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Seiches in a conical lake of large semi-vertical angle

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ABSTRACT. The problem of seiche-oscillations in lakes having the shape of solids of revolution has been considered by Lamb for the particular case of a paraboloidal lake.

This paper deals with a right conical lake of large semi-vertical angle, with and without a circular cylindrical coaxial island. The differential equation of seiche-oscillations, expressed in the vector form is solved by a variational method, first used by Hidaka for seiche-problems. Period equations have been obtained for those modes of vibration, involving nodal diameters and nodal circles. The results obtained for a laks with a central island are applied to Lake Toya, Japan.

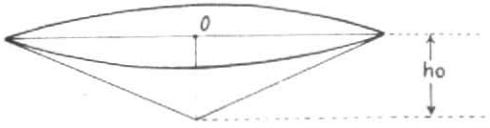

Fig. 1. Conical Lake

3. Solution of the seiche-equation for a conical lake

The type of lake considered is depicted in Fig. 1. For a right circular cone, the depth at (r, θ) is a function of axial distance r only, given by the law—

$$h = h_0 \left(1 - \frac{r}{a} \right),$$

the origin being at the point of intersection of the axis of the cone with the surface. Here h_0 is the central depth of the lake and a the radius of the surface. Equation (2) can now be put into the form—

$$h \nabla^2 \zeta + (\text{grad } \zeta) \cdot (\text{grad } h) - \frac{1}{g} \frac{\partial^2 \zeta}{\partial t^2} = 0$$

$$i.e., \quad \text{div } (h \text{ grad } \zeta) - \frac{1}{g} \frac{\partial^2 \zeta}{\partial t^2} = 0 \qquad (3)$$

If $\zeta \propto \cos \sigma t$, equation (3) reduces to

$$\text{div } (h \text{ grad } \zeta) + k^2 \zeta = 0 \qquad (4)$$

where $k^2 = \sigma^2/g$.

Equation (4) has to be solved under the boundary condition

$$h \frac{\partial \zeta}{\partial r} = 0 \qquad (5)$$

at the periphery of the lake's surface.

Syono (1938) has shown that the solution of div $(h \text{ grad } \zeta) + k^2 \zeta = 0$ under the boundary condition $h . \partial \zeta / \partial r = 0$ equivalent to the problem of making—

$$\left. \begin{array}{l} F = \iint h \, (\text{grad } \zeta)^2 \, dS, \\ \text{a minimum subject to the condition} \\ G = \iint \zeta^2 \, dS = 1 \end{array} \right\} (6)$$

Here dS denotes an element of area of the lake's surface.

Eqn. (6) requires that —

$$\frac{\partial F}{\partial D_i} - k^2 \frac{\partial G}{\partial D_i} = 0 \qquad (7)$$

where the D_i's are parameters occurring in the trial function

$$\zeta = \sum_i D_i \zeta_i$$

In polar co-ordinates

$$(\text{grad } \zeta)^2 = \left(\frac{\partial \zeta}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial \zeta}{\partial \theta} \right)^2$$

and $\quad dS = r \, dr \, d\theta$

Therefore.

$$F = \int_0^{2\pi} \int_0^a h_0 \left(1 - \frac{r}{a} \right) \times \left\{ \left(\frac{\partial \zeta}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial \zeta}{\partial \theta} \right)^2 \right\} r \, dr \, d\theta \qquad (8)$$

$$G = \int_0^{2\pi} \int_0^a \zeta^2 \, r \, dr \, d\theta$$

If for a fixed r and fixed t, $\zeta \propto \cos m\theta$, the solution of (7) will be of the form

$$\zeta = f(r) \cos m \theta \cos \sigma t$$

so that the seiche-oscillation would have m nodal diametral lines. Further if $f(r)$ vanishes n times between 0 and a there would be n nodal circles between 0 and a. A mode of oscillation with m nodal diameters and n nodal circles would break up into $2m(n+1)$ cells if m and n are both non-zero and positive but into $(n+1)$ cells if $m=0$ or into $2m$ cells if $n=0$. The case $m=0$, $n=0$ is of course ruled out. Consecutive cells of a zone bounded by 2 nodal circles will oscillate in opposite phases and this zone as a whole would be oscillating in opposite phase to an adjacent zone.

