MAUSAM, **63**, 4 (October 2012), 573-580

551.573 : 551.509.329

ARIMA model for forecasting of evaporation of Solapur station of Maharashtra, India

D. T. MESHRAM, S. D. GORANTIWAR and A. S. LOHAKARE

National Research Center on Pomegranate, Shelgi, Solapur - 413 006 (M. S.), India

(*Received 13 October 2010, Modified 21 June 2011*)

e mail : gomesh1970@rediffmail.com

सार – इस शोध–पत्र में साप्ताहिक वाष्पोत्सर्जन के प्रसंभाव्य मॉडलिंग का अध्ययन करने के लिए महाराष्ट्र के अर्ध शुष्क सोलापुर स्टेशन के 1987–2008 की अवधि की कुल 1144 रीडिंग्स के आधार पर साप्ताहिक वाष्पोत्सर्जन आँकड़े मौसमी ए. आर. आई. एम. ए. मॉडल का उपयोग किया गया है।

साप्ताहिक वाष्पोत्सर्जन श्रृंखला के स्वसहसंबंध फलन (ए. सी. एफ.) तथा आंशिक स्वसहसंबंध फलन (पी. ए. सी. एफ.) के आधार पर प्रथम आर्डर के ए. आर. आई. एम. ए. मॉडलों का चयन किया गया। मॉडल के प्राचलों को तीन परीक्षणों (यथा मानक त्रुटि, ए. सी. एफ. और पी. ए. सी. एफ. के अवशिष्टों और एकाइके सूचना मापदण्ड) की मदद से अधिकतम संभावना पद्धति का उपयोग करके प्राप्त किया गया है। चयनित मॉडलों के औचित्य का निर्धारण किया गया। जो ए. आर. आई. एम. ए. मॉडल के उपयोगिता परीक्षण के खरे उतरे उन्हें पूर्वानुमान करने के लिए चुना गया। सोलापुर में साप्ताहिक वाष्पोत्सर्जन मानों के पूर्वानुमान के लिए निम्न आर. एम. एस. ई. वाले मौसमी ए. आर. आई. एम. ए. (1,0,1) $(1,0,1)_{52}$ का अंततः चयन किया गया।

ABSTRACT. This paper deals with the stochastic modeling of weekly evaporation by using Seasonal ARIMA model for weekly evaporation data for the period of 1987-2008 with a total of 1144 readings for semi-arid Solapur station in Maharashtra.

ARIMA models of 1st order were selected based on observing autocorrelation function (ACF) and partial autocorrelation function (PACF) of the weekly evaporation series. The model parameters were obtained by using maximum likelihood method with the help of three tests (*i.e.*, standard error, ACF and PACF of residuals and Akaike Information Criteria). Adequacy of the selected models was determined. The ARIMA model that passed the adequacy test was selected for forecasting. The Seasonal ARIMA (1, 0, 1) $(1, 0, 1)_{52}$ with lower RMSE is finally selected for forecasting of weekly evaporation values, at Solapur.

Key words – Stochastic model, Forecasting, Evaporation and Seasonal ARIMA model.

1. Introduction

Several stochastic models have been developed in past (Box and Jenkins, 1994) for modeling of hydrological time series mainly rainfall, runoff and evaporation. These include autoregressive (AR) models of different orders (Davis and Rapport, 1974; Salas *et al*., 1980; Kamte and Dahale, 1984; Gorantiwar, *et al*., 1995; Narulkar, 1995; Mutua, 1998; Singh, 1998; Reddy and Kumar, 1998; Subbaiah and Sahu, 2002 and Patil, 2003), moving average (MA) models for different orders (Gupta and Kumar, 1994 and Verma, 2004), autoregressive moving average (ARMA) models of different orders (Katz and Skaggs, 1981; Chhajed, 2004; Katimon and Demon, 2004) for annual stream flow. For monthly or intra-seasonal flows, seasonal or periodic autoregressive integrated moving average (ARIMA) model (Bender and Simonovle, 1994; Montanari *et al*., 2000; Trawinski and Mackay, 2008), Thomas-Fiering models (Srinivasan, 1995) and fractionally difference ARIMA models (Montanari *et al*., 1997) were used. Often the historical series is short and inadequate for irrigation planning. Hence, stochastic models are useful for generation of long term evaporation data needed for irrigation planning.

