

Drop size distribution in a complete 'warm' shower process

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ABSTRACT. A theoretical discussion of (i) overall raindrop size distribution and (ii) rate of decrease of maximum drop size towards end stages of a 'warm' shower process, based on collision-cum-coalescence theory of raindrop growth is represented. A comparison is made of the theoretical deductions with some of the observational findings.

1. Introduction

Much attention has been paid in recent years to systematic observation of drop size distribution in rain of various types and intensity, with a view to helping confirmation or clarification of some of our present ideas regarding physical processes governing rain formation in clouds. A number of analytical expressions have been suggested (Marshall and Palmer 1948, Spilhaus 1948 and Best 1950) to represent observed drop size distributions as a function of rainfall intensity at the time of sampling. While these are in broad agreement with observation, when averages over a number of samples taken during the middle or relatively steady stage of a rain-shower are considered, drop size distribution shown by individual samples or even groups of samples obtained during the initial or transient stages often reveal marked deviations from the relationships as suggested by these empirical formulae.

An examination of raindrop size distribution from a somewhat different angle has been made in this paper, by limiting consideration to overall distribution of drop sizes in course of a rain-shower and an attempt has been made to find a theoretical basis for the observed size distribution features in the light of our existing knowledge about raindrop growth in 'warm' clouds by the all-water process of collision-cum-coalescence. Consideration has been given in the paper to one commonly

occurring feature of drop sizes during the end stages of a rain-shower, namely, progressive decrease in diameter of the largest drop found on successive samples. Blanchard (1957), in his study of orographic rain in Hawaii, finds that during the last 20 minutes or so of rain the rate of decrease of drop diameter lies usually between 0·07 and 0·1 mm/min. An examination of the feature in relation to monsoon showers in certain parts of India has, however, shown the rate to be somewhat higher, being as much as 0·3 mm/min. in certain instances. A theoretical discussion of this aspect of drop size decay at the later stages of a rain-shower has been included in the paper.

2. Theory

A theoretical treatment of progressive development of drop size distribution in rain from a 'warm' convective cloud has been made in a recent paper by Roy and Srivastava (1958), starting from certain plausible assumptions regarding concentration and size distribution of giant cloud droplets near cloud base, a constant value for liquid water content in cloud, and updraft varying in a certain way with time. In deducing the features relating to overall distribution of drop sizes in course of a complete shower, as has been attempted in the present paper, it has not been necessary to make any detailed quantitative assumptions regarding the above three elements.

The basic equations governing coalescence growth of raindrops in a 'warm' cloud as enunciated by Langmuir (1948), Bowen (1950) and others are —

$$\frac{dr}{dt} = \frac{wvE}{4\rho} \quad (1)$$

$$\text{and} \quad \frac{dz}{dt} = U - v \quad (2)$$

where r is the radius of the growing cloud droplet, w the average liquid water content, ρ the density of water, E the effective collection efficiency for a drop of radius r colliding with smaller cloud droplets, v the terminal velocity of growing droplet, z the height above cloud base reached by the drop at time t and U the updraft rate. Assuming, for simplicity, that liquid water content w is constant and is independent of time and of height above cloud base, and that updraft rate U is same at all heights, although it may vary in any manner with time subject to a restriction mentioned later, we get by integrating equation (2) and substituting from equation (1) —

$$z = \int U dt - \int \frac{4\rho}{wE} dr \quad (3)$$

Taking as a first approximation E to be the same for all drop sizes and equal to unity, we have, after insertion of suitable limits and subscripts,

$$z_1 = \int_t^{t_1} U dt - \frac{4\rho}{w} (\Delta r)_1 \quad (4)$$

where z_1 is height reached by the growing droplet at time t_1 on having started its upward journey from cloud base at time t . $(\Delta r)_1$ in the above equation represents increase in radius of the drop during time interval $(t_1 - t)$ and is given by equation (1) which in its integrated form reads

$$t_1 - t = \frac{4\rho}{w} \int_{r_i}^{r_i + (\Delta r)_1} dr/vE$$

$$= \frac{4\rho}{w} \int_{r_i}^{r_i + (\Delta r)_1} dr/v \quad \text{putting } E = 1$$

r_i being the initial radius of the growing droplet.

