525·6(547·1)

Some computations on Seiches in Lake Fife at Khadakvasla, Poona

S. JANARDHAN

Meteorological Office, Poona

(*Received 4 September 1958*)

ABSTRACT. Chrystal has worked out a number of formulæ for seiche-periods under various assumptions regarding the shape of the bed of the lakes where these seiches are excited. Based upon his method, periods of seiches that could be excited in Lake Fife at Khadakvasla have been computed. The periods of the uninodal, binodal, trinodal and quadrinodal seiches are obtained approximately as 1¼ hours, 2/3 of an hour, 5/12 of an hour and 1/3 of an hour respectively.

Hidaka's method of solving Chrystal's differential equation by the use of the theorem of Ritz in the calculus of variation, has been used for the case, where the lake's bottom is assumed parabolic. The uninodal period thereby worked out for Lake Fife comes out to be 1¼ hours.

For a test of these computed values, experimental observations are required and it is suggested that a suitable limnograph be set up for the purpose of recording these oscillations when natural phenomena excite them.

1. Introduction

The term 'seiches' was originally applied to rhythmic variations of the surface level, observed at the Geneva end of Lake Leman in 1730. Subsequently they have been observed at a great many places and it is now known that they are excited either by seismic activity or by certain meteorological factors such as passage of a storm, barometric oscillations, rapid changes in wind direction and magnitude etc, over bodies of water not sufficiently big by themselves to be directly influenced by astronomical tide-generating forces. In a seiche-oscillation, the entire water mass heaves up and down as one mass, the period of oscillation depending only on the dimensions of the water-body excited, except in those cases where the body of water opens into a sea.

The theoretical determination of ordinary longitudinal seiches in lakes, the shape and size of whose transverse sections vary only slowly, has been reduced by Chrystal (1905 a) to the solution of a linear differential equation of second order, in the normal form, with certain boundary conditions. The solution was then applied by Chrystal (1905 b), to Loch Earn and Loch Treig, with encouraging results. In a subsequent paper Chrystal (1908) has investigated the effect of a number of types of pressure disturbances on a special lake.

Wedderburn (1910, 1912) who discovered the temperature seiche (where the water-body has stratification due to temperature differences) has reduced the problem of such seiches to the solution of a second order differential equation, very similar to that of Chrystal for ordinary seiches.

The problem of seiches in its most general aspects, involving any kind of configuration for the bottom of a lake has been tackled by Proudman (1914) and an application of Proudman's method to Lake Geneva by Doodson, Carey and Baldwin (1920) has yielded highly satisfactory results.

Chrystal's seiche-equation has also been solved in various other ways by Defant (1918), Ertel (1933) and Hidaka (1936). The concept of the equivalent electric and acoustical circuit for a lake has been used by Darbyshire and Darbyshire (1957) in determining the seiches of Lough Neagh.

Seiches generated by earthquakes are called 'seismic seiches'. An account of such seiches generated by the Lisbon earthquake of 1 November 1755 is given by Davison (1936) and of those by the Assam earthquake of 15 August 1950 by Kvale (1955) and by Mukherjee (1955).

The object of the present paper is to apply the results obtained by Chrystal for determining the seiches that could be excited in Lake Fife at Khadakvasla, under simplifying assumptions. The seiche periods are also determined by Hidaka's method for one of the cases and the results compared.

2. Chrystal's method

This method is applicable when the length of the lake is considerably greater than the breadth and the depth.

Let v denote the surface area of the lake from one end up to any transverse section. Let this transverse section be at distance x measured from any arbitrary fixed transverse section along the lake's length. The range for v is 0 to a, where a denotes the total surface area of the lake.

