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Size distribution of Raindrops—Part II

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ABSTRACT. Average values of the number of raindrops per m^3 of air, grouped according to diameters, are given in the form of a table for various average values of precipitation rates ranging from 0·20 mm/hr to 34 mm/hr. The distribution of raindrops amongst different diameter groups has been shown in the form of histograms for six different values of the rate of precipitation. The-variations of the number of raindrops per m^3 of air, of particular diameter groups as well as the total number of drops of all diameters with rate of precipitation are shown graphically. A relation useful for correlation with radar observation is also obtained. If x is the diameter of drop in mm, the relation $Z = \sum x^6 = 189\, I^{1\cdot 35}$ mm^6/m^3 has been obtained by the method of least squares. Radar reflectivities of rain of various intensities have been calculated for $\lambda = 3\cdot 2$ cm and $\lambda = 10$ cm and given in the form of a table. The median drop diameters have been obtained from the drop frequency curves and correlated with intensity of precipitation. The relation $d_{\text{median}} = 0\cdot 6\, I^{0\cdot 28}$ mm is obtained.

1. Introduction

In Part I of this series (Kelkar 1959) the author had reported the results of measurements of the number of raindrops of different diameters per m^2 per second for various intensities of precipitation, *i.e.*, the dropsize distribution on the ground. On account of the variation of the terminal velocity of a water drop with size the dropsize distribution on the ground differs from the dropsize distribution in air. The latter is of importance in correlation with radar observations and is also of interest as a matter of pure meteorology.

2. Results

The results are based on data reported earlier in Part I (Kelkar 1959) and is confined to general rains. The number of drops per m^3 of air (n_1) is obtained from the number of drops received at the ground level per m^2 per second (N_1) of diameter d_1, by dividing the latter by the terminal velocity V_1 in m/sec. Thus

$$n_1 = N_1 / V_1 \, .$$

The values of the terminal velocities were obtained from a curve based upon the values quoted by Best (1950).

Table 1 gives the average number of drops of different diameter ranges per m^3 of air for 21 different average rates of precipitation ranging from 0·20 mm/hr to 34 mm/hr. The averages are based on five individual records of rain except the last one which is a mean of four records. The total number of drops of all sizes per m^3 of air is also shown in a separate column. The distribution of raindrops per m^3 of air amongst different diameter groups is shown in Fig. 1 in the form of histograms for six different intensities of precipitation, *viz.*, 0·20, 0.57 1·5, 5·4, 13, and 34 mm/hr. It is seen that with

TABLE 1

Average number of raindrops per m^3 of air grouped according to diameters

Av. Intensity of precipitation (mm/hr)	Diameter ranges (in mm)																Av. No. of raindrops of all sizes per m^3 of air
	0·00—0·25	0·25—0·50	0·50—0·75	0·75—1·00	1·00—1·25	1·25—1·50	1·50—1·75	1·75—2·00	2·00—2·25	2·25—2·50	2·50—2·75	2·75—3·00	3·00—3·25	3·25—3·50	3·50—3·75	3·75—4·00	
0·20	80	**125**	25	8	2·7	1·5	0·03										242
0·33	22	67	**85**	19	4·1	1·4	0·2	0·2	0·2								198
0·46	..	49	**111**	34	3·5	1·4	0·8	0·3	0·2								200
0·57	34	69	**82**	41	8	2·8	1·2	0·3	0·1								238
0·64	44	81	**123**	47	8·3	2	0·6	0·5	0·2								307
0·76	66	55	**99**	34	8	2·9	1·8	1·2	0·5	0·1	0·06						269
0·87	40	41	**89**	70	13	3·9	1·5	0·6	0·08	0·04	0·04						259
1·1	17	51	**83**	60	17	6·2	2·4	1·2	0·2	0·1	0·04						238
1·5	..	46	**126**	92	28	6·9	2·2	1·7	1·9	0·1	..						305
1·9	..	59	**136**	76	31	14	3·8	1·5	0·9	0·7	0·4						323
2·3	..	54	110	**180**	79	29	5·5	1·9	0·4	0·08	..						460
3·5	..	50	85	93	**100**	42	9·4	2·1	0·5	0·2	..						382
4·1	..	29	72	**168**	88	38	8·8	6·8	1·4	0·2	..						402
4·6	19	60	**130**	107	101	72	8·6	2·7	0·2	0·1	..						501
5·4	..	15	**96**	85	87	59	22	14	5·1	0·4	0·08						383
6·2	..	66	**141**	124	108	72	30	8	1·0	0·5	0·1	..	0·2				552
8·6	..	..	37	47	**123**	98	47	19	1·6	0·9	..	0·1	..				374
9·9	5	60	**121**	83	76	**80**	52	21	7	4·5	1	0·3	..	..	0·1	0·1	511
13	..	18	**104**	74	85	**120**	69	27	12	5·8	1·0	..	0·1	..	..	..	516
18	..	32	**94**	55	88	**128**	70	52	17	7·2	4·4	1·7	0·1	0·9	..	..	546
34	**592**	561	388	118	105	**156**	117	100	51	23	10	1·4	0·4	..	..	..	2222