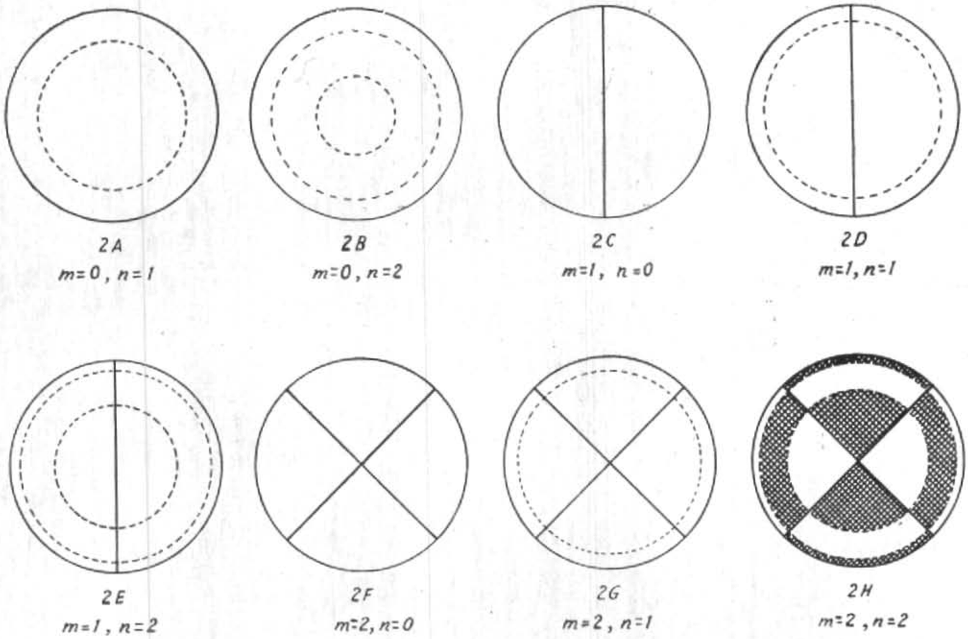

Fig. 2 A–2H. Positions of the Nodal circles in the case of Conical Lake with and without Nodal diameters

A few typical examples are illustrated in Figs. 2(A) to 2(H). For example, the number of cells into which the surface is divided for the case $m=2$, $n=2$ is 12. Those cells which are shown hatched in Fig. 2 (H) oscillate in the same phase, while those left unhatched oscillate in the opposite phase.

The boundary condition

$$\left(h \frac{\partial \zeta}{\partial r} \right)_{r=a} = 0$$

implies that if $h=0$ at $r=a$, then

$$\left(\frac{\partial \zeta}{\partial r} \right)_{r=a}$$

must be finite and non-zero. Further ζ must be finite at the origin $r=0$. We therefore choose

$$f(r) = \sum_{i=p}^{\infty} A_i \left(\frac{r}{a} \right)^i$$

where the values of p are as yet undetermined. Under this assumption we have

$$\zeta = \sum_{i=p}^{\infty} A_i \left(\frac{r}{a} \right)^i \cos m\,\theta \cos \sigma t \qquad (9)$$

Substitution of (9) in (2), gives for the lowest power of r/a, *i.e.*, for $(r/a)^{p-2}$ the coefficient $A_p\,(p^2/a^2 - m^2/a^2)$. Since $A_p \neq 0$, this can vanish only if $p = \pm m$. Ruling out $p = -m$ as ζ has to be finite at the origin, we have—

$$\zeta = \sum_{i=m}^{\infty} A_i \left(\frac{r}{a} \right)^i \cos m\theta \cos \sigma t \qquad (10)$$

(*i*) As a first step, we take for ζ the form—

$$\zeta = \left[A_m \left(\frac{r}{a}\right)^m + A_{m+1} \left(\frac{r}{a}\right)^{m+1} + A_{m+2} \left(\frac{r}{a}\right)^{m+2} \right] \cos m\,\theta \cos \sigma\, t \qquad (11)$$