Let $A(v)$ and $b(v)$ represent the area and surface breadth respectively, in the undisturbed state, of the transverse section corresponding to v. Further let ξ and ζ denote the forward displacement of a particle and the elevation of the free surface at the same section so that ξ and ζ are functions of v and time t only. If now u denotes the product $A(v)\ \xi$, *i.e.*, the total volume of water which has passed the section $A(v)$ upto a time t, we have the following equations —

$$\frac{dv}{dx} = b(v) \tag{1}$$

$$\xi = \frac{u}{A(v)} \tag{2}$$

$$\zeta = -\frac{\partial u}{\partial v} \tag{3}$$

Equation (3) represents the equation of continuity. Assuming the atmospheric pressure over the entire lake to be uniform and neglecting the squares and products of displacements, as is the case with small displacements, one obtains the equation of free motion as

$$\frac{\partial^2 \xi}{\partial t^2} = -g\frac{\partial \zeta}{\partial x} \tag{4}$$

Introducing now another variable $\sigma = A(v)\ b(v)$, we obtain from equations (3) and (4), a single differential equation —

$$\frac{\partial^2 u}{\partial v^2} - \frac{1}{g\sigma(v)}\frac{\partial^2 u}{\partial t^2} = 0 \tag{5}$$

$\sigma(v)$ is always positive and we will consider only those types of lakes for which $\sigma(v)$ is continuous, and $d\sigma/dv$ does not vanish at an end point, where v vanishes.

The curve which has σ as the 'vertical' and v as the 'horizontal' co-ordinates, is called the 'normal' curve of the lake.

Let the motion be periodic with angular velocity n, so that

$$u = \sum_n \phi_n \sin n(t-\tau)$$

where ϕ_n is a function of v only. Equation (5) then reduces to —

$$\frac{d^2\phi_n}{dv^2} + \frac{n^2\phi_n}{g\sigma(v)} = 0 \tag{6}$$

In solving (6), Chrystal assumes the three simple forms for the normal curve—linear, parabolic and quartic. He also assumes a uniform surface breadth for the lake, and rectangular transverse sections. These assumptions further simplify (6) to —

$$\frac{d^2\phi_n}{dx^2} + \frac{n^2\phi_n}{g\sigma(x)} = 0 \tag{7}$$

where x can be regarded as measured from the origin of the (σ, v) curve. Further $\sigma(x)$ is now equal to $h(x)\ b(x)$, where $h(x)$ is the uniform depth of the transverse section at x. The results obtained for various forms of the $[h(x), x]$ curve are summarised below.

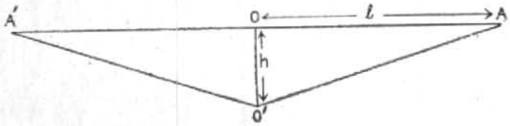

Fig. 1

(*i*) *Rectilinear symmetric lake, with two slopes shelving at both ends* (Fig. 1)— Let OA=OA'=l, be the semi-length of the lake. Let depth OO' at the origin be h. If x is measured along OA and x' along OA', the 2 parts OA and OA' of the normal curve AOA' will have the equations —

$$h(x) = h(1-x/l) \text{ and } h(x') = h(1-x'/l)$$

$$\text{Let } W = \frac{2nl(1-x/l)^{\frac{1}{2}}}{(gh)^{\frac{1}{2}}} \text{ and } W' = \frac{2nl(1-x'/l)^{\frac{1}{2}}}{(gh)^{\frac{1}{2}}}$$

then the period of seiche-oscillation of the lake is given by $T_\nu = \dfrac{4\pi l}{j_\nu\sqrt{gh}}$ where ν is the number of nodes ($\nu = 1, 2, 3, \ldots$). j_ν is a root of $J_0(z) = 0$ for $\nu = 1, 3, 5\ldots$ and of $J_1(z) = 0$ for $\nu = 2, 4, 6\ldots$ $J_n(z)$ being a Bessel function of the n^{th} order.

The nodal points are given by —

$$J_0(W) = 0 \text{ for OA}$$

$$\text{and } J_0(W') = 0 \text{ for OA}'$$

(*ii*) *Rectilinear lake, with one slope, shelving at one end* (Fig. 2)—The seiches for such a lake are the same as the seiches of even nodality for the symmetric lake shelving at both ends. The period is, therefore, given by

$$T_\nu = \frac{4\pi l}{j_{2\nu}\sqrt{gh}}$$

where $\nu = 0, 1, 2, \ldots$, the number of nodes. The nodal equation remains the same as for the first case.