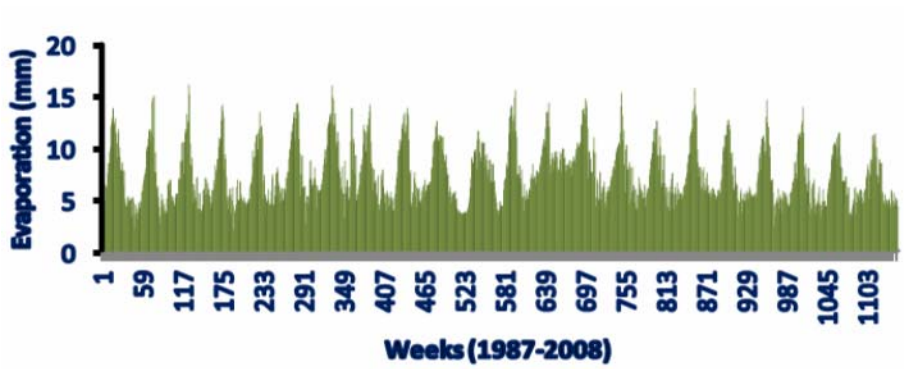


Fig. 1. Weekly evaporation series in Solapur station from 1987 to 2008

The models used for generation and forecasting of the annual evaporation series were Thomson-Firing, AR, MA, ARMA and ARIMA models of different orders. The models SARIMA, PARMA and FARIMA were used for seasonal and periodic evaporation series. The studies indicated that stochastic models can be successfully used for the generation of the synthetic sequence of rainfall, runoff and evaporation. ARIMA class of models was also used for forecasting of runoff/evaporation few time periods ahead. For appropriate planning of the water resources available to farmers, they must match with the water requirement. The reasonable forecast of water requirement at least one year ahead water would enable them to manage water resources efficiently, as the ARIMA models showed possibility to forecast the other hydrological events.

In this study, the applicability of the ARIMA models to forecast evaporation for Solapur station were investigated and finally the appropriate ARIMA model was identified for the forecast of evaporation for Solapur station and then used for assessing water requirement.

2. Material and methods

This study was concerned with the forecast of evaporation by using ARIMA class of models.

2.1. *Development of ARIMA model*

Seasonal autoregressive integrated moving average (SARIMA) are useful for modeling seasonal time series in which the mean and other statistics for a given season are not stationary across the year. The basic ARIMA model in its seasonal form is described as (Hipel *et al*., 1976; Box and Jenkins, 1994) a straightforward extension of the non-seasonal ARIMA models.

The different approaches involved in fitting of ARIMA models to historical hydrological series as suggested by Hipel *et al*., 1976 and Box and Jenkins, 1994 are standardization and normalization of time series, identification of the models, determination of the parameters, diagnostic checking and selection of the best model.

2.1.1. *Standardization and normalization of time series variables*

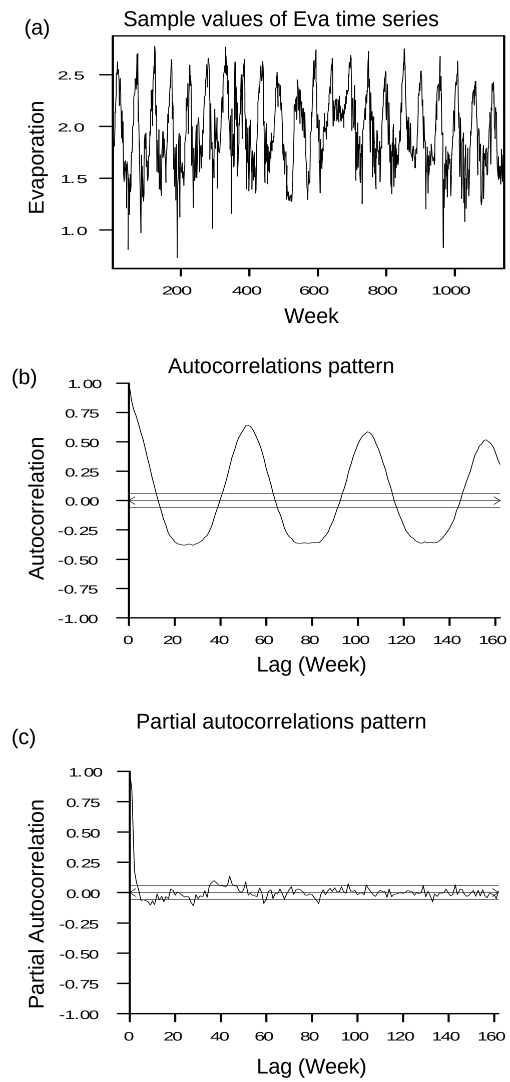
The first step in time series modeling is to standardize and transform the time series. In general, standardization is performed by normalizing the series as follows.

