligible while in the forward differencing the errors are further enhanced by these correlations.

(3) The effect of correlations on growth of random errors has to be investigated in the case of two and three dimensional grids, and for a more generalised motion in the atmosphere.

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APPENDIX A

Explanation of Symbols

σ_A — Standard deviation of errors in A.

$\sigma_{(n)}$ — Standard deviation of errors of temperature after n time steps, by centred time differences method.

$\sigma_{(n,j)}$ — Standard deviation of errors of temperature after n time steps of integration at j^{th} grid point, by centred time differences method.

$\sigma'_{(n)}$ — Standard deviation of errors of temperature after n time steps, by forward differences.

$\sigma'_{(n,j)}$ — Standard deviation of errors of temperature after n time steps at j^{th} grid point by forward differences.

$T(n)$ — Temperature after n time steps of numerical integration.

$T_j(n)$ — Temperature at the j^{th} grid point after n time steps.

t — Time.

$(\triangle T)_x$ — Difference in temperature between two grid points in the x-direction.

$(\triangle T)_y$ — Difference in temperature between two grid points in the y-direction.

G — Horizontal grid distance.

G_Z — Half the vertical interval used to calculate the lapse rate.

$r(t_j, s_k)$ — C. C. between temperature values at j^{th} point after t time steps and k^{th} point after s time steps.

Var A — Variance of error in $A = \sigma_A{}^2$.

Var T — Variance of error in temperature.

Var(n) — Variance of error in temperature after n time steps.

Var(n,j) — Variance of error in temperature after n time steps at j^{th} grid point by centred time differences.

Varm(n) — Variance of error in temperature after n time steps by forward differences.

Varm(n,j) — Variance of error in temperature after n time steps at j^{th} grid point by forward differences.

Γ — Lapse rate of temperature.

Γ_d — Dry-adiabatic lapse rate.

u — Horizontal zonal velocity component of air.

v — Horizontal meridional velocity component of air.

w — Vertical velocity component of air.

APPENDIX B

Let P and Q be two functions of $x, y, \ldots\ldots\ldots\ldots$,

$$P = a_1 x + b_1 y + \ldots\ldots\ldots\ldots$$

$$Q = a_2 x + b_2 y + \ldots\ldots\ldots\ldots$$

where $x, y, \ldots\ldots\ldots\ldots$ take a series of values such that $\Sigma x = \Sigma y = \ldots\ldots\ldots\ldots = 0$ $\Sigma xy = \ldots\ldots\ldots\ldots = 0$, and $\sigma_x = \sigma_y = \ldots\ldots\ldots\ldots = \sigma$. The expression for the correlation coefficient between P and Q then is—

$$r_{P,Q} = \frac{\overline{PQ}}{\sigma_P \, \sigma_Q}$$

$$= \frac{\overline{a_1 a_2 x^2} + \overline{b_1 b_2 y^2} + \ldots.. + \overline{(a_1 b_2 + b_1 a_2) xy} + \ldots..}{(\overline{a_1^2 x^2} + \overline{b_1^2 y^2} + \ldots.. + \overline{2a_1 b_1 xy} + \ldots..)^{\frac{1}{2}} \; (\overline{a_2^2 x^2} + \overline{b_2^2 y^2} + \ldots.. + \overline{2a_2 b_2 xy} + \ldots..)^{\frac{1}{2}}}$$

$$= \frac{a_1 a_2 + b_1 b_2 + \ldots..}{(a_1^2 + b_1^2 + \ldots..)^{\frac{1}{2}} \; (a_2^2 + b_2^2 + \ldots..)^{\frac{1}{2}}}$$