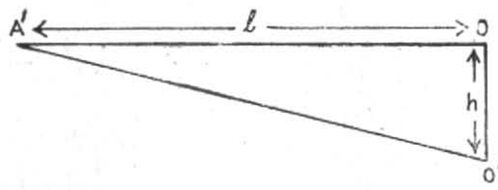

Fig. 2

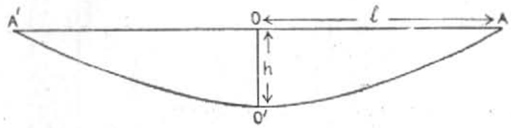

Fig. 3

(*iii*) *Concave symmetric complete parabolic lake* (Fig. 3)—As before semi-length = l and central depth = h. The equation to the normal curve AO'A' is $h(x) = h\,(1-x^2/l^2)$ and if $W = x/l$, the period of the ν-nodal seiche is —

$$T_\nu = \frac{2\pi l}{\sqrt{\nu(\nu+1)gh}}$$

If $c_\nu = n^2_\nu\, l^2/gh$, where n_ν is the angular velocity corresponding to the ν-nodal oscillation, the nodal points are given by $C'(c_\nu, W) = 0$, for seiches of odd nodality, *i.e.*, ν—odd and by $S'(c_\nu, W) = 0$ for seiches of even nodality, *i.e.*, ν—even. $C'(c_\nu, W)$ and $S'(c_\nu, W)$ are the derivatives with respect to W of the seiche-cosine function $C(c_\nu, W)$ and the seiche-sine function $S(c_\nu, W)$, defined by:

$$C(c_\nu, W) = 1 - \frac{c_\nu}{1.2}W^2 + \frac{c_\nu(c_\nu - 1.2)}{1.2.3.4}W^4 - \ldots$$

$$S(c_\nu, W) = W - \frac{c_\nu}{2.3}W^3 + \frac{c_\nu(c_\nu - 2.3)}{2.3.4.5}W^5 - \ldots$$

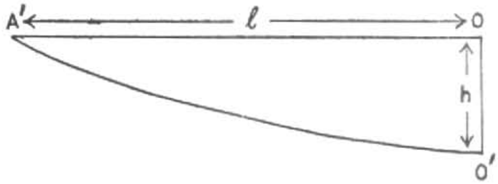

Fig. 4

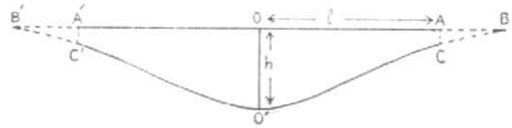

Fig. 5

(*iv*) *Concave semi-parabolic lake* (Fig. 4)— The seiches for such a lake are again the seiches of even nodality for the symmetric complete parabolic case. The period of the ν—nodal seiche is therefore—

$$T_\nu = \frac{2\pi l}{\sqrt{2\nu(2\nu+1)gh}}$$

The nodes are given by—

$S'(\sigma_\nu, W) = 0$ when ν is even;

where $\sigma_\nu = n^2_\nu l^2/gh$ and $n_\nu = 2\pi/T_\nu$.

(*v*) *Concave symmetric quartic lake truncated at the two ends at equal distances from the section of maximum depth* (Fig. 5)—Unlike the above cases, we consider the lake truncated at both ends since the ends of the untruncated lake are essential singularities for the corresponding solution of the seiche differential equation. As before, let l denote the semi-length OA=OA′ and h the central depth OO′. Let BOB′ be the untruncated lake. Let OB=OB′=l'. The equation of the normal curve BO′B′ is $h(x)=h\,(1-x^2/l'^2)^2$. Let the depth at A and A′ be h'.

The period of the ν-nodal seiche is

$$T_\nu = \frac{4\pi l}{\lambda\left\{g h\left(\frac{4\pi^2\nu^2}{K^2}+1\right)\right\}^{\frac{1}{2}}}$$

where $\lambda = 2\sqrt{1-(h'/h)^{\frac{1}{2}}}$,

$$K = 2\log\frac{2+\lambda}{2-\lambda}$$

h' is obtained from the equation of the normal curve as $h(1-l^2/l'^2)^2$. The equation determining the nodes of the lake AO′A′ is $\zeta=-\partial u/\partial x = 0$. Here $u = h(x)\xi$. It can be shown that u is of the form :

$$u = [A\,(l'+x)^e\,(l'-x)^f + B(l'+x)^f\,(l'-x)^e\,]\sin nt$$

where A, B are constants and e, f are the roots of $\rho^2-\rho+c/4l'^2 = 0$; $c = n^2/gh = 4\pi^2/ghT^2$.