On the right hand side of (4) the first term gives the height as would be reached by a drop if it floated in the cloud air and was simply carried up by the prevailing updraft for a period $t_1 - t$, while the second term gives the distance downward through which the drop would fall by virtue of its terminal fall speed during the same time interval, if there was no updraft. The difference between the two gives z_1 the height actually reached by the drop at a particular instant t_1 . This increases at first and, after attaining maximum value at the instant when the terminal velocity of the drop equals updraft, begins to decrease and becomes zero when the finally formed raindrop reaches cloud base, at time t' . At this stage,

$$\int_t^{t'} U dt = \frac{4\rho}{w} (\Delta r)_f = \frac{4\rho}{w} r_f \quad (5)$$

$$\text{and } t' - t = \frac{4\rho}{w} \int_{r_i}^{r_i + (\Delta r)_f} dr/v \quad (6)$$

In the above equations $(\Delta r)_f$ gives total radius increment of the 'giant' cloud droplet in course of its ascent and descent through cloud, and represents very nearly the radius, r_f of the finally formed raindrop, since $r_i \ll r_f$. Using equations (5) and (6) we can get the final radius, r_f and also t' , the time of emergence of the raindrop below cloud base. In general, r_f and t' will depend upon (a) radius r_i of the giant cloud droplet at the time when it starts its life history of growth from near cloud base, (b) the time t when this process starts, (c) amount of liquid water in cloud and (d) rate of updraft. If, however, we now confine our attention to only those raindrops which leave cloud base after a certain time T , and assume that at this instant, T ,

updraft in the cloud falls to zero and remains so during all subsequent period, equation (5) relating to such drops is simplified considerably, enabling us to calculate r_f independently of equation (6). In this case, as the time interval from T to t' during which $U=0$ makes no contribution to the integral, equation (5) reduces to

$$\begin{aligned} r_f &= \frac{w}{4\rho} \int_t^T U dt \\ &= \frac{w}{4\rho} z_t \end{aligned} \quad (7)$$

$$\text{where } z_t = \int_t^T U dt \quad (8)$$

Equation (7) shows that during this phase of rain shower, that is, from time T onwards, final sizes of raindrops formed on giant cloud droplets, starting their life history at the same instant will be the same, irrespective of the initial size of the growing droplets. The time of emergence of the raindrop formed on a particular giant cloud droplet will, however, depend upon its initial radius r_i , and can be obtained from equation (6).

The derivations as above may now be used to deduce the form of the overall raindrop size distribution in a complete shower process, which satisfies the condition implied above, namely, the condition that raindrops start emerging below the cloud base only after updraft has ceased. Although this may not represent the true position in all actual shower situations, we know that a good part of the rain in many of the shower processes in convective clouds falls usually after updraft in the cloud has ceased, or has decreased substantially. The analysis presented here may, therefore, be taken to represent a useful first approximation to the nature of the overall drop size distribution in a rain shower occurring from a 'warm' cloud.

Under the conditions stipulated above, diameter (D_t) of a raindrop starting its life

history of growth at time t as given by equation (7) is

$$D_t = \frac{w}{2\rho} z_t \quad (9)$$

Since z_t decreases as t increases (eq. 8), we see that a giant cloud droplet on which the process of growth commenced at an earlier instant will give rise to a correspondingly larger raindrop. This becomes apparent as we consider that a giant cloud droplet starting its process of growth at an earlier instant will be under the influence of the updraft for a longer time and, as such, have correspondingly greater chance of collisions and coalescence with cloud particles. It thus follows, that the number N_{D_t} of raindrops falling below the cloud base with diameter larger than D_t is the number of giant cloud droplets which enter cloud base before time t . Assuming that total concentration of giant cloud droplets near cloud base is constant and is n per unit volume, we may write

$$\begin{aligned} N_{D_t} &= \int_0^t n U dt \\ &= \int_0^T n U dt - \int_t^T n U dt \\ &= n (z_0 - z_t) \end{aligned} \quad (10)$$