increasing intensity, the spectrum extends towards larger diameter groups and that there is a progressive increase in the number of raindrops larger than 1 mm in diameter. At rates of precipitation above 13 mm/hr the distribution curve has two maxima, one in the diameter group less than 1 mm and the other between 1·25—1·50 mm. There is an abnormal increase in the number of small drops at the intensity of 34 mm / hr, but this has actually occurred only in one out of four records used in calculating the average values.

The variation of the total number of raindrops per m^3 of air $n = \Sigma n_1 = \Sigma N_1/V_1$ with intensity of precipitation I in mm/hr is shown semilogarithmically in Fig. 2. Average values based on five records are shown by circles and the values from the individual records are shown by dots. The average graph is linear but the individual points show a considerable scatter. Small values of n indicate the predominance of large drops whereas large values of n correspond to the predominance of small drops for a particular intensity of precipitation.

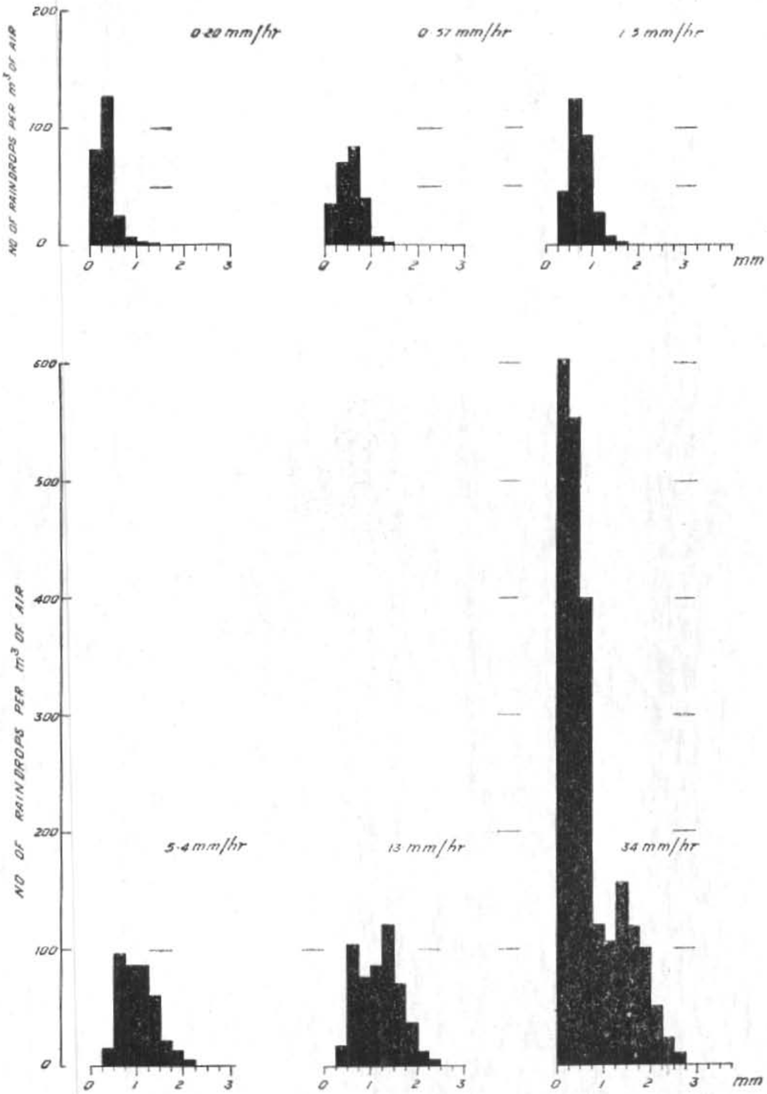