$$y_{i,j} = \frac{x_{i,j} - x_i}{\sigma_i} \tag{1}$$

Where, $y_{i,j}$ - Stationary stochastic component in the mean and variables for week i or the year j; $x_{i,j}$ - Weekly evaporation in the week i of the year j; x_i - Weekly mean and σ_i - Weekly standard deviation.

2.1.2. *Identification of the model*

An important step in the modeling is the identification of a tentative model type to be fitted to the data set. In the present study, the procedure stated by Hipel and McLeod (1994) were adopted for identifying the possible ARIMA models. A time series with the seasonal variation may be considered stationary if the theoretical autocorrelation function (ρ_k) and theoretical partial autocorrelation function (ρ_{kk}) are zero after a lag $k = 2s + 2$ (Where, 's' is the seasonal period; in this study,



Figs. 2 (a-c). Evaporation time series, Autocorrelation and partial autocorrelation pattern of the differenced time series of evaporation (d = 0, D = 0)

s = 52). The requirement of identification procedure is as: *i.e.*, Plot of the original series, Plot of the standardized series, ACF analysis and PACF analysis. The estimates of theoretical autocorrelation function (e_m), *i.e.*, r_m is given by equation (2).

$$r_m = \frac{\sum_{i=1}^{n-m}\left(x_i - \bar{x}\right)\left(x_{i+m} - \bar{x}\right)}{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \tag{2}$$

Where, n - The number of observations; $\bar{x}$ - The average of the observations, r_m - Autocorrelation function at lag m.

The estimate of theoretical partial autocorrelation function (e_{kk}), *i.e.*, ϕ_{mm} is given by the equation (3). The partial autocorrelation function varies between - 1 and + 1, with values near ± 1 indicating stronger correlation. The partial autocorrelation function removes the effect of shorter lag autocorrelation from the correlation estimates at longer lags.

$$\phi_{mm} = \frac{r_m - \sum_{i=1}^{m-1}\left(\phi_{m-1}\right)\left(r_{m-1}\right)}{1 - \sum_{i-1}^{m-1}\left(\phi_{m-1}\right)_r} \tag{3}$$

Where,

ϕ_{mm} - Partial auto correlation function at lag k.

It is considered that ρ_k and ρ_{kk} equal to zero if (Maier and Dandy, 1995)

$$\rho_k = 0,\ i.e.,\ \left|r_k\right| \le \frac{2}{T^{0.5}} \tag{4}$$

$$\rho_{kk} = 0,\ i.e.,\ \left|r_{kk}\right| \le \frac{2}{T^{0.5}} \tag{5}$$

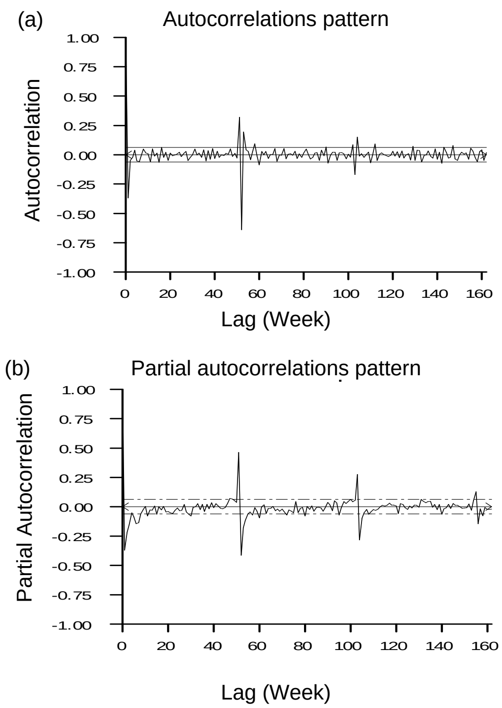
Where, r_k - Sample autocorrelation at lag k; r_{kk} - Sample partial autocorrelation at lag k; T - Number of observation.