Eliminating z_t between this and the preceding equation we get

$$N_D = n \left(z_0 - \frac{2\rho}{w} D \right) \quad (11)$$

The suffix t in D_t and N_{D_t} may now be dropped, being no longer necessary. Expressing the number of rain drops with diameter greater than D as a fraction, F_D , of the total number of drops, we have

$$F_D = 1 - \frac{2\rho}{w z_0} D \quad (12)$$

3. Comparison with observation

In Figs. 1, 2 and 3 the fraction, F_D , of total number of drops above a given size has

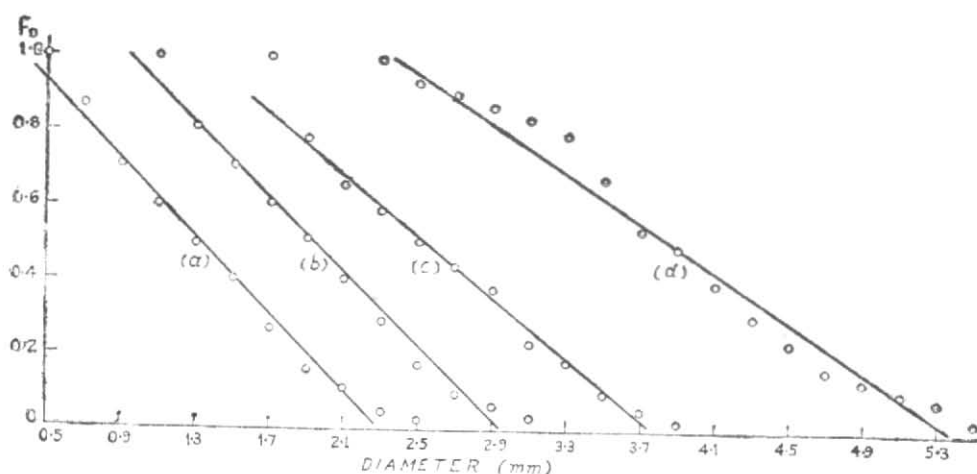


Fig. 1. F_D - D plot for four showers

Mean intensity—Curves (a) to (d) 14.2, 16.9, 17.2 and 14.1 mm/hr respectively. Place of observation—for (a) to (c) Khandala (at cloud base) and (d) Delhi.

Drops smaller than 0.5 mm diameter have not been considered. Abscissae for successive curves have been shifted by 0.6 mm diameter. Mean intensity of rainfall is computed from rain drop size samples.

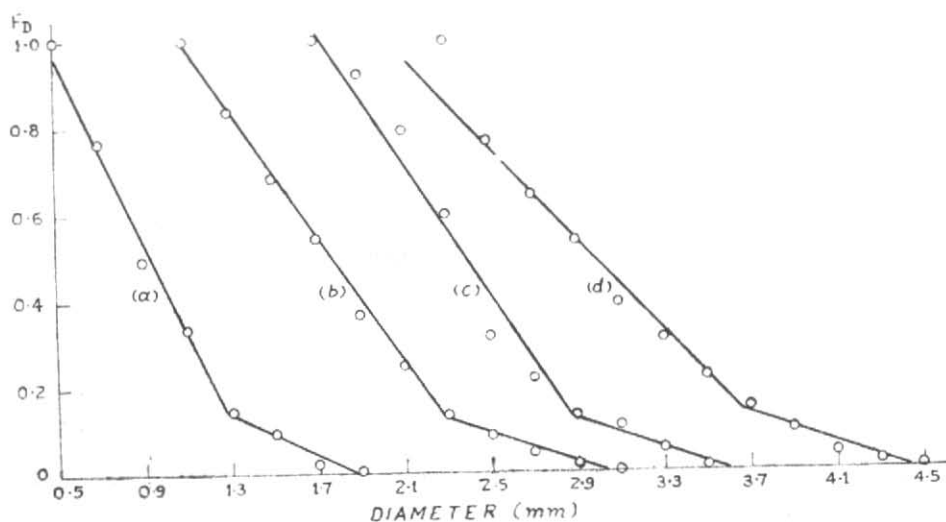


Fig. 2. F_D - D plot for four showers

Mean intensity—Curves (a) to (d) 4.4, 7.8, 1.7 and 7.7 mm/hr respectively. Place of observation—Khandala (at cloud base).

