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The structure of the Twilight Ray in different spectral regions

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Bigg's report (Bigg 1959) on his having distinctly detected isolated stratiform cloud, situated at an altitude of 10 km, with the help of twilight observations using 6000-8000 Å light is of great interest.

In the present article we should like to discuss some questions of twilight phenomena theory, and in particular, to touch on the problem of solar radiation absorption by water vapour in the earth's atmosphere.

1. Let us first consider the dependence of height of an effectively scattering layer h_{eff} on the wave length λ.

The formula given in the work of one of us (Megrelishvili 1958) is

$$h_{\text{eff}} = h + \frac{H_0}{\sin Z} \qquad (1)$$

and approximate value of $H_0=20$ km refer to the case, when observations are fulfilled in zenith and in visible spectral region.

Staude (1936) pointed out that in a clear atmosphere (Rayleigh scattering) H_0 decreases with the increase of λ, and at about $\lambda = 1\,\mu$ it becomes practically equal to zero.

Consequently, for a sufficiently great λ it may be such a type of earth shadow in which "there is a large discontinuous jump in illumination at the lower boundary, with a slow continuous increase above" (Bigg 1959).

Such a structure of the earth shadow can be easily found out by means of observations made near the horizon, in the direction of the sun.

Our previous article (Megrelishvili 1958) concerned observations carried out in the zenith, therefore, we shall consider, in addition, the question of the structure of the earth's shadow under conditions of observation near the horizon (at a height of 20°).

Let us take AC (Fig. 1) for the earth's surface, and O for its centre. The observer is at C, observing the brightness of the twilight sky in the direction of CE. We suppose, that CE is situated in the vertical of the sun and it is directed to a zenith angle γ.

Let us consider the ray SS_1 proceeding from below the observer's horizon (Z is the zenith distance of the sun). The shortest distance between the ray and earth level being $AB=x$. This ray illuminates point E of the atmosphere, situated in the direction of CE, we are interested in.

Point E is situated above the earth level at the altitude $KE=y$. Let us calculate attenuation of the sun ray SS_1, on its way through the earth's atmosphere, due to molecular scattering of light.

If the ray has passed through the entire atmosphere (points S and S_1 being beyond the atmosphere), then the total number of molecules N on the way of ray may be approximately computed according to Hulburt's formula (Hulburt 1938)—

$$N_x = n_x\,[\,2\pi\,(\,r + x\,)/p\,]^{\frac{1}{2}} \qquad (2)$$

where $n_x = n_0 e^{-px}$

$$n_0 = 2{\cdot}61 \times 10^{19}\ \text{cm}^{-3}.$$

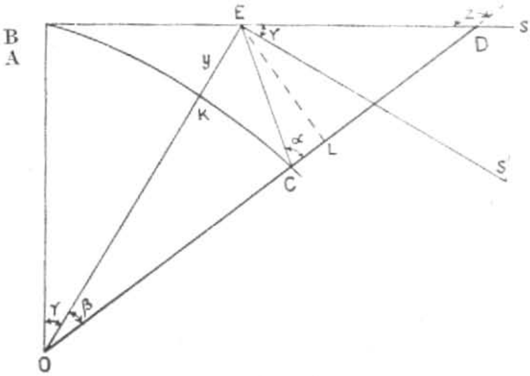

Fig. 1

We are interested in the ray, which has passed not the entire atmosphere, but only part of it, SE.

The number of molecules N_E on the way of such a ray can be represented as follows—

$$N_E = N_x - N' \qquad (3)$$

where N' is the number of molecules on the way of ray ES_1. In order to calculate N', let us consider the subsidiary ray ES′, whose direction makes an angle of 90° with OE.

Then according to the structure, the angle $S_1ES' = \gamma$ (γ and β are the angles with their apexes at point O as is shown in Fig. 1). The number of molecules N_y in the way of ray ES′ according to formula (2) may be expressed as follows—

$$N_y = \tfrac{1}{2} n_y [\,2\pi(r + y)/p\,]^{\frac{1}{2}} \qquad (4)$$

where $y = KE$ (Fig. 1)

$$n_y = n_0\, e^{-py}\,;$$

further we have : $N' = N_y / F_B(\gamma)$

where $F_B(\gamma)$ is the numerical value of the function of the angle γ which is equal to the ratio of Bemporad's function B (90°) for the zenith angle 90°, to the function of Bemporad $B(90° - \gamma)$. Thus, we have finally—

$$N_E = N_x - N_y / F_B\ (\gamma)$$

$$= N_x - N_y\, \frac{B\,(90° - \gamma)}{B\,(90°)}. \qquad (5)$$

With these values of angle α and the earth radius r, the angles β and γ depend on Z and x in the following way :

$$\beta + \gamma = Z - \pi/2$$

$$\tan \alpha = \frac{(r + x) \sin \beta}{r \cos \beta - \cos \gamma + x \cos \beta}\,.$$

The latter formula is obtained from a consideration of triangles OBE, OEL and CEL, where EL is the subsidiary line perpendicular to OD.