Fig. 1. Diameter spectrum of raindrops per m³ of air

The variation of the number of drops of a particular diameter group with intensity of precipitation is also of interest and is shown in Fig. 3. The individual curves show the contribution made by drops of different sizes at various intensities of precipitation. Average values based on mean of ten records are shown by circles and the individual observatons are indicated by dots to bring out the variations from the average. The very small drops within the diameter range 0·00—0·25 mm seldom occur and there is no systematic and continuous variation with rate of precipitation. These are, therefore, omitted. Drops of diameter 0·25—0·50 mm on the whole diminish towards larger intensities as is seen from Fig 3 (a). The number of drops of diameter 0·50—0·75 mm rises to a

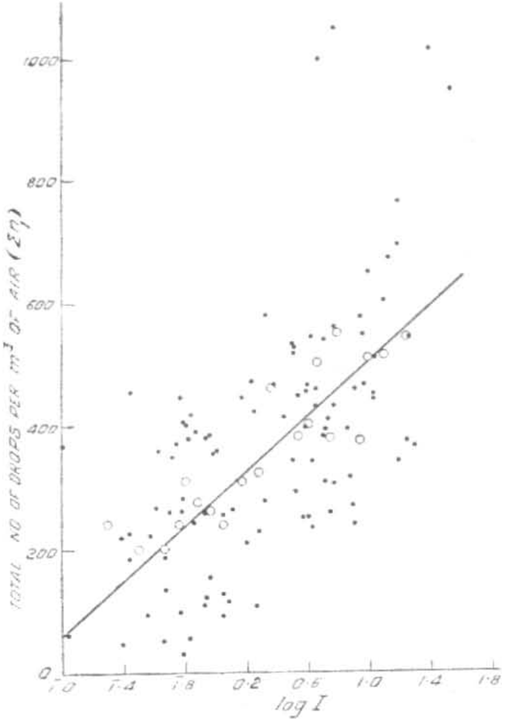

Fig. 2. Total number of raindrops per m³ of air as a function of the intensity of precipitation

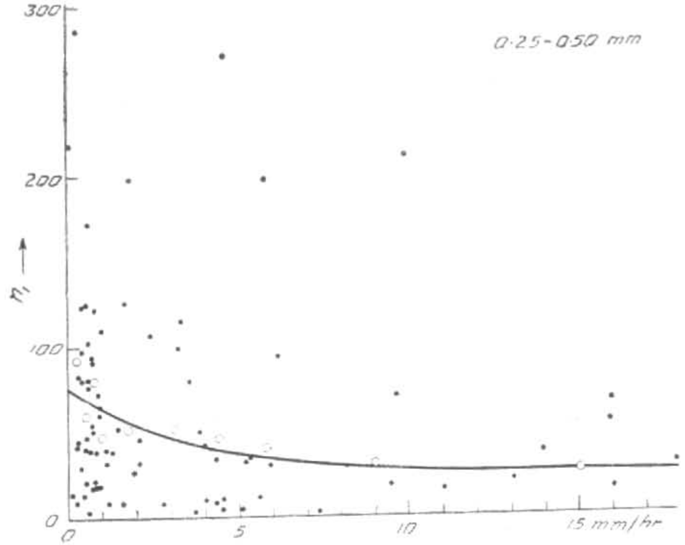

Fig. 3 (a)

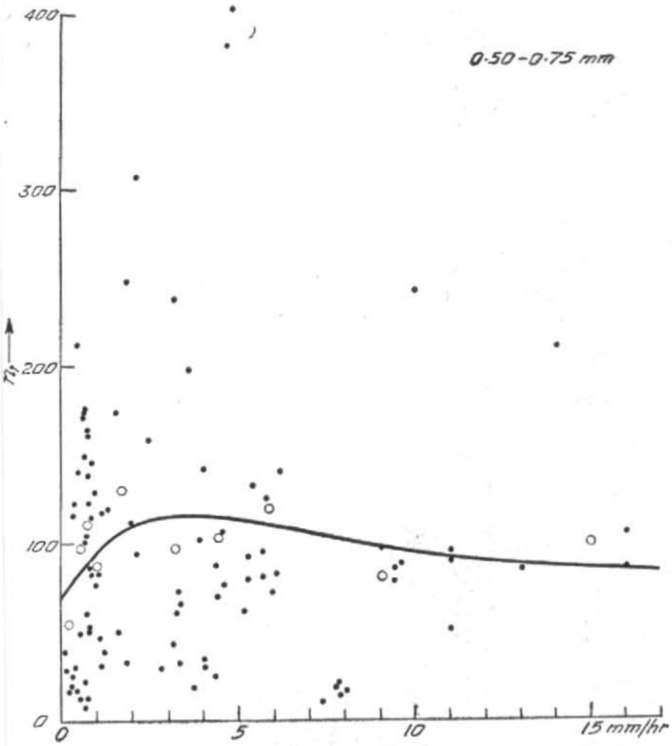
0·50–0·75 mm
n_1 →
0
100
200
300
400
0
5
10
15 mm/hr