Substituting (11) in (8), we obtain, after integration

$$F = h_0\, \pi \cos^2 \sigma\, t \left[A^2_m \left\{ \frac{m}{2m+1} \right\} + A^2_{m+1} \left\{ \frac{(m+1)^2 + m^2}{(2m+2)(2m+3)} \right\} + A^2_{m+2} \left\{ \frac{(m+2)^2 + m^2}{(2m+4)(2m+5)} \right\} + A_m\, A_{m+1} \left\{ \frac{2m}{2m+2} \right\} + A_{m+1}\, A_{m+2} \left\{ \frac{2(m+1)(m+2) + 2m^2}{(2m+3)(2m+4)} \right\} + A_m\, A_{m+2} \left\{ \frac{2m}{2m+3} \right\} \right] \qquad (12)$$

Similarly,

$$G = a^2\, \pi \cos^2 \sigma\, t \left[\frac{A^2{}_m}{2m+2} + \frac{A^2{}_{m+1}}{2m+4} + \frac{A^2{}_{m+2}}{2m+6} + \frac{2A_m\, A_{m+1}}{2m+3} + \frac{2A_{m+1}\, A_{m+2}}{2m+5} + \frac{2A_m\, A_{m+2}}{2m+4} \right] \qquad (13)$$

The conditions $[F - k^2 G]\,.\,\partial/\partial A_i = 0$ give three simultaneous equations in A_m, A_{m+1} and A_{m+2}. These are—

$$A_m \left[\frac{h_0 m}{2m+1} - \frac{k^2 a^2}{2m+2} \right] + A_{m+1} \left[\frac{h_0 m}{2m+2} - \frac{k^2 a^2}{2m+3} \right] + A_{m+2} \left[\frac{h_0 m}{2m+3} - \frac{k^2 a^2}{2m+4} \right] = 0 \qquad (14)$$

$$A_m \left[\frac{h_0 m}{2m+2} - \frac{k^2 a^2}{2m+3} \right] + A_{m+1} \left[\frac{h_0 (2m^2 + 2m+1)}{(2m+2)(2m+3)} - \frac{k^2 a^2}{2m+4} \right] + A_{m+2} \left[\frac{h_0 (2m^2 + 3m+2)}{(2m+3)(2m+4)} - \frac{k^2 a^2}{2m+5} \right] = 0 \qquad (15)$$

$$A_m \left[\frac{h_0 m}{2m+3} - \frac{k^2 a^2}{2m+4} \right] + A_{m+1} \left[\frac{h_0 (2m^2 + 3m+2)}{(2m+3)(2m+4)} - \frac{k^2 a^2}{2m+5} \right] + A_{m+2} \left[\frac{h_0 (2m^2 + 4m + 4)}{(2m+4)(2m+5)} - \frac{k^2 a^2}{2m+6} \right] = 0 \qquad (16)$$

The elimination of A_m, A_{m+1}, A_{m+2} from equations (14), (15) and (16), then yields a cubic equation in (k^2a^2/h_0). This equation is then the period equation to determine the oscillatory periods of the lake.

Calling (k^2a^2/h_0) as α, the period equation gives the following results for three simple cases.

(*a*) *Case* $m = 0$

The cubic period equation is—

$$\alpha\,(\alpha^2 - 30\alpha + 120) = 0 \qquad (17)$$

The roots of the equation are 0, 4·75 and 25·25.

(*b*) *Case* $m = 1$

The period equation is —

$$\alpha^3 - 35{\cdot}5\,\alpha^2 + 266{\cdot}75\,\alpha - 291 = 0 \quad (18)$$

The roots of the equation are 1·31, 8·43 and 26·25.

(*c*) *Case* $m = 2$

The period equation is —

$$\alpha^3 - 45{\cdot}6\,\alpha^2 + 472\,\alpha - 878 = 0 \qquad (19)$$

The roots are 2·39, 11·72 and 31·5.

(*ii*) If instead of (11), we take for ζ the simpler form—

$$\zeta = \left[A_m \left(\frac{r}{a}\right)^m + A_{m+1} \times \left(\frac{r}{a}\right)^{m+1} \right] \cos m\,\theta \cos \sigma t \qquad (20)$$

or the more general form

$$\zeta = \left[A_m \left(\frac{r}{a}\right)^m + A_{m+1} \times \left(\frac{r}{a}\right)^{m+1} + A_{m+2}\left(\frac{r}{a}\right)^{m+2} + A_{m+3}\left(\frac{r}{a}\right)^{m+3} \right] \cos m\,\theta \cos \sigma t \quad (21)$$

it is found, as is evident from Table 1, that barring one exception, the two lowest values of α yielded by (20) and (21), for each of the cases $m=0$ and $m=1$ seem to agree reasonably well with the corresponding values obtained from the assumed solution (11).