If the sample autocorrelation function (ACF) of analyzed series does not meet the above condition, the time series needs to the transformed into a stationary one using different differencing schemes. For example, for (d = 0, D = 1, s = 52) according to the expression given by equation

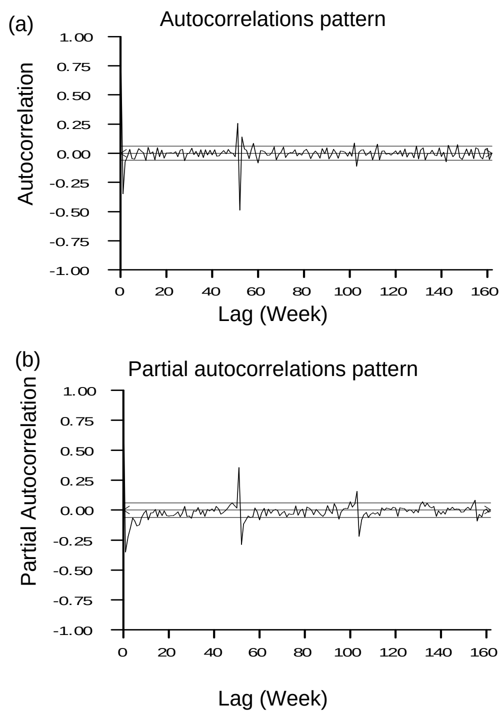
$$y_t = \left(1 - B\right)^d \left(1 - B^s\right)^D x_t = \left(1 - B^s\right) E_{0,t} \tag{6}$$

Where, y_t - Original time series , d - order of non-seasonal differencing operator, D - Order of seasonal differential operator, B - Backshift operator, s - Seasonal length, t - discrete time, $E_{0,t}$ - Evaporation series, k - lag period and x_t - Stationary series formed by differencing series.

The time series y_t is stationary if the ACF and PACF cut off at lags less than k = (2s + 2) seasonal periods. Thus, it is necessary to test the stationary status of the transformed time series obtained by differencing the original times series according to different orders of differencing (seasonal and non-seasonal). The differenced series that pass the stationary criteria needs to be considered for further analysis. The following guidelines



Figs. 3(a&b). Autocorrelation and partial autocorrelation pattern of the differenced time series of evaporation (d = 1, D = 0)



Figs. 4 (a&b). Autocorrelation and partial autocorrelation pattern of the differenced time series of evaporation (d = 1, D = 1)

were used for selecting the orders of AR and MA terms (Gorantiwar, 1984).

(*i*) If the autocorrelation function cuts off, fit ARIMA $(0, d, q)(0, 1, Q)_{52}$ model to the data, where, q is the lag after which the autocorrelation function first cuts off, and Q is the lag after which seasonal ACF cut off.

(*ii*) If the autocorrelation function cuts off, fit ARIMA $(p, d, 0)(P, 1, 0)_{52}$ model to the data, where, p is the lag after which the partial autocorrelation function first cuts off and P is the lag after which seasonal PACF cuts off.

(*iii*) If neither the autocorrelation nor partial autocorrelation functions cuts off, fit the ARIMA $(p, d, q)(P, 1, Q)_{52}$ model for a grid of values of p, P, q and Q.

Thus, on basis of information obtained from the ACF and PACF, several forms of the ARIMA model need to the identified tentatively.

2.1.3. *Estimation of parameters of the model*

After the identification of model, the parameters of the selected models were estimated. The parameters of the identified models are estimated by the statistical analysis of the data series. The most popular approaches of parameters estimation is the method of maximum likelihood.

2.1.4. *Diagnostic checking of the model*

Once a model has been selected and parameters calculated, the adequacy of the model has to be checked (diagnostic checking). Here, following three tests were used.