The altitude of point E above the earth surface y can be calculated according to the formula—

$$y = \frac{r\,(1 - \cos \gamma) + x}{\cos \gamma}.$$

When $x=0$, this formula obtained from a consideration of triangle OBE, will give the altitude of the earth shadow h, at observer's zenith K :

$$h = y_{x=0} = \frac{r\,(1 - \cos \gamma)}{\cos \gamma}. \qquad (6)$$

The above formulæ allow us to take into account attenuation of the sun ray SE passing through the atmosphere till the required point E.

We neglect the attenuation of light, scattered at point E, towards EC on its way to the observer, for it is comparatively small.

The intensity of light i_s scattered at point E is calculated according to the formula (Hulburt 1938)—

$$i_s = i_\lambda\, n\, S_\lambda,$$

where $$S_\lambda = 2\,\pi^2\, \alpha_\lambda^2 / \lambda^4$$

$$\alpha_\lambda = (\beta_\lambda - 1)\, /\, n,$$

the value of β_λ were taken from Table 4 given in the paper by H.C. Van de Hulst in the book edited by Kuiper (Kuiper 1947).

To calculate i_λ the intensity of the rays, illuminating the atmosphere at point E, the following equation has been used—

$$i_\lambda = i_0 e^{-\tau_2 l} \qquad (7)$$

The optical thickness of the atmosphere τ_2 reduced to the sea level can be represented as follows : $\tau_2 = \tau_1 \; H/6\cdot 44$, τ_1 is the optical thickness of atmosphere above Mount Wilson, $H=8\cdot 00$ km height-scale at the sea level, $6\cdot 44$ is the same above Mount Wilson. The optical way (path) l of the ray SE up to point E can be equalled with ratio $l = N_E/n_0 H$, in which $n_0 = 2\cdot 61\times 10^{19}$ cm^{-3}, N_E is found according to formula (5). In formulæ (2) and (4) we regard $p=0\cdot 125$ km^{-1}.

2. Using the above formulæ, we have calculated the intensity of the scattered light i_s as a function of the height y with various zenith distances of the sun Z and various wavelengths λ.

These calculations made for $\alpha = 70°$ correspond to the conditions of Bigg's observation (Bigg 1959).

The values of i_s thus obtained for five various λ (from $0\cdot 4$ to $1\cdot 0\mu$) and six values of Z (from $Z=91°\cdot 6$, when the earth shadow height $h=y_{x=0}=1\cdot 5$ km to $Z=94°\cdot 2$ with $h=15$ km) are represented in Table 1. These data allow us to investigate the structure of 'twilight ray' and some of the earth shadow under various conditions.

As it can be seen from this table, the altitude h_{eff} with maximal intensity of scattered light exceeds the altitude of the earth shadow h at 10-22 km for the wavelength $0\cdot 4$ to $0\cdot 7\mu$. Between $0\cdot 7\,\mu$ and $1\cdot 0\,\mu$ for this case of observation near the horizon $\alpha=70°$ may indeed be noticed a transition from one type of earth shadow which varies continuously (the first type), to another type of earth shadow in which there is a large discontinuous jump in illumination of the lower boundary (the second type).

In order, to clear up more definitely at which wavelengths the transition of earth shadow from one type to the other takes place we made analogous calculations for $Z=93°\cdot 4$ and for a greater number of λ values, namely, from $0\cdot 4$ to $1\cdot 0\,\mu$ after each $0\cdot 1\mu$.

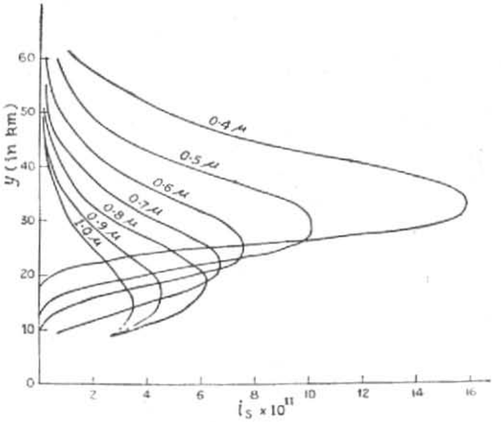

Fig. 2

(The zenith distance of the sun $Z = 93°\cdot 4$ has been chosen taking into consideration that value of $h = y_{x=0} = 9$ km corresponds to it at about 10 km. At this altitude Bigg distinctly found out isolated stratiform cloud). The results of calculations are given in Fig. 2 and concisely in Table 2.

3. The curves (Fig.2) in their lower parts (below maximum) show the structure of earth shadow. According to these data the transition from the first type of shadow to the second one takes place within the $0\cdot 7$-$0\cdot 8\mu$ region.

However, we must remember that the absorption of solar radiation in water vapour bands had yet not been taken into account in our previous calculations and it will greatly change the result. As a matter of fact just about $\lambda = 0\cdot 7\mu$ strong absorption by water steam begins.

