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A simple model to study wave-surge interaction

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सार — इस शोध पत्र में हमने एक गणितीय निदर्श का ढाँचा तैयार करने का प्रयास किया है, जिससे पवन से उत्पन्न होने वाली महासागरीय सतही तरंगों तथा बंगाल की खाड़ी के उथले बेसिन में उठने वाली समुद्री तरंगों के योगदान का पता चलता है। इसके लिए अरैखिकीय प्रणोदन अवधि को छोड़कर ऊर्जा सन्तुलन समीकरण पर विचार किया गया और लैक्स वैडरॉफ समाकलन योजना के माध्यम से इसका समाधान निकाला गया है। कार्डोन के सूत्र को अपनाते हुए सभी ग्रिड बिन्दुओं पर पवन का विवरण दिया गया है। द्रवगतिकीय समीकरणों को रैखिकीय रूप में अपनाते हुए जेलेसनियनस्की द्वारा अपनाए गए ढंग और शूमैन एल्गोथ्मि विधि का प्रयोग करते हुए उन समीकरणों का हल निकाला गया है। समुद्री तरंगों के पवन तरंगों में समविष्ट ऊर्जा योगदान के लिए इन समीकरणों का हल निकालने की प्रक्रिया में ऊर्जा सन्तुलन समीकरण के निष्पादन को तंरग अवस्थित अवधि के रूप में सम्मिलित किया गया है। तरंग के योगदान के साथ-साथ तथा इस योगदान पर विचार किए बिना समुद्री तरंग की अनुमानित ऊँचाई की तुलना की गई है।

ABSTRACT. In this paper we have tried to set up a mathematical model that will show the contribution of wind-induced surface waves of the ocean, on surges in shallow basin of Bay of Bengal. For this, the energy balance equation, excluding non-linear forcing term, is considered and solved by Lax-Wendroff integration scheme. Wind is specified over all the grid points following Cardone's formulation. The hydrodynamic equations in linearised form as used by Jelesnianski have been considered and using Shuman's algorithm, those equations have been solved. In the process of solving these equations, the output of the energy balance equation is included as wave set up term to incorporate energy contribution of wind waves to surges. The estimated surge height is compared with and without considering wave contribution.

Key words — Bay of Bengal, Storm-surge, Wave-surge coupling.

1. Introduction

Storm surges are atmospherically forced oscillations of the water level in a coastal or inland water body, in the period range of a few minutes to a few days (Murthy *et al.* 1986). According to this, storm surges are distinct from wind waves and swell, which have periods of the order of a few to several seconds. Storm surges belong to the same class of wave as tides and tsunamis, *i.e.*, long gravity waves. Amongst many, the two principal factors, responsible for storm surge generation (Murthy *et al.* 1986) in shallow region of sea, *e.g.*, Bay of Bengal are :

(*i*) The atmospheric pressure drop

(*ii*) The wind stress

Most of the model studies of storm surges in the Bay of Bengal, have used idealized model cyclones, with most of them having a circular symmetry and

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movement along straight track with constant speed. There are different assumptions regarding the dependence of surface atmospheric pressure and wind speed on distance from the storm centre. However, we in this paper have taken the assumption of Jelesnianski (1965). Pioneering work of surge modelling in Bay of Bengal was done by Das *et al.* (1974). In 1972, he considered wind to be cyclostrophic. In one of his papers (Das 1981), he tried an analytical solution of the governing equations for a basin of uniform depth. He suggested that divergence of wind stress is more important than curl of wind stresses in shallow basin. Among others, a prominent group comprising Johns *et al.* (1983) made a significant contribution in storm surge prediction. They made a numerical model using non-uniform grid spacing. They showed that near coastal bathymetry is critical in determining the coastal surge and corresponding sea surface elevation. Ghosh (1977) attempted to estimate the peak surge height on the basis of nomogram and linearised hydrodynamic equations.