2.1.4.1. *Examination of standard error*

A high standard error in comparison with the parameter values point out a higher uncertainty in parameter estimation which questions the stability of the model. The model is adequate if it meets the following condition.

$$t = \frac{cv}{se} > 2 \qquad (7)$$

Where,

cv - parameter value and *se* - standard error.

TABLE 1

Basic statistics for Solapur station of weekly evaporation data (mm)

No. of observation	Mean	St. Dev.	Var.	Min	Max
1144	7.5	2.80	7.88	2.1	13.5

2.1.4.2. *ACF and PACF of residuals*

If the model is adequate at describing behaviour of a time series (evaporation), the residuals of the model should not be correlated, *i.e.*, all ACF and PACF should lie within the limits calculated by equations (4) and (5) after lag $k = 2s + 2$, where s = number of periods.

2.1.4.3. *Akaike information criteria (AIC)*

The AIC (Akaike, 1974) are computed as

$$\text{AIC} = 2k + \left\{(\ln\frac{2\pi v_r}{T}) + 1\right\}T \qquad (8)$$

Where, k - Number of model parameters, v_r - Residual variance, T - Total number of observations.

2.2. *Selection of the most appropriate model*

The root mean square error (RMSE) were estimated for each model

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(E_{act} - E_{for})^2}{N}} \qquad (9)$$

Where, E_{act} - Actual value of evaporation (mm), E_{for} - Forecast value of evaporation (mm), N - Total number of observation

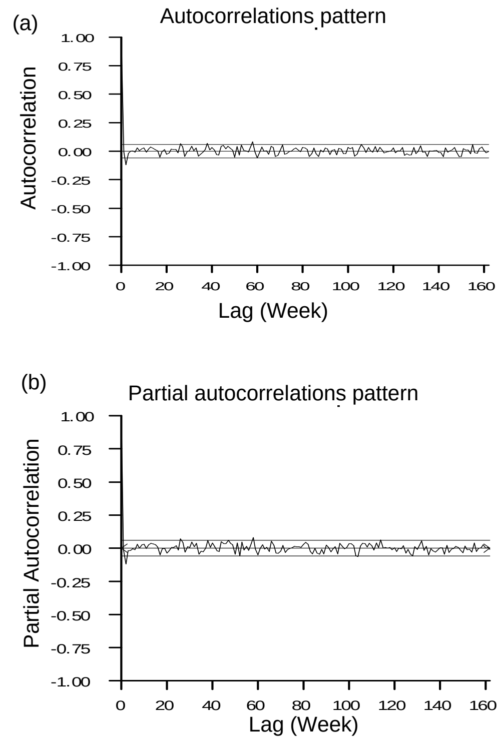
3. Results and discussion

3.1. *Evaporation analysis*

Weekly evaporation (Fig. 1) shows a seasonal cycle. The ACF and PACF of the original evaporation data are not stationary. The descriptive statistics for evaporation data are shown in Table 1.

3.2. *Fitting of ARIMA model*

The weekly evaporation data were used for generating and forecasting of development and validation of stochastic model. The results obtained from the study have been presented and discussed under the following heads.



Figs. 5 (a&b). Autocorrelation and partial autocorrelation pattern of differenced time series of evaporation (d = 0, D = 1)

3.2.1. *Standardization and normalization of time series variables*

The ARIMA model has the provision to differentiate the time series. Hence, standardization and normalization was not performed.

3.2.2. *Identification of the model*

The ACF and PACF of weekly evaporation time series were estimated for different lags. These are shown with upper and lower limits. It is seen from Fig. 2, that ACF lie outside the limit after lag $k = 2s + 2$, *i.e.*, 106. Thus, ARIMA model cannot be applied to the original time series of evaporation. Therefore, the time series was transformed differencing schemes by using $d = 0$; $D = 1$, $d = 1$; $D = 1$, $d = 1$; $D = 0$, $d = 0$; $D = 0$. The ACF and PACF along with the upper and lower limits were estimated by equations (4) & (5). It is observed from the Figs. 3 to 5, that ACF of $d = 0$, $D = 1$ and $d = 1$, $D = 1$ lie within the limits of range specified by equations (4) & (5) after lag 104. However, for $d = 1$, $D = 0$ and $d = 0$, $D = 0$, ACF does not lie within the limits after the lag 104. Therefore, differencing schemes (*i.e.*, $d = 0$; $D = 1$, $d = 1$; $D = 1$) were used for developing ARIMA model for weekly evaporation time series.