This can be seen from the curves in Fig. 3 which we reproduce from Foitzik's and Hinzpeter's book (Foitzik and Hinzpeter 1958). Curve 1 represents outside atmospheric distribution of energy in the spectrum of the sun; curve 2 shows the same after passing through the ozonesphere (absorption in the band of Chappius); curve 3 represents the same after molecular scattering; curve 4 considers additional attenuation with aerosol particles and curve 5 beside the above factors

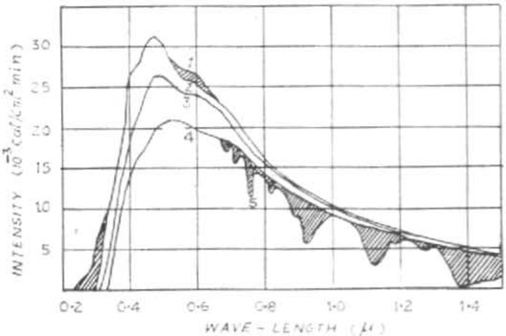

Fig. 3

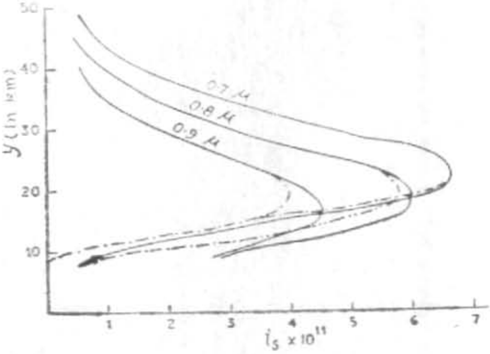

Fig. 4

(absorption by ozone, molecular and aerosol scattering of light) takes into account absorption by water bands. (All the curves are given for the vertical path of the sun rays $Z = 0$ direction).

For a rough evaluation of the discussed effect, we may suppose, that in the region $0{\cdot}7 \leqslant \lambda \leqslant 0{\cdot}8\,\mu$ the vertical ray is attenuated in the atmosphere at an average of 5 per cent owing to absorption by water vapour. (The shaded bands between curves 4 and 5 in the region $0{\cdot}7$–$0{\cdot}8\mu$). That is the ray SE, tangent to the earth surface (x=0. Fig. 1) is attenuated in the atmosphere approximately $1/0{\cdot}95^{40} \cong 7{\cdot}8$ times. Analogously taking the attenuation of the vertical rays for the region $0{\cdot}8\,\lambda < 1\mu$ to be 10 per cent on an average, we see, that the tangent ray SE is attenuated approximately $1/0{\cdot}90^{40} \cong 67$ times.

It is known, that in the free atmosphre the elasticity of water vapour l_x falls with the altitude x considerably quicker, than the denisty n_x does. According to aerological data l can be represented approximately by formula :

$$l_x = l_0\, 10^{-qx} \tag{8}$$

where $q = 0{\cdot}2$ km^{-1}.

Vapour elasticity falls 10 times when x=5 km and 100 times when x=10 km (in comparison with the level x=0). Thus absorption in the bands of H_20 will affect only the lower part of the curves (Fig. 2).

It is easy to see, that this effect changes the structure of earth shadow, transferring the second type of shadow into the first one. The curves in Fig. 4 are plotted for three wave lengths ($0{\cdot}7\mu$, $0{\cdot}8\mu$, $0{\cdot}9\mu$). They are re-computed taking into account absorption of rays by atmospheric water vapour. For this re-computation, we took as fundamental curves those shown in Fig. 2 for these wavelengths. (They are represented for comparison by solid lines in Fig. 4). We have also considered, that the ray SE (Fig. 1) tangent to the earth surface (x=0), is attenuated 7·8 times if, $\lambda = 0{\cdot}7$ and $0{\cdot}8\mu$, or 67 times if $\lambda = 0{\cdot}9\mu$. In particular, these curves, inspite of their tentative character, show that h_{eff} differs from the height of earth shadow h at about 9–14 km for the region $0{\cdot}6$–$0{\cdot}8\mu$ used by Bigg for measurement (Bigg 1959). Without taking into account the following correction $\triangle$

$$\triangle = h_{\text{eff}} - h \tag{9}$$

one can not compare twilight observations with radiosonde readings. As for the second curve, mentioned in Bigg's article for the spectral region 1-3μ, the presence of strong absorption bands are to be noted also (see Fig. 3).

One can, however, select a narrow band of the spectrum of about 1μ (approximately $1{\cdot}01$—$1{\cdot}03\mu$), where there is practically no absorption. Twilight observations by means of photometer provided with an interference light-filter, would not require corrections according to formula (9).

4. As we have already noted at the beginning of the article, Bigg's discovery of isolated stratiform cloud using 6000—8000Å light is of great interest. We think that besides the necessity of taking into account the corrections △ (formula 9) the question of aerosol layer altitude found out by means of twilight observations requires additional consideration. This correction can be introduced if it is calculated beforehand.