At present our objective is to investigate the surge from the coupling point of view, *i.e.*, how the propagating surge gets influenced by wind waves of comparable frequency. Before going into details of coupling, let us discuss the work done on wave studies. Wave forecasting techniques were first designed and developed in 1940's using empirical relationship between local wind speed and wave height (Sverdrup and Munk 1947). These techniques were designed for application at a single point and permitted only a limited allowance for variable winds during wave generation process. The techniques were afterwards developed by Wilson (1955) to accommodate the varying wind but it also had lacking of a sound theoretical basis that limits its usefulness. Such a theoretical formulation was completed by Pierson *et al.* (1955) by treating the water elevation in terms of a spectrum of surface waves. The individual component of spectrum could then be deduced by linear propagation theory and wave height has been obtained by integrating Longuet-Higgins (1952) relations. The theory of energy gain or loss within the different components of the spectrum has been established by Hasselmann (1962) in the form of energy balance equation. Theories of energy loss were put forward by Miles (1957, 1960) focussing on whitecapping. Later on Golding (1983) prepared a model for British Meteorological Office to give forecast.

We are, here, considering this paper to find out the energy of the surface waves generated at a certain point of time. Before dealing with energy balance equation we have generated the wind velocities following Cardone (1969) over the region 87°E to 92°E and 20°N & 22.5°N. The output of energy balance equation is parameterized as Murthy (1984) in the form of a forcing term and by Shuman (1957) algorithm, the surge height before and after coupling is calculated and compared.

2. Wind specification for wave analysis

Wind is the only driving force for all wave models and hence, a wave model will be good when the wind field will be generated properly. Ideally, wind specification must be such as to allow the important physical process of wave generation, growth and dissipation to be appropriately represented in a wave model. In specifying wind, we have followed Cardone's procedure. Cardone (1969) developed a two-layer model of Marine Boundary Layer (MBL) that includes the effects of atmospheric stability, baroclinity and realistic description of the lower boundary. The atmospheric stability measured by the temperature difference between the sea surface and the overlying air and has been identified as an important factor influencing wave growth. Cardone (1969) considered the universal relationships between non-dimensional wind shear and temperature gradients as follows :

$$\Phi_u = \frac{KZ}{U_*}\frac{\partial U}{\partial Z} \tag{1}$$

$$\Phi_t = \frac{Z}{T^*}\frac{\partial \theta}{\partial Z} \tag{2}$$

$$T^* = -\frac{1}{KU_*}\frac{H}{C_P\,\rho_a} \tag{3}$$

where, K = Von-Karman constant

θ = Potential temperature

H = Sensible heat flux from sea to air

C_p = Specific heat of air at constant pressure

ρ_a = air density

The non-dimensional gradients Φ_u and Φ_t are related by $\Phi_u = \alpha_h \Phi_t$

where, $\alpha_h = \frac{k_n \text{ (eddy diffusivity)}}{k_m \text{ (eddy viscosity)}}$

The assumption of similarity of wind and temperature profiles in the surface layer implies that α_h is constant and hence a modified form of the stability length L′ (Monin-Ozukhov length) is written as follows :

$$L' = \frac{U_* \left(\frac{\partial U}{\partial Z}\right) T}{kg \left(\frac{\partial \theta}{\partial Z}\right)} \quad (4)$$

The set of Eqns. (1), (2) and (3) can be integrated to get the following :

$$U_z = \frac{U_*}{k}\left[\ln\left(\frac{Z}{Z_0}\right) - \varphi\left(\frac{Z}{L'}\right)\right] \quad (5)$$

$$\theta_z - \theta_0 = \frac{T^*}{\alpha_h}\left[\ln\left(\frac{Z}{Z_0}\right) - \varphi\left(\frac{Z}{L'}\right)\right] \quad (6)$$

in the above equations,

$$\varphi\left(\frac{Z}{L'}\right) = \int_0^J \frac{1 - \Phi_u(J)}{J}\, dJ$$

Z_0 = Roughness parameter

The expressions chosen for the stability functions are :

$\Phi_u(0) = 1$ neutral

$\Phi_u = 1 + \beta\left(\frac{Z}{L'}\right)$, $\beta = 7$ stable

$\Phi_u^4 - \Gamma\left(\frac{Z}{L'}\right)\Phi_u^3 - 1 = 0$, $\Gamma = 18$ unstable