TABLE 2

Parameter estimates, standard error, corresponding *t* values and AIC values for different ARIMA models

Models	ϕ_1	θ_1	Φ_1	Θ_1	*c*	Models	ϕ_1	θ_1	Φ_1	Θ_1	*c*
ARIMA (1, 1, 1) (1, 1, 0)$_{52}$						*ARIMA (1, 1, 0) (1, 0, 0)$_{52}$*					
Estimate	0.528	0.947		0.51	0.0001	Estimate	0.2007		0.103		0.005
SE	0.031	0.011		0.02	0.0033	SE	0.0297		0.030		0.036
t-value	16.9	80.82		19.4	0.05	*t*-value	6.75		3.43		0.01
AIC	5077					AIC	4863				
ARIMA (1, 0, 0) (1, 0, 1)$_{52}$						***ARIMA (1, 0, 1) (1, 0, 1)$_{52}$***					
Estimate	0.716		0.996	0.950	7.508	**Estimate**	**0.8431**	**0.2658**	**0.99607**	**0.949**	**7.503**
SE	0.023		0.009	0.015	0.372	**SE**	**0.0248**	**0.0395**	**0.00994**	**0.016**	**0.47**
t-value	30.91		110.0	63.49	20.18	***t*-value**	**33.96**	**6.73**	**100.23**	**58.21**	**15.96**
AIC	4690					**AIC**	**4650**				
ARIMA (1, 0, 0) (0, 1, 1)$_{52}$						*ARIMA (1, 0, 1) (0, 1, 1)$_{52}$*					
Estimate	0.646			0.959	0.012	Estimate	0.8431	0.2658	0.99607	0.949	7.503
SE	0.023			0.021	0.016	SE	0.0248	0.0395	0.00994	0.016	0.47
t-value	27.97			43.94	0.73	*t*-value	33.96	6.73	100.23	58.21	15.96
AIC	4675					AIC	4650				
ARIMA (1, 0, 1) (0, 0, 1)$_{52}$						*ARIMA (0, 1, 1) (1, 1, 0)$_{52}$*					
Estimate	0.907	0.178		0.112	7.478	Estimate		0.4861	0.495		0.002
SE	0.014	0.033		0.030	0.378	SE		0.0266	0.0267		0.016
t-value	63.37	5.29		3.71	19.76	*t*-value		18.3	18.52		0.01
AIC	4791					AIC	5209				
ARIMA (0, 0, 1) (1, 0, 1)$_{52}$						*ARIMA (0, 1, 1) (0, 1, 1)$_{52}$*					
Estimate		0.527	0.991	0.850	7.351	Estimate		0.3696	0.9963		0.000
SE		0.025	0.006	0.018	0.32	SE		0.0279	0.0123		0.055
t-value		20.7	163.3	46.26	22.96	*t*-value		14.23	80.77		0.01
AIC	5509					AIC	4746				

On the basis of information obtained from ACF and PACF, the orders of autoregressive (AR) and moving average (MA) terms were identified as one. Based on this, 36 forms of ARIMA models were identified and parameters computed.

3.2.3. *Determination of parameters of model and diagnostic checking*

The following parameters of the selected models were calculated by maximum likelihood method.

1. ϕ_1 2. θ_1 3. Φ_1 4. Θ_1 5. c

Out of the 36 possibilities, ten ARIMA models satisfied the test for all parameters. Standard error and *t* values for these ten models are given in Table 2.