If $\delta = +\left(\frac{c-l'^2}{4l'^2}\right)^{\frac{1}{2}}$,

then $e = \frac{1}{2}+i\delta$, $f = \frac{1}{2}-i\delta$,

where $i = \sqrt{-1}$

We now have—

$$u = (l^2-x^2)^{\frac{1}{2}}\left\{A\left(\frac{l'+x}{l'-x}\right)^{\delta i} + B\left(\frac{l'+x}{l'-x}\right)^{-\delta i}\right\}\sin nt$$

A and B are determined from the conditions $u=0$ at $x=\pm l$. We thus obtain—

$$A = -\tfrac{1}{2}\,D\,(\tan\delta\alpha + i)$$

$$B = -\tfrac{1}{2}\,D\,(\tan\delta\alpha - i)$$

where D is an arbitrary constant and α is defined by

$$\alpha = \log\frac{l'+l}{l'-l}$$

Nodal equation $\partial u/\partial x = 0$ now becomes—

$$(l' + x)^{e-1} \ (l' - x)^{f-1} \ (\tan \delta\alpha + i) \times (x - 2\, l'\delta i) + (l' + x)^{f-1} (l' - x)^{e-1} \times (\tan \delta\alpha - i) \ (x + 2\, l' \, \delta \, i) = 0$$

The period of the half-lake OO′C′A′ is for the ν-nodal seiche,

$$T_\nu = \frac{4\pi\, l}{\lambda \sqrt{gh \left(\frac{16\, \pi^2\, \nu^2}{K^2} + 1 \right)}}$$

3. Application of Chrystal's method to Lake Fife

The following details of the lake are extracted from *Data on High Dams in India*, (Central Board of Irrigation 1950)—

Length of the lake	=11 miles
Surface area	=6 sq. miles
Breadth at dam site	=0·75 mile
Storage capacity	$=4000 \times 10^6$ cu. ft

Observed max. depth at dam =93·5 ft

(*i*) *Lake assumed to have a rectilinear bottom with one slope, shelving at one end; surface breadth is uniform and transverse sections are rectangular*—Fig. 2 is a longitudinal section of the lake, in which OO′ represents the dam.

The volume of the whole lake is obviously 1/2 lbh. Taking the surface area of the lake as 6 sq. miles and the volume as 4000 million cu. ft, the height h at the dam end works out to be 48 ft. This represents, under our assumptions, the average depth of the lake at the dam.

The period of the ν-nodal seiche is given by

$$T_\nu = \frac{4\pi\, l}{j_{2\nu} \sqrt{gh}} \qquad \nu = 0, 1, 2\ldots.$$

Taking $j_2 = 3.8317$, $j_4 = 7{\cdot}0156$, $j_6 = 10{\cdot}1735$, $j_8 = 13{\cdot}3237$,

we get after computation

T_1=1 hour 21 min, T_2 = 44 min,

T_3=31 min, and T_4 = 23 min.

(*ii*) *Lake assumed to have a concave semi-parabolic bottom; surface breadth is uniform and transverse sections rectangular*—Fig. 4 is a longitudinal section of the lake with OO′ representing the dam.

The area of this longitudinal section can be shown by quadrature to be 2/3 (lh). The volume of the lake is, therefore, 2/3 (lbh). Taking the volume of water impounded on the Lake Fife as 4000 million cu. ft and its surface area as 6 sq. miles, we obtain h =36 ft.

The period of the ν-nodal seiche is given by—

$$T_\nu = \frac{2\,\pi\, l}{\sqrt{2\nu\,(2\nu + 1)\,gh}} \qquad \nu = 0, 1, 2\ldots.$$

We thus get

T_1 = 1 hour 13 min, T_2 = 40 min,
T_3 = 28 min, and T_4 = 21 min.