The wind profile parameters U_*, and L' can be calculated from the following equations :

$$L' = \frac{U_*^2\, \bar{\theta}\left[\ln\left(\frac{Za}{Z_0}\right) - \varphi\left(\frac{Za}{L'}\right)\right]}{K^2 g(\theta_a - \theta_s)} \quad (7)$$

$$U_* = \frac{KU_m}{\left[\ln\left(\frac{Zm}{Z_0}\right) - \varphi\left(\frac{Zm}{L'}\right)\right]} \quad (8)$$

where,

$\bar{\theta}$ = mean potential temperature

θ_a = potential temperature for air

θ_s = potential temperature at sea surface

Z_a = Height at which θ_a, the potential temperature for air is determined

Z_m = Height at which wind speed U_m is measured

Z_0 = Roughness parameters

$$Z_0 = \frac{bU_*^2}{g} \qquad \text{[Charnock (1955)]} \quad (9)$$

using Eqns. (5) to (9) the solution for wind profile in the surface layer has been determined.

3. Wave-induced surge

To estimate the wave-induced surge, we have sought to find out the energy of the wind waves in the shallow region that is to be coupled with the governing equations of surge. In this context, we have considered Golding's (1983) approach in BMO model. He took energy balance equation used by Hasselmann (1962).

$$\frac{dE}{dt} = s(f, \theta, \bar{x}, t) \quad (10)$$

$$\frac{dE}{dt} = \frac{\partial E}{\partial t} + \nabla \cdot (C_g E) + \frac{\partial}{\partial \theta}\left(E \frac{d\theta}{dt}\right) = s(f, \theta, \bar{x}, t)$$

$$\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + C_g \cdot \nabla\theta = C_g \cdot \nabla\theta \qquad \text{as } \frac{\partial \theta}{\partial t} = 0$$

Here, E = Energy density of the wave field described as a function of frequency f, direction θ, position x and time t.

C_g = Group velocity of the wave field

S = Source function representing the physical processes that transfer energy to and from the wave spectrum.

So, the energy balance equation is rewritten in the form,

$$\left[\left\{\left(\frac{\partial E}{\partial t} = -\frac{\nabla \cdot (C_g E)}{Propagation}\right) - \frac{\frac{\partial}{\partial \theta}(C_g \cdot \nabla\theta)}{Refraction}\right\} + \frac{S_{in}}{Growth} + \frac{S_{ds}}{Decay}\right] \quad (11)$$

Considering the innermost bracketed term,

$$\frac{\partial E}{\partial t} = -\nabla \cdot (C_g E)$$

A modified Lax-Wendroff integration scheme due to Gadd (1978) is used.

$$E_{j+1/2}^{n+1/2} = \frac{1}{2}(E_j^n + E_{j+1}^n) - \Delta t\,(C_{j+1} E_{j+1}^n - C_j E_j^n)/2\Delta x \qquad (11a)$$

$$E_j^{n+1} = E_j^n - \Delta t\,\Big\{(1+a)\,(C_{j+1/2} E_{j+1/2}^{n+1/2} - C_{j-1/2} E_{j-1/2}^{n+1/2}) - \frac{1}{3} a\,(C_{j+3/2} E_{j+3/2}^{n+1/2} - C_{j-3/2} E_{j-3/2}^{n+1/2}\Big\}/\Delta x \qquad (11b)$$

In our model we have assumed that the bottom slope of the ocean varies very slowly so that no appreciable change will occur in phase velocity of the propagating wave, *i.e.*, refraction will be absent. The group velocity of the above has been considered following Gadd (1978) and has been found to be,

$$C_g = \frac{1}{2}\,[g \tanh (KH)/K]^{0.5}\;\{1 + 2KH/\sinh (2KH)\} \qquad (12)$$

where, K = wave number, H = depth of water

$$(2\pi f)^2 = gK \tanh KH \qquad (13)$$

The energy input Sin is represented by linear and exponential terms following Miles (1957, 1960) and Phillips (1957) and has been written as;

$$S_{in} = A + BE \qquad (14)$$

This term represents a direct forcing of the water surface due to turbulent fluctuation in the surface wind. According to Snyder *et al.* (1981). This linear term has become unimportant, so the following forms of A and B are chosen :

$$A = \frac{6.10^{-8}}{2\pi f_m} U^2 \cos^2 (\theta - \phi) \qquad (15)$$

for $f = fm$ and $|\theta - \phi| < 90$

$= 0$ otherwise

where, U = wind speed, ϕ = wind direction

Now

$$B = 6 \times 10^{-2} \left(2\pi f \frac{\rho_a}{\rho_w}\right) \frac{U \cos (\theta - \phi)}{C} > 1 \qquad (16)$$

for $\dfrac{U \cos (\theta - \phi)}{C} \leq 1$

$= \theta$ otherwise

The dissipation term is prescribed by Hasselmann (1971) considering whitecapping as the principal process for this, as

$$S_{ds}(f, \theta) = 4.10^{-4} f^2 E(f, \theta) \left\{ \iint E(f, \theta)\, d\theta df \right\}^{0.25} \qquad (17)$$