Fig. 3(b)

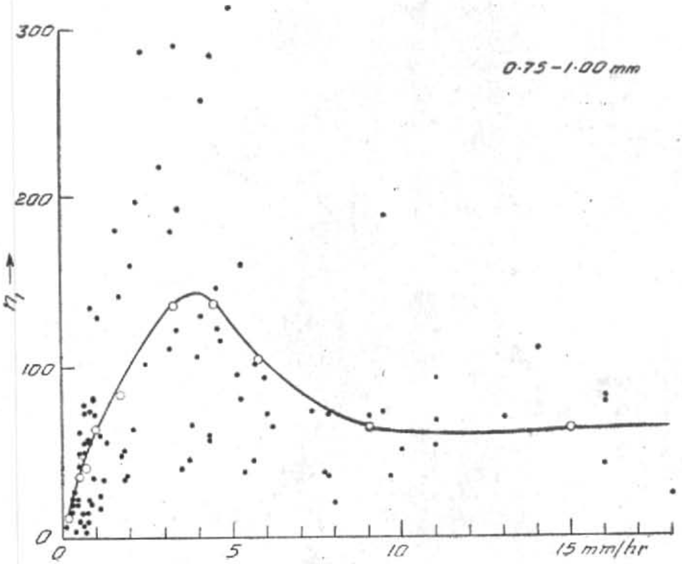
0·75–1·00 mm
n_1 →
0
100
200
300
0
5
10
15 mm/hr

Fig. 3(c)

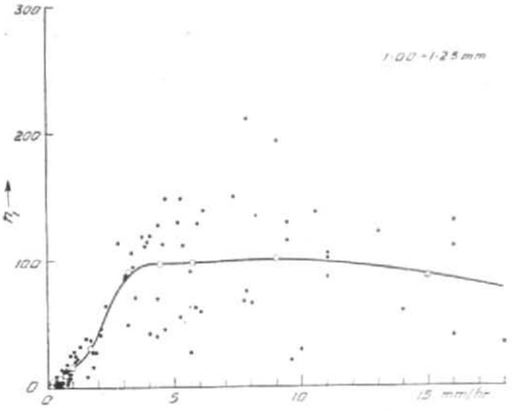

Fig. 3(d)

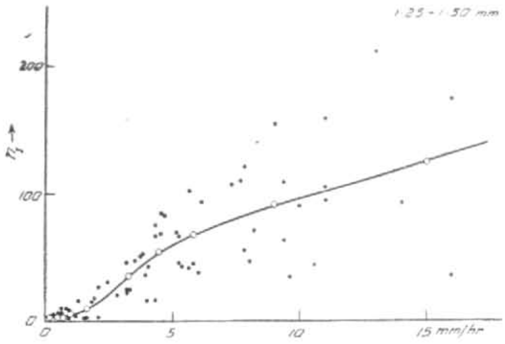

Fig. 3(e)

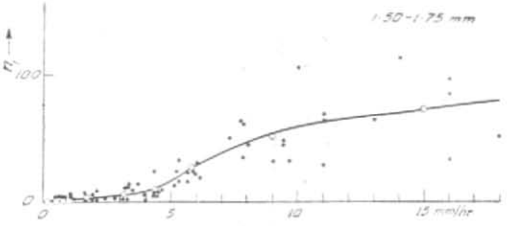

Fig. 3(f)

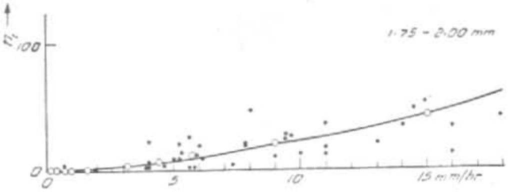

Fig. 3(g)