(*iii*) *Lake assumed to have a concave semi-quartic bottom; surface breadth is uniform and transverse sections rectangular*—For reasons already stated, the case of the complete quartic lake cannot be considered.

In Fig. 5, OO′C′A′ represents the longitudinal section of the lake under consideration, OO′ denoting the dam. It can be shown by quadrature, that if OA′=l, OB′ =l', OO′=h, the area of the section OO′C′A′ is —

$$\frac{hl}{15}\left[\, 15 - 10\, \frac{l^2}{l'^2} + 3\, \frac{l^4}{l'^4} \right]$$

The volume of the lake is therefore

$$\frac{hbl}{15}\left[\, 15 - 10p^2 + 3\,p^4 \right]$$

where $p = l/l'$ is the ratio of the lengths of the truncated and complete lake. Since the surface area and the volume impounded are known for Lake Fife, we can work out h, if p is known.

(*a*) Let $p = \frac{3}{4}$. Then h works out to be 36 ft. This gives depth $h' = A'C'$ at the shallow end $= h\,(1 - p^2)^2 = 7$ ft. The seiche-oscillation with ν-nodes has a period

$$T_\nu = \frac{4\pi l}{\lambda \sqrt{gh\left(\frac{4\pi^2 \nu^2}{K^2} + 1\right)}}$$

where, $\lambda = 2\sqrt{1 - (h'/h)^{\frac{1}{2}}}$ and

$$K = 2 \log \frac{2 + \lambda}{2 - \lambda}$$

For $h' = 7$ ft, $h = 36$ ft, we have

$\lambda = 1{\cdot}5$ and $K = 1{\cdot}69$.

These values yield—

$T_1 = 1$ hour 2 min, $T_2 = 32$ min,

$T_3 = 21$ min and $T_4 = 16$ min.

(*b*) Let $p = 0{\cdot}95$. We then obtain

$h = 42{\cdot}6$ ft, $h' = 4{\cdot}15$ ft,

$\lambda = 1{\cdot}67$, $K = 2{\cdot}08$

and these values yield —

$T_1 = 1$ hour 2 min, $T_2 = 33$ min,

$T_3 = 22$ min and $T_4 = 16$ min.

4. Computation of seiche-oscillation periods by Hidaka's method

In Hidaka's method, the solution of Chrystal's differential equation (5), namely,

$$\frac{\partial^2 u}{\partial v^2} - \frac{1}{g\sigma(v)} \cdot \frac{\partial^2 u}{\partial t^2} = 0$$

is subject to

$$u(0) = u(a) = 0 \qquad (8)$$

and $$\sigma(0) = \sigma(a) = 0 \qquad (9)$$

The substitution $z = v/a$, where a is the total surface area of the lake, then reduces the above differential equation to

$$\frac{d^2u}{dz^2} + \frac{\lambda u}{\gamma(z)} = 0 \qquad (10)$$

where u is assumed periodic, $\lambda = 4\pi^2 a^2/T^2 gh$ and $\gamma(z)$, a dimensionless function of z, is given by

$$\sigma(z) = h\gamma(z) \qquad (11)$$

*The solution of (10), subject to (8), is equivalent to finding the stationary value of the integral

$$I(u) = \int_0^1 \left\{ \left(\frac{du}{dz} \right)^2 - \frac{\lambda}{\gamma(z)} \right\} dz \qquad (12)$$

Hidaka assumes that

$$u = \sum_{i=0}^{m} z(1 - z)\, z^i A_i$$

where the A_i's are constants. The $(m+1)$ constants $A_0, A_1, \ldots\ldots A_m$, must be so chosen as to make the integral (12) a minimum. This necessitates that

$$\frac{\partial I}{\partial A_0} = \frac{\partial I}{\partial A_1} = \ldots\ldots = \frac{\partial I}{\partial A_m} = 0 \qquad (13)$$