Drops smaller than 0.5 mm diameter have not been considered. Abscissae for successive curves have been shifted by 0.6 mm diameter. Mean intensity of rainfall is computed from rain drop size samples.

been plotted against corresponding diameter. To save space, drop size distributions showing similar features have been plotted in the same figure, the abscissae having been shifted by 0.6 mm diameter for each successive curve for sake of clarity. The data relate to observations made in monsoon showers at Khandala during August 1956, and at Delhi during July to October 1956 when some of the showers were completely scanned by exposing sensitised Whatman filter paper at fairly regular and close intervals. A full discussion of observations made is being published elsewhere (Ramana Murty and Gupta). It is seen that curves (a), (b), (c) and (d) in Fig. 1 are nearly straight lines, conforming largely to the theoretical deduction. Curves (a) to (d) in Fig. 2 on the other hand, show an initial straight portion, followed by another straight line having a different slope, while curve in Fig. 3 is an example of drop size distribution which does not conform to either of the two patterns.

Considering the various simplified assumptions made in deriving relationship (12) and also possible errors in drop size sampling, certain departure from the theoretical formula as shown by the size distribution curves in Figs. 2 and 3 are only to be expected. The derivations from (12), as shown by curves in Fig. 2 are, however, of a fairly regular nature and may be explained, at least qualitatively by taking into consideration more realistic conditions in regard to (i) time variation of updraft and (ii) changes in cloud liquid water content with height. If, for example, we consider a situation in which updraft persists for some time during the initial stages of the shower, the number of drops at the large size end will be more than in the hypothetical case treated in section 2, giving rise to a bend in the distribution curve towards its large size end, as in the case of curves in Fig. 2. Again, the possible effect on the slope of the curve due to liquid water content increasing with height as is generally observed in cumulus clouds (Warner and Newnham 1952) will be apparent from examination of equation (12). The theoretic-

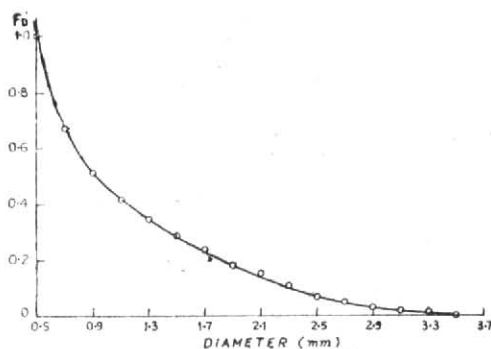


Fig. 3. F_D - D plot for a shower sampled at Delhi

Mean intensity of rainfall, 12.4 mm/hr

cal slope of F_D , D plot being given by $2\rho/wz_0$, the result of increasing w with elevation will be that, in the size range covering small raindrops whose trajectories will be confined to a relatively small depth of the cloud compared to those of large raindrops, the magnitude of the slope will be larger. The net effect of the slope decreasing with increasing D will be a continuously turning size distribution curve, so oriented as to present its convex side towards the axis of D .

The slopes of the distribution curves plotted in Figs. 1 and 2 have been used to compute the values of wz_0 for the corresponding rain situations. In case of curves (a) to (d) of Fig. 2 the computed values have been based on the slopes of the initial straight portions. Values of wz_0 so derived, range from 0.2 to 0.6 gm/cm². In the absence of any definite knowledge concerning liquid water content in

the cloud, and also of z_0 ($= \int_0^T U dt$), that is, the total flux upwards of cloud air through the cloud base, all that we may consider is whether the values calculated are of the right order. Taking $w = 1$ gm/m³ or 10^{-6} gm/cc, and $T = 30$ minutes, and assuming (a) $U = 1$ m/s and (b) $U = 2$ m/s, we see that the corresponding values of wz_0 come out to be (a) 0.18 gm/cm² and 0.36 gm/cm² respectively.