3.2.4. *ACF and PACF of residual series*

For a model to be considered by adequate at describing behaviour of evaporation time series, the residuals of model should be correlated, *i.e.*, all ACF and

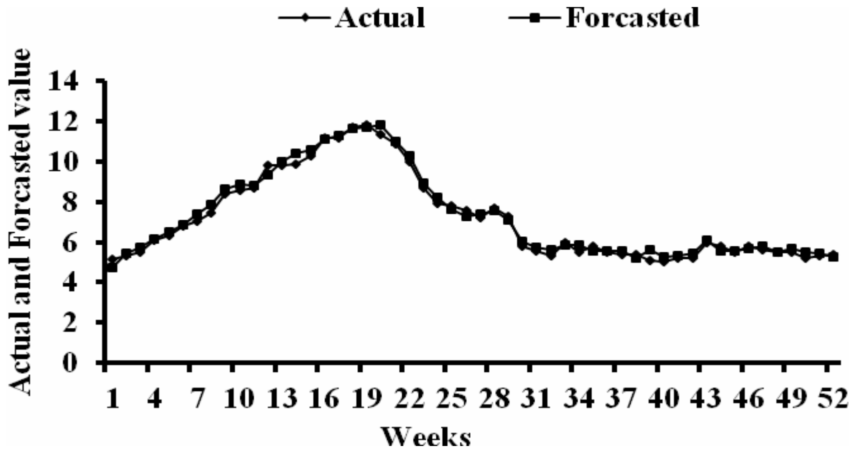


Fig. 6. Comparison of forecasted and actual values of evaporation by using ARIMA (1, 0, 1) $(1, 0, 1)_{52}$ model

TABLE 3

Root mean square error values of first five models

Models	RMSE
ARIMA(1, 0, 0)$(1, 0, 1)_{52}$	0.18
ARIMA(1, 0, 1)$(1, 0, 1)_{52}$	**0.16**
ARIMA(1, 0, 1)$(0, 1, 1)_{52}$	0.45
ARIMA(0, 0, 1)$(1, 0, 1)_{52}$	0.17
ARIMA(1, 0, 1)$(0, 0, 1)_{52}$	0.29

PACF should lie within the limits calculated by equations (4) & (5) after lag $k = 2s + 2$, where s = number of periods. In this case, for the value of k is 106, computations showed that, ACF and PACF residual series of 15 models lie within the prescribed limits.

3.2.5. *Selection of the best model*

First 5 models with less AIC that satisfied the standard error and ACF and PACF of residuals criteria were finally used (Table 2) for generation of weekly evaporation values. For this purpose, the evaporation values were forecast for one year with the help of identified ARIMA models. These values were compared with the actual values for one year by calculating the root mean square error (RMSE) between them (Table 3). It is observed from the Fig. 6, that seasonal pattern of evaporation series is maintained in generated values by all the ARIMA models.

Based on RMSE, the ARIMA (1, 0, 1) $(1, 0, 1)_{52}$ of models are selected for forecasting. The values of the parameters of the ARIMA model which is finalized for forecasting of parameters are: ϕ_1 = 0.8431, θ_1 = 0.2658, Φ_1 = 0.9960, Θ_1 = 0.9498 and C = 7.503.

3.3. *Comparison of forecast and actual values of evaporation*

The ARIMA models that were finalized to forecast the values of evaporation for Solapur station are presented in Fig. 6. These values were developed using the climatological data up to 2008. The evaporation values were forecasted with the help of best model and forecast weekly evaporation values were calculated with the help of weekly evaporation series. Forecasted values were compared (Fig. 6) with actual values of evaporation of 2009.

4. Conclusions

The study indicates that the seasonal ARIMA model is a viable tool for forecasting the evaporation at Solapur location. The system studies reveal that, if sufficient length of data is used in model building, then frequent updating of model may not be necessary. This forecasted evaporation can be advantageously used in deriving the optimal irrigation system. The ARIMA (1,0,1) $(1,0,1)_{52}$ gave the lower values of RMSE and hence is the best stochastic model for generation and forecasting of weekly evaporation values for Solapur, Maharashtra, India. It is concluded that seasonal ARIMA models can be successfully used for forecasting of evaporation having inbuilt seasonal pattern.

References

Akaike, H., 1974, "A new look at the statistical model identification", *IEEE Trans. Autom. Control*., AC - **19** (6), 716-723.

Box, G. E. P. and Jenkins, G. M., 1994, "Time Series Analysis, Forecasting and Control", Revised Edition, Holden-Day, San Francisco, California, United States.

Bender, M. and Simonovle, S., 1994, "Time-series modeling for long-range stream flow forecasting", *Journal of Water Resources Planning and Management*, ASCE, **120** (6), 857-870.