$\frac{\partial I}{\partial A_j} = 0$ leads to the condition,

---

*For an application of the calculus of variations to eigen-value problems, a general treatment is found in *Methods of Mathematical Physics* by R. Courant and D. Hilbert, 1, Chap. VI, 1953

$$\sum_{i=0}^{m} \left\{ \frac{(i+1)\ (j+1)}{(i+j+1)} - \frac{(i+1)\,(j+2) + (i+2)\,(j+1)}{(i+j+2)} + \right.$$

$$\left. \frac{(i+2)\ (j+2)}{(i+j+3)} - \lambda I_{i+j} \right\} A_i = 0 \tag{14}$$

$$\text{Here } I_{i+j} = \int_0^1 \frac{z^2 (1-z)^2 \times z^{i+j}}{\gamma(z)} \, dz \tag{15}$$

Equation (14) gives rise to the following $(m+1)$ simultaneous equations—

$$\left.\begin{array}{l}
\left(\frac{1}{3} - I_0 \lambda\right) A_0 + \left(\frac{1}{6} - I_1 \lambda\right) A_1 + \left(\frac{1}{10} - I_2 \lambda\right) A_2 + \ldots \\
\quad \ldots + \left\{ \frac{2\,(m+1)}{(m+1)\,(m+2)\,(m+3)} - I_m \lambda \right\} A_m = 0 \\
\left(\frac{1}{6} - I_1 \lambda\right) A_0 + \left(\frac{2}{15} - I_2 \lambda\right) A_1 + \left(\frac{1}{10} - I_3 \lambda\right) A_2 + \ldots \\
\quad \ldots + \left\{ \frac{4\,(m+1)}{(m+2)\,(m+3)\,(m+4)} - I_{m+1} \lambda \right\} A_m = 0 \\
\left(\frac{1}{10} - I_2 \lambda\right) A_0 + \left(\frac{1}{10} - I_3 \lambda\right) A_1 + \left(\frac{3}{35} - I_4 \lambda\right) A_2 + \ldots \\
\quad \ldots + \left\{ \frac{6\,(m+1)}{(m+3)\,(m+4)\,(m+5)} - I_{m+2} \lambda \right\} A_m = 0 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\left\{ \frac{2\,(m+1)}{(m+1)\,(m+2)\,(m+3)} - I_m \lambda \right\} A_0 + \left\{ \frac{4\,(m+1)}{(m+2)\,(m+3)\,(m+4)} - \right. \\
\quad \left. I_{m+1} \lambda \right\} A_1 + \left\{ \frac{6\,(m+1)}{(m+3)\,(m+4)\,(m+5)} - I_{m+2} \lambda \right\} A_2 + \ldots \\
\quad \ldots + \left\{ \frac{(2\,m+2)\,(m+1)}{(2\,m+1)\,(2\,m+2)\,(2\,m+3)} - I_{2m} \lambda \right\} A_m = 0
\end{array}\right| \tag{16}$$

Eliminating $A_0, A_1 \ldots\ldots\ldots A_m$ between these $(m+1)$ equations, we obtain an equation in λ of the $(m+1)$ th order and this is the period equation. This equation is—

$$\begin{vmatrix} \frac{1}{3} - I_0\lambda & \frac{1}{6} - I_1\lambda & \frac{1}{10} - I_2\lambda & \ldots\left\{\frac{2(m+1)}{(m+1)(m+2)(m+3)} - I_m\lambda\right\} \\ \frac{1}{6} - I_1\lambda & \frac{2}{15} - I_2\lambda & \frac{1}{10} - I_3\lambda & \ldots\left\{\frac{4(m+1)}{(m+2)(m+3)(m+4)} - I_{m+1}\lambda\right\} \\ \frac{1}{10} - I_2\lambda & \frac{1}{10} - I_3\lambda & \frac{3}{35} - I_4\lambda & \ldots\left\{\frac{6(m+1)}{(m+3)(m+4)(m+5)} - I_{m+2}\lambda\right\} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ \ldots\ldots\ldots \\ \left\{\frac{2(m+1)}{(m+1)(m+2)(m+3)} - I_m\lambda\right\} & \left\{\frac{4(m+1)}{(m+2)(m+3)(m+4)} - I_{m+1}\lambda\right\} & \left\{\frac{6(m+1)}{(m+3)(m+4)(m+5)} - I_{m+2}\lambda\right\} & \ldots\left\{\frac{(2m+2)(m+1)}{(2m+1)(2m+2)(2m+3)} - I_{2m}\lambda\right\} \end{vmatrix} = 0 \quad (17)$$