However, measurements in the infra-red region of the spectrum as Bigg did, require a more accurate computation of absorption by water vapour.

The calculations represented above, can be only considered as preliminary.

There may be another way—the experimental one. For instance, by using search light probing technique aerosol layer altitude may be directly measured (Khvostikov 1945). Local measurements at various altitudes by means of apparatus taken up in an airplane could also give direct answer. Some measurements of aerosol layers by two independent methods taken parallel at about the same time would be of great interest and would help to make more accurate the most promising method of aerosol layer investigations by means of twilight observations.

In theoretical part of this problem, it would be desirable, beside a more exact determination of the correction of △ (see formula 9) taking into account absorption of radiation by atmospheric humidity, to consider also the theory of twilight phenomena in optically heterogeneous atmosphere.

The matter is, that in theory of twilight observations an optically homogeneous atmosphere is usually taken into consideration; the scattering power of it changes monotonously with height; *e.g.*, according to the law $n_x = n_0 e^{-px}$ used in our calculations (see formula 2).

However it may be easily seen, that the appearance in the atmosphere of an aerosol layer of limited height transfers the atmosphere into such a medium, which (in the sense, mentioned above) is optically heterogeneous.

In this case some results of the present theory of twilight phenomena may require revision. This problem will be discussed by us in another paper.

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TABLE 1

x	y	i_s				
(km)	((km)	$\lambda=0\cdot4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=1\cdot0\mu$
		$Z=91^\circ\cdot6$				
0	1·5	$2\cdot13\times10^{-15}$	$1\cdot51\times10^{-12}$	$1\cdot30\times10^{-11}$	$6\cdot54\times10^{-11}$	$1\cdot00\times10^{-10}$
2	3·5	2·26	1·61	1·50	7·94	1·08
3	4·5					1·12
4	5·5	$6\cdot85\times10^{-14}$	8·32	3·92	$1\cdot22\times10^{-10}$	1·12
5	6·5					1·12
6	7·5	$7\cdot78\times10^{-13}$	$2\cdot70\times10^{-11}$	7·57	1·61	1·09
7	8·5					1·05
8	9·5	$4\cdot90\times10^{-12}$	6·36	$1\cdot20\times10^{-10}$	1·88	
9	10·5				1·97	$9\cdot56\times10^{-11}$
10	11	$2\cdot25\times10^{-11}$	$1\cdot29\times10^{-10}$	1·76	2·16	
11	12					8·96
12	13·5	6·36	1·91	2·03	2·05	
13	14			2·22		7·58
14	15	$1\cdot35\times10^{-10}$	2·58	2·35	2·02	
15	16			2·30		6·32
16	17	2·20	2·94	2·29	1·81	5·20
17	18					5·20
18	19	3·19	3·17	2·19	1·59	
19	20		3·10			4·22
20	21	3·76	3·08	1·97	1·34	
21	22					3·40
22	23	4·12	2·88	1·73	1·11	
23	24	4·29				2·71
24	25	4·35	2·62	1·48	$9\cdot21\times10^{-11}$	
25	26	4·29				2·15
26	27		2·31	1·25	7·49	
27	28	4·23				1·71
28	29	3·80	1·93	1·01	5·98	
29	30					1·34

TABLE 1 *(contd)*

x (km)	y (km)	i_s				
		$\lambda=0\cdot4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=1\cdot0\mu$
30	31	$3\cdot50\times10^{-10}$	$1\cdot64\times10^{-10}$	$8\cdot40\times10^{-11}$	$4\cdot81\times10^{-11}$	
31	32					$1\cdot06\times10^{-11}$
32	33	2·93	1·33	6·71	3·80	
33	34					$8\cdot29\times10^{-12}$
34	35	2·46	1·08	5·36	3·01	
35						6·50
36	37	2·04	$8\cdot65\times10^{-11}$	4·35	2·36	
37						5·42
38	38·5	1·79	7·43	3·62	2·00	
39						4·23
40	40·5	1·43	5·84	2·82	1·55	
42	42·5	$1\cdot14\times10^{-11}$	4·62	2·23	1·21	2·91
44	44·5	$9\cdot03\times10^{-12}$	3·62	2·04		
45						2·01
46	46·5	7·21	2·85	1·37		
48	48·5	5·70	2·24	1·07		1·14
50	50·5	4·75	1·85	$8\cdot87\times10^{-12}$	$7\cdot82\times10^{-12}$	
54	54·5			7·62		7·23
55	55·5	3·21	1·27			
60	60·5	1·82	$7\cdot79\times10^{-12}$	4·86	2·77	3·61
				$Z=92^\circ\cdot2$		
0	3	$2\cdot56\times10^{-16}$	$5\cdot47\times10^{-13}$	$7\cdot55\times10^{-12}$	$5\cdot57\times10^{-11}$	$7\cdot00\times10^{-11}$
2	5	4·73	6·64	7·87	5·11	8·03
3	6					8·10
5	7·5			$3\cdot56\times10^{-11}$	$1\cdot05\times10^{-10}$	9·09
6	8·5	$3\cdot26\times10^{-13}$	$1\cdot62\times10^{-11}$			9·10
7	9·5			6·87	1·37	8·83
8	10·5	$2\cdot45\times10^{-12}$	4·17			
9	11·5			$1\cdot07\times10^{-10}$	1·57	8·08