Using the above algorithm and using numerical integration scheme we have calculated the energy of the propagating swell. After knowing the energy of the swell, we have taken the following well known equations of hydrodynamics following Jelesniansky (1965) and Murthy (1984).

$$\frac{\partial U}{\partial t} = -gD(x, y)\frac{\partial h}{\partial x} + fV + \tau^{(x)}(x, y, t) \qquad (18)$$

$$\frac{\partial V}{\partial t} = -gD(x, y)\frac{\partial h}{\partial y} - fU + \tau^{(y)}(x, y, t) \qquad (19)$$

$$\frac{\partial h}{\partial t} = -\frac{\partial U}{\partial x} - \frac{\partial V}{\partial y} + \alpha \int_D^h \frac{\partial}{\partial x}(u')^2 \qquad (20)$$

where, D = Depth of the basin from the equilibrium surface

f = Coriolis parameter

$\tau^{(x)}, \tau^{(y)}$ = Wind stresses per unit mass

h = Surface elevation from equilibrium

U = Tidal component of velocity along zonal direction

V = Tidal component of velocity along meridional direction

The numerical scheme for the above equations is given below :

$$U_{i,j}^{m+1} = U_{i,j}^{m-1} - gD_{i,j}\frac{\Delta t}{4\,\Delta S}\begin{vmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{vmatrix} h_{i,j}^m + \frac{f\Delta t}{8}\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{vmatrix} V_{i,j}^m + 2\Delta t^{(x)}\,\tau_{i,j}^m \qquad (18a)$$

$$V_{i,j}^{m+1} = V_{i,j}^{m-1} - gD_{i,j}\frac{\Delta t}{4\,\Delta S}\begin{vmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{vmatrix} h_{i,j}^{m}$$

$$-\frac{f\Delta t}{8}\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{vmatrix} U_{i,j}^{m} + 2\Delta t^{(y)}\,\tau_{i,j}^{m} \qquad (19a)$$

$$h_{i,j}^{m+1} = h_{i,j}^{m-1} - \frac{\Delta t}{4\,\Delta S}\left\{\begin{vmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{vmatrix} U_{i,j}^{m}\right.$$

$$\left. + \begin{vmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{vmatrix} V_{i,j}^{m}\right\} + 2\Delta t F_{i,j}^{m} \qquad (20a)$$

where, forcing term $F = \int_{D}^{h}\left[\frac{\partial}{\partial x}(u')^2\right] \mathrm{d}Z$

The boundary conditions are :

(*i*) The transport U on left closed boundary is zero

(*ii*) U and V are non-zero on right open boundary

(*iii*) $\frac{\partial V}{\partial y} = 0$ at the top and bottom boundary

(*iv*) A static height boundary condition was applied to the right boundary in deep water

The above equations are solved numerically using the well known Shuman's algorithm in a nine point grid system.

4. Results and discussion

For calculating the surge height for a model storm as prescribed by Das *et al.* (1974), heating Bangladesh coast, the pressure profile is considered as per Das *et al.* (1974).