In practice, accurate results even for complicated forms of the normal curve, are obtained by taking $m = 2$, which then reduces (17) to

$$\begin{vmatrix} \frac{1}{3} - I_0\lambda & \frac{1}{6} - I_1\lambda & \frac{1}{10} - I_2\lambda \\ \frac{1}{6} - I_1\lambda & \frac{2}{15} - I_2\lambda & \frac{1}{10} - I_3\lambda \\ \frac{1}{10} - I_2\lambda & \frac{1}{10} - I_3\lambda & \frac{3}{35} - I_4\lambda \end{vmatrix} = 0 \quad (18)$$

On development, equation (18) gives the cubic in λ

$$A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0 \quad (19)$$

where $A_1 = \begin{vmatrix} I_0 & I_1 & I_2 \\ I_1 & I_2 & I_3 \\ I_2 & I_3 & I_4 \end{vmatrix}$

$$A_2 = -\left[\frac{3}{35}(I_0 I_2 - I_1^2) + \frac{1}{5}(-I_0 I_3 + I_1 I_2 + I_1 I_3) + \frac{2}{15} I_0 I_4 + \frac{1}{3}(-I_1 I_4 - I_2^2 + I_2 I_3 + I_2 I_4 - I_3^2)\right]$$

$$A_3 = \frac{1}{700} I_0 - \frac{3}{350} I_1 + \frac{53}{2100} I_2 - \frac{1}{30} I_3 + \frac{1}{60} I_4$$

$$A_4 = -\frac{1}{10500}$$

The three values of λ given by (19) when substituted in $T = 2\pi a/\sqrt{\lambda g h}$ determine the periods of the uninodal, binodal and trinodal seiches.

5. Application of Hidaka's method

We consider a lake whose normal curve is as shown in Fig. 3. If the portion A'OO' of this lake corresponds to Lake Fife, then

$l=11$ miles, $h = 36$ ft and surface breadth is uniform.

Chrystal's formula $T_\nu = 2\pi l/\sqrt{\nu(\nu+1)gh}$ yields for the periods of the uninodal, binodal and trinodal seiches of the complete lake A'O'A.

$$\left.\begin{aligned} T_1 &= 2 \text{ hours } 7 \text{ min.} \\ T_2 &= 1 \text{ hour } 13 \text{ min.} \\ T_3 &= 52 \text{ min.} \end{aligned}\right\} \quad (20)$$

The equation of the normal curve in Fig. 3 is $h(x) = h(1-x^2/l^2)$. Since breadth is assumed uniform, the variable z of equation (10) may be regarded as given by $z = x/2l$, so that the normal curve will be given by

$$h(x) = h\,(1-4z^2)$$

Transferring now, the origin in Fig. 3 to the end $A'(x=-l)$, the equation of the normal curve becomes

$$h(x) = h\{1-4(z-\tfrac{1}{2})^2\}$$

The differential equation (10) now becomes

$$\frac{d^2u}{dz^2} + \frac{\lambda u}{\{1-4(z-\frac{1}{2})^2\}} = 0$$

From (15), we then have

$$I_n = \int_0^1 \frac{z^2(1-z)^2 z^n}{\{1-4(z-\frac{1}{2})^2\}}\, dz$$

$$= \frac{1}{4}\left[\frac{1}{n+2} - \frac{1}{n+3}\right]$$

Thus $I_0 = \frac{1}{24}$, $I_1 = \frac{1}{48}$, $I_2 = \frac{1}{80}$

$$I_3 = \frac{1}{120} \text{ and } I_4 = \frac{1}{168}$$

With these values the equation (19) becomes

$$\lambda^3 - 80\lambda^2 + 1728\,\lambda - 9216 = 0$$

The substitution $\lambda = \beta + 80/3$ leads to a cubic equation not involving β^2, namely, $\beta^3 - 405\beta - 1062 = 0$ and the roots of this equation are $\beta_1 = -18{\cdot}65$, $\beta_2 = -2{\cdot}670$ and $\beta_3 = 21{\cdot}33$. The corresponding values of λ are $\lambda_1 = 8$, $\lambda_2 = 24$ and $\lambda_3 = 48$.