TABLE 1 *(contd)*

x (km)	y (km)	i_s $\lambda=0{\cdot}4\mu$	$\lambda=0{\cdot}5\mu$	$\lambda=0{\cdot}6\mu$	$\lambda=0{\cdot}7\mu$	$\lambda=1{\cdot}0\mu$
10	12·5	$1{\cdot}12\times10^{-11}$	$8{\cdot}11\times10^{-11}$		$1{\cdot}58\times10^{-10}$	
11	13·5			$1{\cdot}44\times10^{-10}$	1·67	$7{\cdot}24\times10^{-11}$
12	14·5	3·46	$1{\cdot}31\times10^{-10}$		1·65	
13	15·5			1·62	1·61	6·13
14	16·5	7·73	1·76	1·69	1·57	
15	17·5			1·77		5·21
16	18·5	$1{\cdot}38\times10^{-10}$	2·12	1·74	1·43	
17	19·5			1·72		4·24
18	20·5	2·10	2·28		1·26	
19	21·5		2·38	1·63		3·47
20	22	2·91	2·53		1·15	
21	23		2·51	1·57		2·97
22	24	3·33	2·44		$9{\cdot}69\times10^{-11}$	
23	25	3·48		1·38		2·39
24	26	3·51	2·21		7·96	
25	27			1·02		1·66
26	28					
27	29	3·33	1·80		5·87	1·49
28	30	3·06		$8{\cdot}74\times10^{-11}$		
29	31		1·53		4·74	1·18
30	32	2·86		7·24		
32	34	2·50	1·15	5·83	3·34	$8{\cdot}22\times10^{-12}$
34	36	2·11	$9{\cdot}37\times10^{-11}$	4·69	2·64	6·47
40	42	1·16	4·80	2·33	1·28	3·08
48	49·5	4·92	1·95	$9{\cdot}38\times10^{-12}$	$6{\cdot}29\times10^{-12}$	
60	62	$7{\cdot}80\times10^{-12}$	$3{\cdot}52\times10^{-12}$			
				$Z=92^\circ.6$		
0	5	$8{\cdot}01\times10^{-19}$	$2{\cdot}31\times10^{-14}$	$9{\cdot}21\times10^{-13}$	$1{\cdot}51\times10^{-11}$	$4{\cdot}80\times10^{-11}$
2	6·5		$3{\cdot}34\times10^{-13}$	$4{\cdot}77\times10^{-12}$	3·54	6·18
3	7·5	$1{\cdot}18\times10^{-15}$				

TABLE 1 *(contd)*

x	y	i_s				
(km)	(km)	$\lambda=0\cdot4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=1\cdot0\mu$
				$Z=92°\cdot6$		
4	8·5		$2\cdot12\times10^{-12}$	$1\cdot41\times10^{-11}$	$5\cdot90\times10^{-11}$	$6\cdot67\times10^{-11}$
5	9·5	$3\cdot09\times10^{-14}$				6·79
6	10·5		8·62	3·20	8·44	6·70
7	11·5	$3\cdot81\times10^{-13}$				6·51
8	12·5		$2\cdot31\times10^{-11}$	5·47	$1\cdot02\times10^{-10}$	
9	13	$2\cdot74\times10^{-12}$				6·11
10	14·5		$4\cdot83\times10^{-11}$	8·20	1·16	
11	15·5	$1\cdot08\times10^{-11}$				5·38
12	16·5		8·40	$1\cdot05\times10^{-10}$	1·19	
13	17	3·28				4·93
14	18		$1\cdot28\times10^{-10}$	1·28	1·26	
15	19	6·98		1·30	1·23	4·18
16	20		1·59	1·35	1·14	
17	21	$1\cdot22\times10^{-10}$		1·35		3·46
18	22		1·78	1·32	1·01	
19	23	1·71	1·82	1·29		2·84
20	24		1·85	1·17	$8\cdot78\times10^{-11}$	
21	25	2·14	1·81			2·29
22	26	2·19	1·78		7·36	
23	27	2·44		1·04		
24	28	2·50	1·64		6·13	1·65
25	29	2·50		$8\cdot83\times10^{-11}$		
26	30	2·48	1·45		4·99	
27	30·5					1·24
28	31·5	2·30	1·27	6·91	4·17	
30	33·5	2·21	1·12	5·82	3·42	$8\cdot64\times10^{-12}$
32	35·5	1·91	$9\cdot03\times10^{-11}$	4·62	3·66	
33	36·5					6·02
34	37·5	1·69		3·83	2·18	