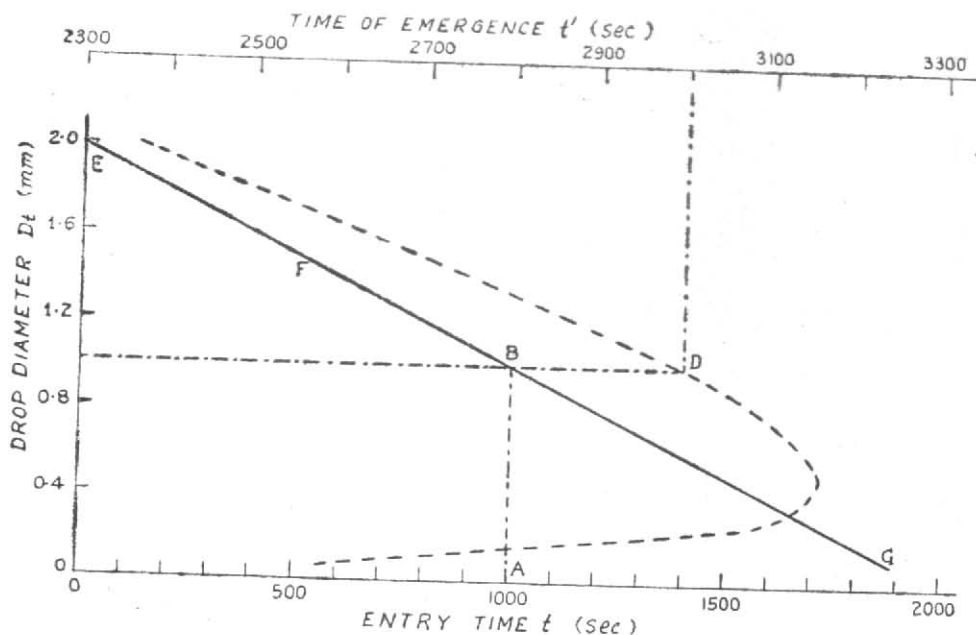


Fig. 4. (a)

For details see text. To trace the history of growth of a 'giant' cloud droplet starting at $t=1000$ sec we proceed from the point A vertically along the dash-dot line to its point of intersection with the full line at B. Ordinate corresponding to this point B gives final diameter, D_t , as 1.0 mm. From B, we proceed horizontally to the right along the dash-dot line to its point of intersection with the dashed curve at D. Abscissa corresponding to this point, on the top scale, gives time of emergence t' as 3000 sec.

4. Rate of decrease of raindrop size

To find the rate at which the diameter of the largest amongst the raindrops decreases with time, it is necessary to take into account the time of arrival of each raindrop below cloud base. This can be computed from equation (6). A case in which the updraft continues at a steady rate of 2 m/s from $t=0$ to $t=T=2000$ sec, and then drops abruptly to zero has been worked out numerically, assuming that $w=1$ gm/m³ and that the initial sizes of giant cloud droplets which grow finally to raindrops range from 25μ to 50μ in radius. Incidentally, the calculations in this case cover also the sizes of largest raindrops, which fall during the initial stages of the shower, when updraft is still persisting. Calculations show that the first raindrop to leave cloud base is the one which is formed

by coalescence growth of a giant cloud droplet of radius 50μ , starting its life history at a time, $t=0$. This leaves the cloud base at time $t'=1500$ sec, with a diameter of 1.5 mm. Subsequently, for a time we get progressively larger raindrops grown on smaller giant cloud droplets till the biggest raindrop of diameter 2.0 mm, originating on giant cloud droplet of 33μ radius, reaches cloud base at time $t'=2000$ sec. After 2000 sec, at which instant the updraft is assumed to cease completely the maximum drop diameter does not increase any further but remains steady till we get at time $t'=2350$ sec, a drop of diameter 2.0 mm grown on a giant cloud droplet of radius 25μ starting its life history at time $t=0$ sec. After this instant the maximum drop size decreases with time. Results of the computation during this stage