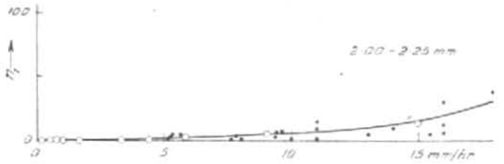

Fig. 3(h)

Figs. 3(a)—3(h). Variation of the number of raindrops of particular diameter group per m^3 of air with intensity of precipitation

maximum at about 3 mm/hr intensity of precipitation as shown in Fig. 3 (b). Fig. 3(c) shows that the number of drops of diameter 0·75—1·00 mm vary systematically with intensity, developing a distinct maximum between 3—4 mm/hr intensity of precipitation. Fig. 3(d) shows that the number of drops of diameter 1·00—1·25 mm rise to a flat maximum slightly diminishing towards higher intensities. Figs. 3(e), 3 (f), 3 (g) and 3(h) show the variation of the number of drops of diameters 1·25–1·50mm, 1·50–1·75 mm, 1·75–2·00 mm, and 2·00–2·25 mm respectively. All these curves are similar in general character having a well developed ankle which extends towards larger intensities for larger diameters, showing that larger drops become significant only at higher intensities of precipitation. Drops of diameter greater than 2·25 mm are always few in number but the frequency of occurrence is greater at larger intensities of precipitation. On the whole the variation from the average are larger for smaller drops and comparatively smaller for larger drops.

The radar response is very sensitive to drop diameter and varies as the sixth power of the diameter. The quantity $Z = \Sigma x^6$ (summation being extended over all the drops in a cubic metre of air) where x is the drop diameter in mm has been calculated for various intensities of precipitation. Fig. 4 is a logarithmic plot of Z against I, which is linear. Average values based on mean of ten records are shown by circles whereas the values from individual records are shown by dots. The scatter of points forms a narrow band rising linearly towards higher intensities. By the method of least squares the relation between Z and I is found to be

$$Z = 189\, I^{1 \cdot 35} \text{ mm}^6/\text{m}^3,$$

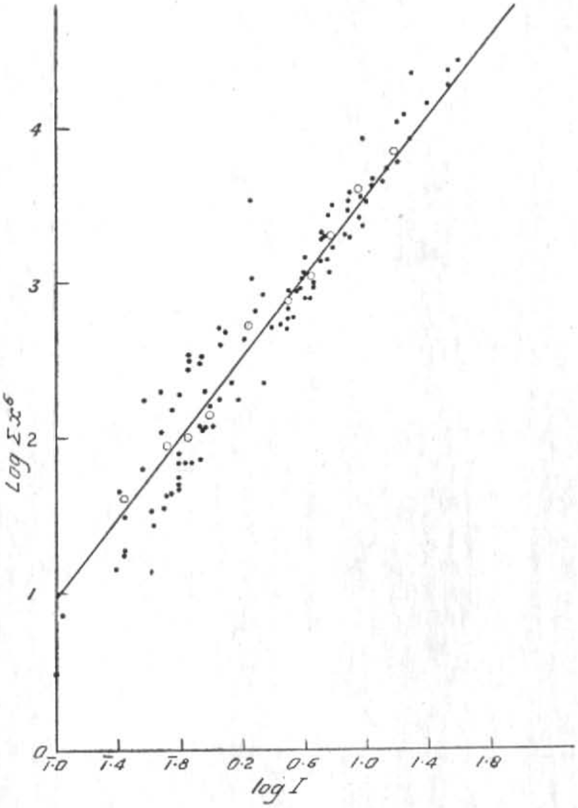

Fig. 4. Radar reflectivity factor $Z=\sum x^6$ as a function of the intensity of precipitation

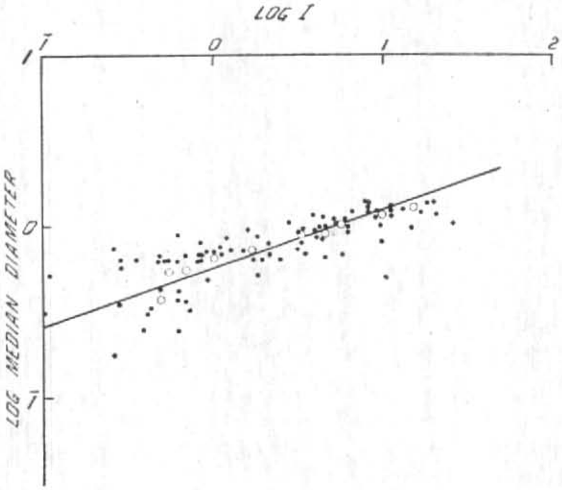

Fig. 5

TABLE 2

Radar reflectivities $\eta = (\pi^4/4\,\lambda^4)\,[(\epsilon-1)/(\epsilon+2)]^2 \times \Sigma\, x^6$ **of rain, of various intensities**