TABLE 1 (*contd*)

x (km)	y (km)	i_s $\lambda=0\cdot4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=1\cdot0\mu$
35			$7\cdot32\times10^{-11}$			
37	40	$1\cdot38\times10^{-10}$	$5\cdot93$	$2\cdot90\times10^{-11}$	$1\cdot62\times10^{-11}$	$3\cdot94\times10^{-12}$
39	42	$1\cdot13$	$4\cdot71$	$2\cdot30$		
41	44	$9\cdot06\times10^{-11}$	$3\cdot76$	$1\cdot83$	$1\cdot00$	
43	46	$7\cdot30$	$2\cdot95$		$7\cdot84\times10^{-12}$	$1\cdot87$
45	47·5	$6\cdot22$	$2\cdot48$		$6\cdot54$	
47	49·5	$4\cdot93$	$1\cdot95$	$9\cdot45\times10^{-12}$	$5\cdot11$	$1\cdot22$
55	57·5	$2\cdot35$				
57	60		$6\cdot77\times10^{-12}$	$3\cdot84$	$2\cdot17$	$2\cdot46\times10^{-13}$
60	62·5	$4\cdot99\times10^{-12}$				
				$Z = 93^\circ\cdot2$		
0	7·5	$1\cdot61\times10^{-19}$	$9\cdot67\times10^{-15}$	$4\cdot73\times10^{-13}$	$8\cdot90\times10^{-12}$	$3\cdot24\times10^{-11}$
2	9·5		$1\cdot30\times10^{-13}$	$2\cdot27\times10^{-12}$	$1\cdot98\times10^{-11}$	$3\cdot89$
3	10·5	$3\cdot08\times10^{-15}$				
4	11·5		$9\cdot39$	$7\cdot36$	$3\cdot46$	$4\cdot25$
5	12·5	$9\cdot71\times10^{-14}$				
6	13·5		$4\cdot19\times10^{-12}$	$1\cdot77\times10^{-11}$	$5\cdot12$	$4\cdot39$
7	14·5	$1\cdot41\times10^{-13}$				$4\cdot28$
8	15·5		$1\cdot24\times10^{-11}$	$3\cdot22$	$6\cdot47$	
9	16·5	$1\cdot06$				
10	17		$2\cdot94$	$5\cdot23$	$7\cdot82$	$4\cdot05$
11	18	$5\cdot49\times10^{-12}$				
12	19		$5\cdot23$	$6\cdot92$	$8\cdot29$	$3\cdot50$
13	20	$1\cdot67\times10^{-11}$				
14	20·5		$8\cdot28$	$8\cdot71$	$8\cdot69$	$3\cdot29$
15	21·5	$4\cdot23$			$8\cdot43$	
16	22·5		$9\cdot73$	$8\cdot82$		$2\cdot72$
17	23·5	$6\cdot04$		$9\cdot08$	$7\cdot77$	
18	24·5		$1\cdot20\times10^{-10}$	$9\cdot21$		$2\cdot27$

TABLE 1 *(contd)*

x	y	i_s				
(km)	(km)	$\lambda=0\cdot4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=0\cdot1\mu$
19	25·5	$1\cdot10\times10^{-10}$		$9\cdot01\times10^{-11}$	$6\cdot75\times10^{-11}$	
20	26·5		$1\cdot32\times10^{-10}$			
21	27	1·52	1·35	8·84	6·14	$1\cdot78\times10^{-11}$
22	28		1·33			
23	29	1·75	1·31	7·85	5·16	
24	30	1·81	1·23			1·27
25	31	1·84		6·77	4·25	
26	32	1·84	1·10			
27	33	1·81		5·70	3·47	$8\cdot99\times10^{-12}$
28	34	1·67	$9\cdot33\times10^{-11}$			
29	34·5			5·05	2·99	
30	35·5	1·67	8·40			6·54
31	36·5			4·10	2·38	
32	37	1·62	7·71			
33	38			3·55	2·02	4·99
34	39	1·39	6·29			
35	40		5·67	2·84	1·59	
37	42	1·06	4·58	2·25	1·26	3·07
39	44	$8\cdot71\times10^{-11}$	3·67	1·79	$9\cdot99\times10^{-12}$	
41	46		2·90	1·41	7·76	1·87
43	47·5	6·04	2·45	1·18	6·51	
45	49·5	4·82				
46	50	4·55	1·94	$9\cdot36\times10^{-12}$	5·09	1·22
50	55	2·89	1·17			
55	60		$7\cdot79\times10^{-12}$	3·00	1·67	$5\cdot81\times10^{-13}$
60	65	$9\cdot64\times10^{-12}$		2·20		
				$Z=93^\circ\cdot8$		
0	12	$3\cdot83\times10^{-20}$	$3\cdot13\times10^{-15}$	$1\cdot86\times10^{-13}$	$4\cdot17\times10^{-12}$	$1\cdot68\times10^{-11}$
2	13·5	$1\cdot02\times10^{-17}$	$5\cdot35\times10^{-14}$	$1\cdot10\times10^{-12}$	$1\cdot05\times10^{-11}$	2·25