Now, the two lowest values of α, for any value of m, pertain to the two simplest modes of oscillation, in which there is either no nodal circle or there is only one nodal circle. Since these two modes of oscillation are far more important than those consisting of 2 or more nodal circles, it is obvious that the first three terms on the right hand side of equation (10) could adequately represent the actual mode of oscillation of the lake.

Lamb (1930) dealt with a lake having the shape of a paraboloid of revolution, for which the law of depth is $h=h_0\,[1-(r^2/a^2)]$. He solved the differential equation of seiches by the series solution method. The values he obtained for the dimensionless number $\alpha=\sigma^2a^2/gh_0$ are shown in Table 2 for the cases $m=0$, $m=1$ and $m=2$. The values for the same three cases obtained by the author for a conical lake are also shown in the table for the purpose of comparison.

4. Determination of the nodes

The nodal diameters are determined by the equation : $\cos m\theta=0$.

The equation to determine the nodal circles is $\zeta=0$, *i.e.*,

$$\rho^m\,(A_m + A_{m+1}\,\rho + A_{m+2}\,\rho^2) = 0$$

where $\rho=r/a$. The ratios A_{m+1}/A_m and A_{m+2}/A_m can be obtained from equations (14) and (15). The values of these ratios, obtained for the three cases $m=0$, $m=1$ and $m=2$, then yield the following results—

(1) *Case* $m=0$: The equation, whose roots give the radii of the nodal circles as fractions of a, the radius of the lake's circular surface, is

$$1 + \frac{A_1}{A_0}\,\rho + \frac{A_2}{A_0}\,\rho^2 = 0$$

TABLE 1

Values of α yielded by three different assumed solutions of the seiche differential equation

Form of the solution	Values of $\alpha = k^2a^2/h_0 = \sigma^2a^2/gh_0$							
	$m=0$ (no nodal diameter)				$m=1$ (one nodal diameter)			
	No nodal circle	One nodal circle	Two nodal circles	Three nodal circles	No nodal circle	One nodal circle	Two nodal circles	Three nodal circles
First two terms of solution (10)	0	·35	—	—	1·31	9·52	—	—
First three terms of solution (10)	0	4·75	25·25	—	1·31	8·43	26·25	—
First four terms of solution (10)	0	4·72	15·25	70·04	1·35	8·42	20·35	63·08

TABLE 2

Values of α for a conical lake and for a paraboloid of revolution

Nature of the lake	$m=0$			$m=1$			$m=2$		
	No nodal circle	One nodal circle	Two nodal circles	No nodal circle	One nodal circle	Two nodal circles	No nodal circle	One nodal circle	Two nodal circles
Conical lake (Author)	0	4·75	25·25	1·31	8·43	26·25	2·39	11·72	31·5
Paraboloid of revolution (Lamb)	0	8	24	2	14	34	4	20	44

Solving this equation we have—

(*i*) For $\alpha=4\cdot75$: The positive root, the only admissible root is ·706.

(*ii*) For $\alpha=25\cdot25$: The roots are ·413 and ·787.

(2) *Case* $m=1$: The solution of the equation

$$1 + (A_2/A_1)\,\rho + (A_3/A_1)\,\rho^2 = 0$$

gives —

(*i*) For $\alpha=1\cdot31$: Roots are imaginary.

(*ii*) For $\alpha=8\cdot433$: The positive root, the only admissible root is ·84.

(*iii*) For $\alpha=26\cdot25$: The roots are ·571 and ·909.

(3) *Case* $m=2$: Proceeding as in (1) and (2), we obtain from the solution of

$$1 + (A_3/A_2)\,\rho + (A_4/A_2)\,\rho^2 = 0$$

(*i*) For $\alpha=2\cdot39$: Roots are imaginary.

(*ii*) For $\alpha=11\cdot72$: The positive root, the only admissible root is ·877.