I (mm/hr)	Σx^6 (mm^6/m^3)	η (cm^{-1}) $\lambda = 3 \cdot 2$ cm	η (cm^{-1}) $\lambda = 10$ cm
1	189	$4 \cdot 08 \times 10^{-11}$	$4 \cdot 28 \times 10^{-13}$
2	482	$1 \cdot 04 \times 10^{-10}$	$1 \cdot 09 \times 10^{-12}$
3	832	$1 \cdot 80 \times 10^{-10}$	$1 \cdot 89 \times 10^{-12}$
4	1228	$2 \cdot 66 \times 10^{-10}$	$2 \cdot 79 \times 10^{-12}$
5	1660	$3 \cdot 59 \times 10^{-10}$	$3 \cdot 79 \times 10^{-12}$
6	2125	$4 \cdot 59 \times 10^{-10}$	$4 \cdot 81 \times 10^{-12}$
7	2615	$5 \cdot 65 \times 10^{-10}$	$5 \cdot 92 \times 10^{-12}$
8	3130	$6 \cdot 76 \times 10^{-10}$	$7 \cdot 08 \times 10^{-12}$
9	3668	$7 \cdot 93 \times 10^{-10}$	$8 \cdot 30 \times 10^{-12}$
10	4193	$9 \cdot 15 \times 10^{-10}$	$9 \cdot 57 \times 10^{-12}$
11	4814	$1 \cdot 04 \times 10^{-9}$	$1 \cdot 09 \times 10^{-11}$
12	5401	$1 \cdot 17 \times 10^{-9}$	$1 \cdot 23 \times 10^{-11}$
13	6017	$1 \cdot 30 \times 10^{-9}$	$1 \cdot 36 \times 10^{-11}$
14	6661	$1 \cdot 44 \times 10^{-9}$	$1 \cdot 51 \times 10^{-11}$
15	7320	$1 \cdot 58 \times 10^{-9}$	$1 \cdot 66 \times 10^{-11}$

It may be useful to compare corresponding results obtained by other workers. Marshall and Palmer (1948) have obtained the relation

$$Z = 220\, I^{1 \cdot 60} \text{ mm}^6/\text{m}^3.$$

Wexler (1948) from Anderson's data (1948) obtains

$$Z = 209\, I^{1 \cdot 53} \text{ mm}^6/\text{m}^3.$$

Laws and Parsons (1943) give

$$Z = 228\, I^{1 \cdot 43} \text{ mm}^6/\text{m}^3.$$

Twomey (1953) obtains

$$Z = 127\, I^{2 \cdot 29} \text{ mm}^6/\text{m}^3.$$

Of course no agreement is to be expected as the results should depend upon meteorological conditions.

For ready reference the radar reflectivities

$$\eta = (\pi^4/4\lambda^4)\ [(\epsilon-1)/(\epsilon+2)]^2\ \Sigma\, x^6$$

of rain at various intensities have been calculated and are given in Table 2 for two different wavelengths, $\lambda = 3 \cdot 2$ cm and $\lambda = 10$ cm. The value of $[(\epsilon-1)\,/\,(\epsilon+2)]^2$ is taken to be $0 \cdot 93$ both for $\lambda = 3 \cdot 2$ cm and $\lambda = 10$ cm as given by Mason (1957).

The median drop diameters have been obtained from the drop frequency curves and Fig. 5 is a logarithmic plot of the median diameter in mm against the intensity of precipitation and is seen to be linear. The figure is based on averages of ten records shown by circles and individual points shown by dots. The scatter diagram is a band fairly following the average graph. This median drop diameter is that diameter above and below which the number of drops per m^3 of air is equally divided. By the method of least squares the relation obtained is

$$d_{\text{median}} = 0 \cdot 6\, I^{0 \cdot 28} \text{ mm}.$$

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