TABLE 1 (*contd*)

x	y	i_s				
(km)	(km)	$\lambda = 0{\cdot}4\mu$	$\lambda = 0{\cdot}5\mu$	$\lambda = 0{\cdot}6\mu$	$\lambda = 0{\cdot}7\mu$	$\lambda = 1{\cdot}0\mu$
				$Z = 93^\circ.8$		
3	14·5					$2{\cdot}37\times 10^{-11}$
4	15·5	$6{\cdot}41\times 10^{-16}$	$4{\cdot}11\times 10^{-13}$	$3{\cdot}62\times 10^{-12}$	$1{\cdot}86\times 10^{-11}$	2·45
5	16					2·69
6	17	$1{\cdot}77\times 10^{-14}$	$2{\cdot}13\times 10^{-12}$	9·76	3·03	2·71
7	18					2·66
8	19	$1{\cdot}96\times 10^{-13}$	6·68	$1{\cdot}85\times 10^{-11}$	3·89	
10	21	$1{\cdot}22\times 10^{-12}$	$1{\cdot}54\times 10^{-11}$	2·89	4·50	
12	22·5	5·20	3·01	4·16	5·12	2·27
14	24·5	$1{\cdot}41\times 10^{-11}$	4·50	4·86	5·03	
15						1·92
16	26	3·19	6·30	5·62	5·05	
18	28	5·34	7·35	5·73	4·57	
19	29			5·62		1·31
20	30	7·55	7·75	5·44	3·97	
22	31·5	9·78	8·19	5·24	3·59	
23	32·5		8·00			$9{\cdot}12\times 10^{-12}$
24	33·5	1·10		4·64	3·00	
25	34·5	1·11				
26	35·5	1·10	6·74	3·84	2·38	
27	36					6·17
28	37	1·10	6·31	3·48	2·11	
30	39	1·09	5·49	2·91	1·71	
31	39·5					4·09
32	40·5	1·02	4·96	2·54	1·46	
34	42·5	8·73	4·00	2·02	1·45	
35						2·70
36	44	7·90	3·48	1·74	$9{\cdot}74\times 10^{-12}$	
38	46	6·52	2·79	1·37	7·65	
39						1·65

TABLE 1 *(contd)*

x (km)	y (km)	i_s $\lambda=0\cdot4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=1\cdot0\mu$
40	48	$5\cdot33\times10^{-11}$	$2\cdot33\times10^{-11}$	$1\cdot09\times10^{-11}$	$6\cdot03\times10^{-12}$	
42	49·5	4·60	1·89	$9\cdot14\times10^{-12}$		$1\cdot23\times10^{-12}$
47	55			5·99		
50	58	1·89				
52	60		$8\cdot66\times10^{-12}$	3·84	2·21	$6\cdot41\times10^{-13}$
57	65		5·72			
60	68	$6\cdot33\times10^{-12}$				
				$Z=94°.2$		
0	15	$1\cdot90\times10^{-20}$	$1\cdot79\times10^{-15}$	$1\cdot14\times10^{-13}$	$2\cdot68\times10^{-12}$	$1\cdot15\times10^{-11}$
2	16·5	$5\cdot14\times10^{-18}$	$3\cdot09\times10^{-14}$	6·61	6·67	1·49
4	18	$3\cdot84\times10^{-17}$	$2\cdot72\times10^{-13}$	$2\cdot48\times10^{-12}$	$1\cdot31\times10^{-11}$	1·77
6	20	$9\cdot86\times10^{-15}$	$1\cdot31\times10^{-12}$	6·25	1·99	1·83
8	21·5	$1\cdot25\times10^{-13}$	4·54	$1\cdot29\times10^{-11}$	2·79	1·93
10	23·5	$7\cdot93\times10^{-12}$	1·06	2·03	3·22	1·74
12	25	3·37	2·10	2·94	3·68	1·66
14	27	9·65	3·18	3·51	3·63	
15						1·31
16	28·5	$2\cdot21\times10^{-11}$	4·49	4·13		
18	30·5	3·69	5·21	4·13	3·36	1·06
19	31·5		5·45	4·06		
20	32	5·63	5·95		3·13	
21	33		5·98	4·01		$8\cdot27\times10^{-12}$
22	34	6·96	5·90	3·80	2·61	
23	35		5·79			
24	35·5	8·16		3·48	2·26	
25	36·5	8·51				5·65
26	37·5		5·37	3·07	1·91	
27	38·5	8·60				
28	39	8·55	4·88	2·69	1·64	