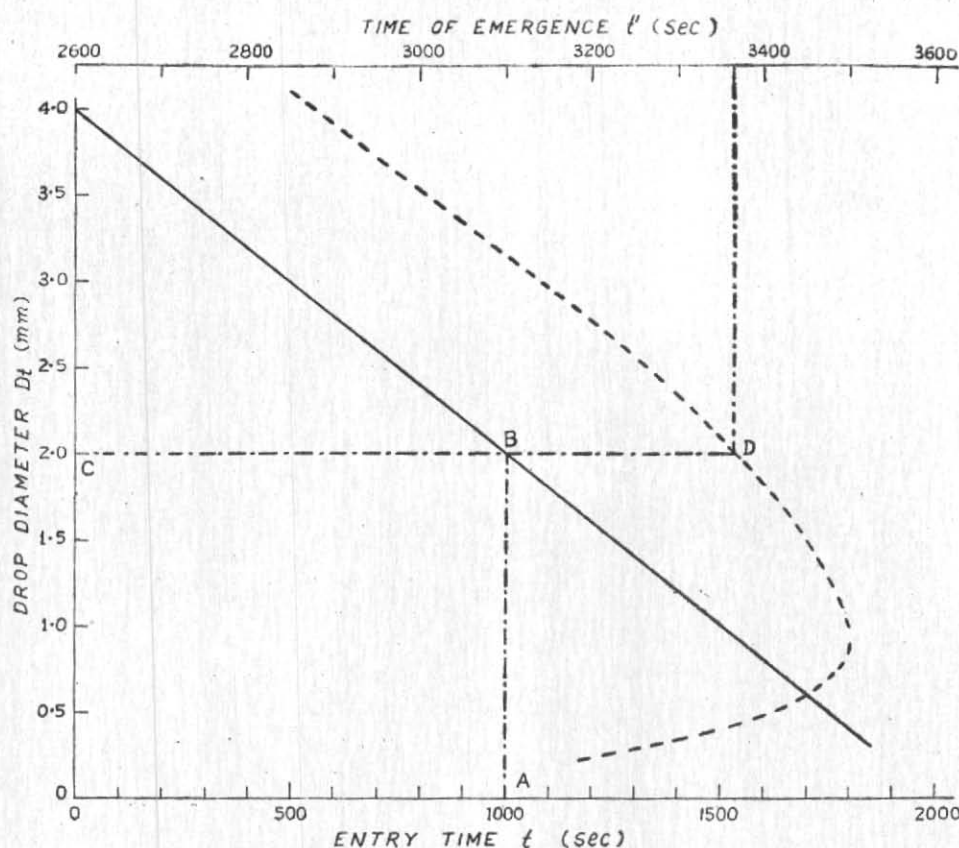


Fig. 4 (b)

For details see text. To trace the history of growth of a 'giant' cloud droplet starting at $t=1000$ sec we proceed from the point A vertically along the dash-dot line to its point of intersection with the full line at B. Ordinate corresponding to this point B gives final diameter, D_t , as 2.0 mm. From B, we proceed horizontally to the right along the dash-dot line to its point of intersection with the dashed curve at D. Abscissa corresponding to this point, on the top scale, gives time of emergence t' as 3360 sec.

of the shower are shown graphically in Fig. 4(a). The full line EFG representing the final diameter against entry time t , holds for giant cloud droplets in the radius range 25μ to 33μ , irrespective of their initial sizes, as the products of growth on them leave the cloud base only after the updraft has ceased. Further, as any giant cloud droplet starting its life history of growth at $t=500$ sec or later leaves the cloud base only after time $t'=2000$ sec, when updraft is zero, the portion

FG of the full line applies to raindrops formed on giant cloud droplets in the radius range $25\mu-50\mu$. The dashed curve represents the time of arrival t' , of raindrop at cloud base, against its diameter. The calculations pertaining to this curve are with reference to giant cloud droplet of radius 25μ , the product of growth on which, amongst others starting their life history at the same instant, is the last to reach the cloud base as a finally formed raindrop. It is seen from Fig. 4(a)

that the last raindrop leaves the cloud base at $t' = 3160$ sec with a diameter of 0.5 mm giving an average rate of decrease of diameter of the largest amongst raindrops as approximately 0.11 mm/min. Calculations show that this rate of decrease will be higher if the updraft is stronger. A case in which updraft continues at a steady rate of 4 m/sec upto 2000 sec and then ceases completely has also been worked out numerically. Results of this calculations are presented in graphical form in Fig. 4(b). It is seen that the maximum drop diameter decreases from 4.0

mm to 0.9 mm in course of $10\frac{1}{2}$ minutes giving the average rate of decrease as 0.30 mm/min.

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