(*iii*) For $\alpha=31\cdot5$: The roots are ·667 and ·926.

The diameters of the nodal circles increase as the number of nodal diameters increases. Figs. 2A to 2H represent the eight cases, ($m=0$, $n=1$), ($m=0$, $n=2$), ($m=1$, $n=0$), ($m=1$, $n=1$), ($m=1$, $n=2$), ($m=2$, $n=0$), ($m=2$, $n=1$) and ($m=2$, $n=2$).

5. Conical lake with a circular, coaxial, cylindrical island

Let a and b be the outer and inner radii of the surface of the lake as shown in Fig. 3.

Proceeding as before, we have now to obtain F and G given by equations (6) and (11), for r ranging from b to a, and θ as before ranging from 0 to 2π. Carrying out the integrations, we have

$$\begin{aligned} F = h_0 \pi \cos^2 \sigma t \Bigg[& A^2_m \left\{ \frac{m}{2m+1} - \left(\frac{b}{a}\right)^{2m} \left(m - \frac{b}{a} \cdot \frac{2m^2}{2m+1} \right) \right\} + \\ & A^2_{m+1} \left\{ \frac{(m+1)^2+m^2}{(2m+2)(2m+3)} - \left(\frac{b}{a}\right)^{2m+2} \left(\frac{m^2+(m+1)^2}{2m+2} - \frac{b}{a} \cdot \frac{m^2+(m+1)^2}{2m+3} \right) \right\} + \\ & A^2_{m+2} \left\{ \frac{(m+2)^2+m^2}{(2m+4)(2m+5)} - \left(\frac{b}{a}\right)^{2m+4} \left(\frac{m^2+(m+2)^2}{2m+4} - \frac{b}{a} \cdot \frac{m^2+(m+2)^2}{2m+5} \right) \right\} + \\ & A_m A_{m+1} \left\{ \frac{2m}{2m+2} - \left(\frac{b}{a}\right)^{2m+1} \left(2m - \frac{b}{a} \cdot \frac{2m(2m+1)}{2m+2} \right) \right\} + \\ & A_{m+1} A_{m+2} \left\{ \frac{2m^2+2(m+1)(m+2)}{(2m+3)(2m+4)} - \left(\frac{b}{a}\right)^{2m+3} \left(\frac{2m^2+2(m+1)(m+2)}{2m+3} - \right.\right. \\ & \left.\left. \frac{b}{a} \cdot \frac{2m^2+2(m+1)(m+2)}{2m+4} \right) \right\} + A_m A_{m+2} \left\{ \frac{2m}{2m+3} - \left(\frac{b}{a}\right)^{2m+2} \times \right. \\ & \left. \left(2m - \frac{b}{a} \cdot \frac{2m(2m+2)}{2m+3} \right) \right\} \Bigg] \end{aligned} \tag{22}$$

and

$$\begin{aligned} G = a^2 \pi \cos^2 \sigma t \Bigg[& A^2_m \left\{ \frac{1}{2m+2} - \frac{(b/a)^{2m+2}}{2m+2} \right\} + A^2_{m+1} \left\{ \frac{1}{2m+4} - \frac{(b/a)^{2m+4}}{2m+4} \right\} + \\ & A^2_{m+2} \left\{ \frac{1}{2m+6} - \frac{(b/a)^{2m+6}}{2m+6} \right\} + 2A_m A_{m+1} \left\{ \frac{1}{2m+3} - \right. \\ & \left. \frac{(b/a)^{2m+3}}{2m+3} \right\} + 2A_{m+1} A_{m+2} \left\{ \frac{1}{2m+5} - \frac{(b/a)^{2m+5}}{2m+5} \right\} + \\ & 2A_m A_{m+2} \left\{ \frac{1}{2m+4} - \frac{(b/a)^{2m+4}}{2m+4} \right\} \Bigg] \end{aligned} \tag{23}$$

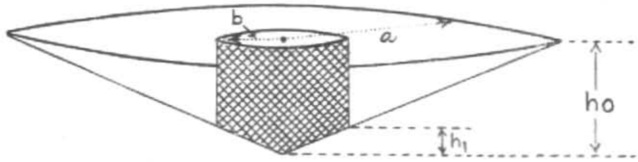

Fig. 3. Conical Lake with a nearly-cylindrical coaxial Island

The conditions $[F - k^2 G] \cdot \partial/\partial A_i = 0$, give rise to three long simultaneous equations. On eliminating the constants A_m, A_{m-1}, A_{m+2} from these equations, we obtain the following period equations :