Chhajed, N., 2004, "Stochastic modeling for forecasting Mahi river inflows", M. E. Thesis submitted to MPUAT, Udaipur.

Davis, J. M. and Rappoport, P. N., 1974, "The use of time series analysis techniques in forecasting meteorological drought", *Monthly Weather Review*, **102**, 176-180.

Gupta, R. K. and Kumar, R., 1994, "Stochastic analysis of weekly evaporation values", *Indian Journal of Agricultural Engineering*, **4** (3-4), 140-142.

Gorantiwar, S. D., 1984, "Investigating applicability of some operational hydrology models to WB streams", M. Tech. Thesis submitted to I.I.T., Kharagpur.

Gorantiwar, S. D., Majumdar, M. and Pampattiwar, P. S., 1995, "Application of autoregressive models of different orders to annual stream flows of Barkar river with their logarithmic transformation", *Journal of Applied Hydrology*, **8**, 33-39.

Hipel, K. W., McLeod, A. I. and Lennox, W. C., 1976, "Advances in Box Jenkins modeling: 1. Model construction", *Water Resource Research*, **13**, 567-575.

Hipel, K. W. and McLeod, A. I., 1994, "Time series modeling of water resources and environmental systems", Elsevier, Amsterdam, The Netherland, p1013.

Katz, R. W. and Skaggs, R. H., 1981, "On the use of autoregressive moving average processes to model meteorological time series", *Monthly Weather Review*, **109**, 479-484.

Kamte, P. P. and Dahale, S. D., 1984, "A stochastic model on drought", *Mausam*, **35**, 387- 390.

Katimon, A. and Demun, A. S., 2004, "Water use trend at university technology Malaysia: Application of ARIMA model", *Journal Technology*, **41**(B), 47-56.

Maier, H. R. and Dandy, G. C., 1995, "Comparison of Box-Jenkins procedure with artificial neural network methods for univariate time series modeling, Research", Report No. R 127, Department of Civil and Environmental Engineering, University of Adelaide, Australia, p120.

Montanari, A., Rosso, R. and Taqqu, M. S., 1997, "Fractional differenced ARIMA models applied to hydrological time series: Identification, Estimation and Simulation", *Water Resources Research*, **33**(5), 1035-1044.

Montanari, A., Rosso, R. and Taqqu, M.S., 2000, "A seasonal fractional ARIMA model applied to the Nile river monthly flows at Aswan", *Water Resources Research*, **36** (5), 1249-1259.

Mutua, F. M., 1998, "Transfer function hydrologic modeling: A case Study", *Journal of Applied Hydrology*, **11** (2), 11-15.

Narulkar, M. S., 1995, "Optimum real time operation of multi-reservoir systems for irrigation scheduling", Ph. D. Thesis submitted to I.I.T., Bombay.

Patil, R. M., 2003, "Stochastic modeling of water deficit for Rahuri region", M. E. Thesis submitted to MPUAT, Udaipur.

Reddy, K. M. and Kumar, D., 1998, "Time series analysis of monthly rainfall for Bimo watershed of Ramganga river", *Journal of Agriculture Engineering*, ISAE, **36**(4), 19-29.

Salas, J. D., Dellur, J. W., Yevjevich, V. and Lane, W. L., 1980, "Applied modeling of Hydrological Time Series", *Water Resources Publication*, Littleton, Coloredo, p484.

Srinivasan, K., 1995, "Stochastic modeling of Monsoon River flows", *Journal of Applied Hydrology*, **8**, 51-57.

Singh, C. V., 1998, "Long term estimation of monsoon rainfall using stochastic models", *International Journal of Climatology*, **18**, 1611-1624.

Subbaiah, R. and Sahu, D. D., 2002, "Stochastic model for weekly rainfall of Junagadh", *Journal of Agrometeorology,* **4**, 65-73.

Trawinski, P. R. and Mackay, D. S., 2008, "Meteorologically conditioned time series predictions of West Nile virus vector mosquitoes", *Vector-Borne and Zoonotic Diseases*, **8** (4), 505-522.

Verma, A., 2004, "Stochastic modeling on monthly rainfall of Kota, Rajasthan", M. E. Thesis Submitted to GBPUAT, Pantnagar, India.