$$P_a = 1010 - \Delta P/[1 + (r/r_0)^2] \qquad (21)$$

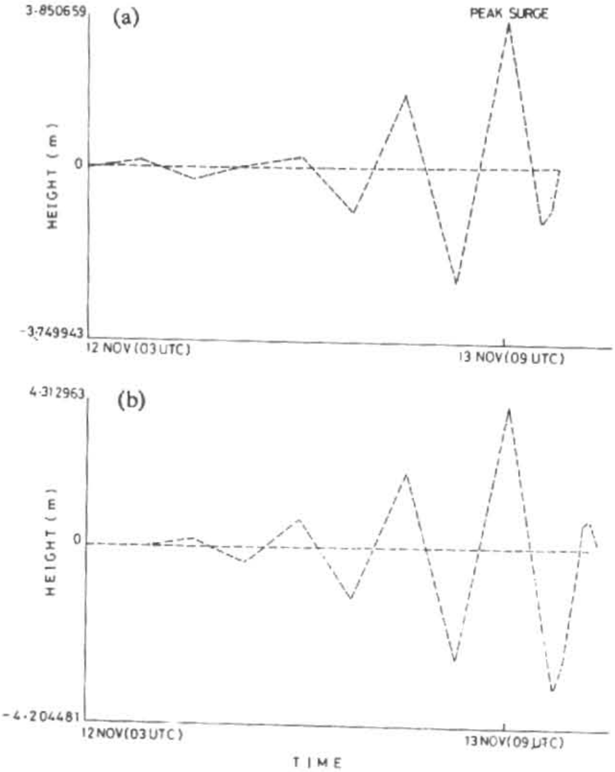
where, r = radial distance from storm centre

r_0 = radial distance at which wind attained its maximum velocity

Δp= pressure difference between centre of the storm and its outer periphery.

The cyclostrophic wind is prescribed as follows :

$$V^2 = 4\,V_m^2\,\mu^2/(1+\mu^2)^2 \qquad (22)$$



Figs.1(a&b). Graph of (a) surge height and time without wave and (b) surge height and time with wave contribution

where, $\mu = r/r_0$

V_m = maximum wind at r_0

By using Eqns. (21) and (22), the maximum wind speed (V_m) is related to the pressure deficit (Δp) by the approximate relation,

$$V_m^2 = (13)^2\ \Delta P \qquad (23)$$

V_m is in knot and ΔP in hPa. For a central pressure of 960 hPa, *i.e.*, ΔP = 50 hPa, Eqn. (23) fields a maximum wind of 100 knots at r_0 (30 km) from the storm centre.

The wind stresses $\tau^{(x)} = k\rho_a \mid V_a \mid V_a$ and $\tau^{(y)} = k\rho_a \mid V_a \mid V_a$ are calculated using Eqn. (22).

In Fig. 1(a) the graph of surge height and time without wave and Fig. 1(b) graph of surge height and time with wave contribution is plotted.

The storm track as per Das *et al.* (1974) is considered and shown in Fig. 2. A fixed speed of 20 km hr^{-1} is chosen for the storm. A twenty four hour time run

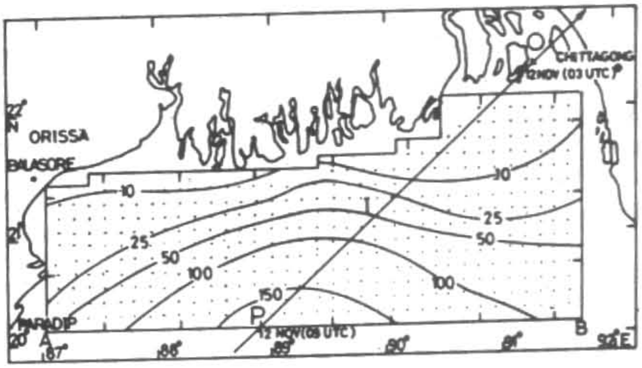


Fig. 2 Storm track (After Das *et al.* 1974)

assuming the storm to be at 20° N latitude and 88.8°E longitude shows a peak surge of 3.8 m when the contribution of wave is not considered. With the wave contribution the figure is improved to 4.3 m, but still it is lower than the observed 6 to 9 or 5 m as calculated by Das *et al.* (1974).

The result is tabulated below :

Date	Speed (km hr^{-1})	V_m (kt)	Observed (m)	Das *et al.* (m)	Model without wave (m)	Model with wave (m)
13 Nov 1970	20	87	5.6 to 9.0	5.0	3.8	4.3

In the present study we tried to concentrate on wave-surge interaction. Although the approach is idealistic in nature, the results are showing that the wave contribution in final sea level elevation is significant. The value of the surge height has been found to be below the actual. One main reason may be that we for simplicity did not consider the advective term in the depth-averaged hydrodynamic equation. Besides, the astronomical tide also has been kept aside to give emphasis in surge-wave interaction. Considera- tion of Tide-Surge-Wave coupled model will surely give better result.

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