(1) *Case* $m = 0$

The period equation is—

$$\begin{vmatrix} -k^2a^2(1-b^2/a^2) & -\frac{2}{3}k^2a^2(1-b^3/a^3) & -\frac{1}{2}k^2a^2(1-b^4/a^4) \\ -\frac{2}{3}k^2a^2(1-b^3/a^3) & \left[2h_0\left\{\frac{1}{6}-\frac{b^2}{a^2}\left(\frac{1}{2}-\frac{b}{3a}\right)\right\} - \frac{k^2a^2}{2}(1-b^4/a^4)\right] & \left[h_0\left\{\frac{1}{3}-\frac{b^3}{a^3}\cdot\left(\frac{4}{3}-\frac{b}{a}\right)\right\} - \frac{2}{5}k^2a^2(1-b^5/a^5)\right] \\ -\frac{1}{2}k^2a^2(1-b^4/a^4) & \left[h_0\left\{\frac{1}{3}-\frac{b^3}{a^3}\left(\frac{4}{3}-\frac{b}{a}\right)\right\} - \frac{2}{5}k^2a^2(1-b^5/a^5)\right] & \left[2h_0\left\{\frac{1}{5}-\frac{b^4}{a^4}\left(1-\frac{4}{5}\frac{b}{a}\right)\right\} - \frac{k^2a^2}{3}(1-b^6/a^6)\right] \end{vmatrix} = 0 \qquad (24)$$

(2) *Case* $m=1$

The period equation is—

$$\begin{vmatrix} \left[2h_0\left\{\frac{1}{3}-\frac{b^2}{a^2}\left(1-\frac{2}{3}\frac{b}{a}\right)\right\} - \frac{1}{2}k^2a^2\left(1-b^4/a^4\right)\right] & \left[h_0\left\{\frac{1}{2}-\frac{b^3}{a^3}\left(2-\frac{3}{2}\frac{b}{a}\right)\right\} - \frac{2}{5}k^2a^2(1-b^5/a^5)\right] & \left[h_0\left\{\frac{2}{5}-\frac{b^4}{a^4}\left(2-\frac{8}{5}\frac{b}{a}\right)\right\} - \frac{1}{3}k^2a^2(1-b^6/a^6)\right] \\ \left[h_0\left\{\frac{1}{2}-\frac{b^3}{a^3}\left(2-\frac{3}{2}\frac{b}{a}\right)\right\} - \frac{2}{5}k^2a^2(1-b^5/a^5)\right] & \left[2h_0\left\{\frac{1}{4}-\frac{b^4}{a^4}\left(\frac{5}{4}-\frac{b}{a}\right)\right\} - \frac{1}{3}k^2a^2(1-b^6/a^6)\right] & \left[h_0\left\{\frac{7}{15}-\frac{b^5}{a^5}\left(\frac{14}{5}-\frac{7}{3}\frac{b}{a}\right)\right\} - \frac{2}{7}k^2a^2(1-b^7/a^7)\right] \\ \left[h_0\left\{\frac{2}{5}-\frac{b^4}{a^4}\left(2-\frac{8}{5}\frac{b}{a}\right)\right\} - \frac{1}{3}k^2a^2(1-b^6/a^6)\right] & \left[h_0\left\{\frac{7}{15}-\frac{b^5}{a^5}\left(\frac{14}{5}-\frac{7}{3}\frac{b}{a}\right)\right\} - \frac{2}{7}k^2a^2(1-b^7/a^7)\right] & \left[2h_0\left\{\frac{5}{21}-\frac{b^6}{a^6}\left(\frac{5}{3}-\frac{10}{7}\frac{b}{a}\right)\right\} - \frac{1}{4}k^2a^2(1-b^8/a^8)\right] \end{vmatrix} = 0 \qquad (25)$$

TABLE 3

Values of α and the corresponding periods of oscillation of lake Toya for the cases $m = 0$ and $m = 1$