TABLE 1—(*contd*)

x	y	i_s				
(km)	(km)	$\lambda=0{\cdot}4$	$\lambda=0{\cdot}5\mu$	$\lambda=0{\cdot}6\mu$	$\lambda=0{\cdot}7\mu$	$\lambda=1{\cdot}0\mu$
				$Z=\mathbf{94}^\circ{\cdot}2$		
29	40					$3{\cdot}78\times10^{-12}$
30	41	$8{\cdot}26\times10^{-11}$	$4{\cdot}25\times10^{-12}$	$2{\cdot}25\times10^{-11}$	$1{\cdot}33\times10^{-11}$	
32	42·5	7·39	3·58	1·84	1·06	
33	43·5					2·51
34	44·5	6·84	3·14	1·59		
35	45·5				$8{\cdot}02\times10^{-12}$	
36	46	6·10	2·70	1·34		
37	47				6·70	1·64
38	48	5·05	2·17	1·07		
39	49				5·35	
40	49·5	4·44	1·85	$9{\cdot}06\times10^{-12}$		1·21
45	55		1·11			
50	60	1·35	6·18	3·74	2·21	$5{\cdot}91\times10^{-13}$
55	65	$8{\cdot}05\times10^{-12}$	4·36			
60	70	5·23				

TABLE 2

x	y	i_s						
(km)	(km)	$\lambda=0\cdot4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=0\cdot8\mu$	$\lambda=0\cdot9\mu$	$\lambda=1\cdot0\mu$
0	9	$1\cdot26\times10^{-19}$	$6\cdot97\times10^{-15}$	$3\cdot56\times10^{-13}$	$7\cdot09\times10^{-12}$	$2\cdot87\times10^{-11}$	$2\cdot75\times10^{-11}$	$2\cdot62\times10^{-11}$
2	11	$2\cdot14\times10^{-18}$	$9\cdot02\times10^{-14}$	$1\cdot69\times10^{-12}$	$1\cdot54\times10^{-11}$	3·69	3·57	3·14
4	13	$1\cdot28\times10^{-16}$	$6\cdot84\times10^{-13}$	5·63	2·74	5·14	4·22	3·48
5								3·53
6	15	$2\cdot86\times10^{-14}$	$3\cdot10\times10^{-12}$	$1\cdot35\times10^{-11}$	4·04	5·88	4·51	3·52
7								3·47
8	17	$3\cdot04\times10^{-13}$	$1\cdot06\times10^{-11}$	2·53	5·19	6·15	4·51	
9								3·27
10	18·5	$2\cdot06\times10^{-12}$	2·40	4·35	6·20		4·73	
11						6·31	4·35	3·14
12	20·5	7·56	4·12	5·55	6·70	5·99	4·12	
14	22	$2\cdot18\times10^{-11}$	6·60	7·01		5·74	3·86	2·69
15	23	3·21						
16	24	4·44	8·45	7·53	6·59	4·98	3·29	2·26
17	25	5·40		7·59				
18	25·5	7·74	$1\cdot04\times10^{-10}$		6·31	4·50	2·93	2·00
20	28			7·13	5·16	3·52	2·26	
21	28·5	$1\cdot22\times10^{-10}$	1·10					1·47
22	29		1·15			3·29	2·09	
23	30	1·50	1·12	6·89				
24	31	1·56	1·08		4·14	2·64	1·68	1·20
25	32	1·58		5·96				
26	33	1·59	$9\cdot66\times10^{-11}$		3·39	2·12	1·34	
27	33·5	1·59		5·38				
28	34·5	1·57	8·78		2·90	1·80	1·13	$7\cdot52\times10^{-12}$
29	35·5	1·52		4·45				
30	36·5	1·50	7·62		2·35	1·43	$8\cdot98\times10^{-12}$	
31	37			3·85				5·61
32	38		6·76		2·00	1·20	7·55	

TABLE 2 *(contd)*

x	y	i_s						
(km)	(km)	$\lambda=0.4\mu$	$\lambda=0\cdot5\mu$	$\lambda=0\cdot6\mu$	$\lambda=0\cdot7\mu$	$\lambda=0\cdot8\mu$	$\lambda=0\cdot9\mu$	$\lambda=1\cdot0\mu$
33	39	$1\cdot32\times10^{-10}$		$3\cdot11\times10^{-11}$				
34	40		$5\cdot53\times10^{-11}$		$1\cdot59\times10^{-11}$	$9\cdot49\times10^{-12}$	$5\cdot91\times10^{-12}$	$5\cdot06\times10^{-12}$
35	41	1·12		2·49				
36	42		4·48		1·24	7·45	4·64	
37	43	$9\cdot32\times10^{-11}$		2·00				2·73
38	43·5		3·84		1·03	6·23	3·89	
39	44·5	8·20		1·69				
40	45·5		3·07		$8\cdot27\times10^{-12}$	4·87	3·03	1·99
41	46	7·04		1·41				
42	47			1·25	6·87	4·05	2·52	
43	48	5·66	2·30					
44	49				5·41			1·29
45	50	4·52	1·81	$8\cdot71\times10^{-12}$		3·22	1·74	
50	55	2·89	1·15		3·75	2·72	1·48	1·01
55	60	1·40	$6\cdot51\times10^{-12}$	3·84	2·21	1·61	$9\cdot01\times10^{-13}$	$6\cdot41\times10^{-13}$
60	65	$9\cdot63\times10^{-12}$			1·56			
65	70	6·76	3·10	1·73				