These values, when inserted in the formula, $T = 4\pi l/\sqrt{\lambda gh}$ determine T_1, T_2, T_3 for the uninodal, binodal and trinodal seiches, as

$$\left.\begin{aligned} T_1 &= 2 \text{ hours } 7 \text{ min.} \\ T_2 &= 1 \text{ hour } 13 \text{ min.} \\ T_3 &= 52 \text{ min.} \end{aligned}\right\} \quad (21)$$

These values are identical with those in (20). Out of the three values in (20) or (21) the second one, corresponding to the binodal oscillation of the complete lake would be the period of the uninodal seiche for Lake Fife.

The binodal and trinodal oscillations of Lake Fife, by Hidaka's method, would require consideration of higher order terms in the power series expression for u.

6. Conclusion

It is seen from the computed seiche periods for Lake Fife that, surface breadth remaining uniform, changes in the configuration of the bed of the lake cause only slight variations in the seiche periods, provided the volume impounded remains constant and there are no large irregularities in the bed. However it has to be pointed out, that as far as actual lakes like Lake Fife are concerned, the assumption of uniform breadth is rather much of an approximation. It is likely that seiche periods are predominantly determined by the first few miles upstream of the dam, where changes in surface breadth are likely to be marked. This difficulty can, however, be surmounted, if recourse is taken to the numerical methods, now available, in the solution of Chrystal's equation.

The numerical computation of seiche periods and nodal positions requires detailed data of the three-dimensional configuration of the lake such as the values for a number of transverse sections of (*i*) the surface breadth, (*ii*) the surface area from an end and (*iii*) the dimensions of the transverse section.

Actual experimental observations of seiche oscillations are not available for Lake Fife for a comparison with the above computed values. The author would plead for the setting up of limnographs, for a continuous recording of water levels at Lake Fife, as well as at the numerous dam sites recently constructed.

7. Acknowledgements

The author gratefully acknowledges the encouragement and assistance of Dr. P. R. Pisharoty, who initiated him into this work and who contributed time and effort in helpful and critical discussions. He is also grateful to the Government of India, Ministry of Scientific Research and Cultural Affairs, for encouragement with a scholarship to carry out this investigation.

REFERENCES

Central Board of Irrigation, India	1950	*Data on High Dams in India*, **1**, C.B.I. Publ. No. 48.
Chrystal, G.	1905(a)	*Trans. roy. Soc., Edinb.*, **41**, p. 599.
	1905(b)	*Proc. roy. Soc., Edinb.*, **25**, p. 328, p. 637.
	1908	*Trans. roy. Soc., Edinb.*, **46**, p. 499.
Chrystal, G. and Wedderburn, E. M.	1905	*Ibid.*, **41**, p. 823.
Darbyshire, J. and Darbyshire, M.	1957	*Quart. J. R. met. Soc.*, **83**, p. 93.
Davison, C.	1936	*Great Earthquakes*, p. 14.
Defant, A.	1918	*Ann. hydrogr., Hamburg.*
Doodson, A. T., Carey, R. M. and Baldwin, R.	1920	*Trans. roy. Soc., Edinb.*, **52**.
Ertel, H.	1933	*S. B. preuss. Akad.*, Wiss.
Hidaka, K.	1936	*Mem. Imp. Mar. Obs. Kobe*, **6**.
Kvale, A.	1955	*Bull. seismol. Soc. Amer.*, **45**.
Mukherjee, S. M.	1955	*Indian J. Met. Geophys.*, **6**, 2, p. 149.
Proudman, J.	1914	*Proc. Lond. math. Soc.*, **14**.
Wedderburn, E. M.	1910	*Trans. roy. Soc., Edinb.*, **47**, p. 619.
	1912	*Ibid.*, **48**, p. 629.