Computed quantitity	$m=0$ (No nodal diameter)			$m=1$ (One nodal diameter)		
	No nodal circle	One nodal circle	Two nodal circles	No nodal circle	One nodal circle	Two nodal circles
α	0	5·35	35·31	0·96	6·9	17·02
Period T	—	3 min 28 sec	1 min 21 sec	8 min 10 sec	3 min 3 sec	1 min 57 sec

6. Evaluation of the periods of the various modes of seiche–oscillations of lake Toya, Japan

Lake Toya is situated in South Hokkaido, Japan. It has a circular form of radius 4770 metres with a nearly circular island at the centre, of radius 1240 metres. The lake is made up of 2 sectional regions (Fig. 4) of mean depths 60·4 metres and 131·5 metres respectively, the sector of mean depth 60·4 metres, subtending an angle of 67° at the centre.

If lake Toya is treated as a conical lake having (*i*) a circular surface of outer radius $a=4770$ metres and (*ii*) an almost cylindrical coaxial circular island of radius $b=1240$ metres, shown hatched in Fig. 3, we have—

(a) $h_1 = h_0\,(b/a)$

(*b*) Volume of the island at the centre $= \pi\, b^2\,(h_0 - \frac{2}{3}\, h_1)$

(*c*) Volume of the rest of the cone $= (h_0\pi/3a)\,(a^3 - 3a^2b + 2b^2)$

In lake Toya, the depth of water varies at different points of the lake. The volume of water computed from the mean depths of the lake in the two different sectors works out to be $78\cdot86\times10^8$ cubic metres.

Equating this volume to the volume

$$\frac{h_0\pi}{3a}\left[a^3 - 3a^2b + 2b^2\right]$$

for a conical bottom we obtain h_0=the central depth of the equivalent conical lake with a central island as in Fig. 3=397·4 metres. Substituting $h=397\cdot4$ metres and $b/a=\cdot26$ in the period equations (24) and (25), the period equations reduce to

$$\alpha^2 - 40\cdot66\,\alpha + 189\cdot1 = 0$$

and $\alpha^3 - 24\cdot88\,\alpha^2 + 140\cdot46\,\alpha - 113.15 = 0$

where, $\alpha = k^2a^2/h_0 = \sigma^2a^2/gh_0$

The values of α, yielded by these two equations and the corresponding oscillation periods, obtained from the formula

$$T = \frac{2\pi a}{\sqrt{\alpha}\,\sqrt{gh_0}}$$

are given in Table 3.

7. Comparison with observations

According to Koenuma (1934), Honda has observed a period of oscillation of 9·3 min for lake Toya, while Mori has observed a period of oscillation of 4·5 min for the same lake.

The values computed by the author for the conical approximation of lake Toya, with a circular cylindrical island are (*i*) 8·17 min for an oscillation with one nodal diameter and no nodal circle and (*ii*) 3·65 min for an oscillation with one nodal circle and no nodal diameter.

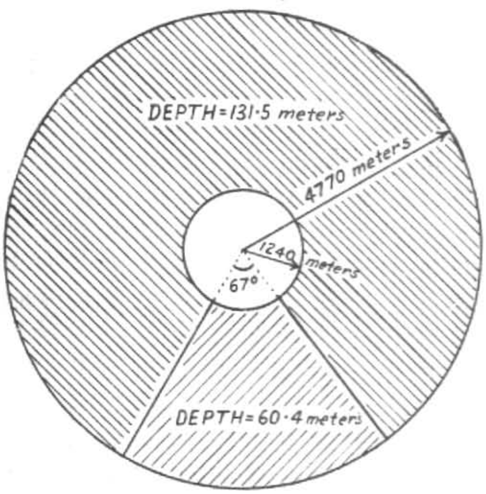

Fig. 4. Assumed Top-view of Lake Toya, Japan

Assuming a cylindrical form for the lake with a circular cylindrical island at the centre, Syono computed a value of 10·25 min for the oscillation with one nodal diameter and no nodal circle and a value of 3·9 min for the oscillation with one nodal circle and no nodal diameter (*cf.* Koenuma, 1934). He concluded that the former corresponds to the period observed by Honda and the latter to that observed by Mori.

8. Acknowledgement

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