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A conceptual model of water transport through unsaturated soil zone

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ABSTRACT. A simple conceptual model of water transport in the unsaturated soil zone is presented. The transport of water is visualized in pulses of infiltrating sheet through a series of interconnected expandable mixing cells. By applying mass balance equation, successively to each hypothetical mixing cell sub-dividing the soil profile, at discrete time intervals, the process of dispersion and mixing during transport of water through the unsaturated zone is mathematically approximated. Simulation of tritium tracing experiments in Indo-Gangetic plains of north India, using the above model, is shown to closely reproduce the field data. A close correlation is seen to exist between the rainfall and the estimated number of recharge pulses from the field data.

1. Introduction

In a typical soil profile (Fig. 1) *the unsaturated upper part* of the soil water regime may be usefully sub-divided into three main zones: (*i*) the root zone, (*ii*) the intermediate zone, and (*iii*) the capillary zone. The *saturated lower part* may also be further sub-divided into (*i*) capillary fringe, and (*ii*) ground water zone.

Starting from the groundwater zone upward, water occupies all the pores upto the capillary fringe level. Water content at the top of capillary zone declines to a value corresponding to field capacity (FC), *i.e.,* maximum water that can be retained in the soil in unsaturated condition when water infiltrates under gravity.

The water content of the soil generally remains at field capacity throughout the intermediate zone. Moisture conditions in this zone are relatively constant and seasonal fluctuations are seldom observed (Meinzer 1942). In contrast, the water content of the root zone fluctuates between field capacity at the base to even below the wilting point at the top layers after a prolonged dry spell. However, during periods of excessive precipitation, the moisture content of the top of the root zone may equal the saturation value.

2. Infiltration of water through the unsaturated soil profile

Transfer of rainwater to the watertable through the unsaturated soil zone is often described as sheet infiltration. This situation arises when the rainfall intensity is sufficient to form excess liquid water on the surface of soil. Such conditions occur not only in humid regions but also in arid regions during a storm of high intensity even though of short duration. Here the rate of rainfall exceeds the infiltration capacity of the soil or is just equal to it. A saturated water sheet is formed within the soil and infiltration processes are largely one of movement and dissipation of this sheet within the soil mass.

When a given infiltration sheet travels through a soil matrix having moisture at field capacity (FC), (as in the intermediate zone) it merely picks up unsaturated water bodies ahead of it and leaves an equal number of such bodies behind. In either case the unsaturated moisture is at field capacity — in front as well as behind the sheet. The result is that all water in the sheet in excess of the field capacity is transmitted to the watertable as recharge. When, however, the infiltration sheet travels through the soil at moisture content (S) less than the field capacity (FC) (as through the root zone that has previously been subjected to prolonged desiccation) the sheet picks up the moisture content (S) ahead of it and deposits unsaturated water bodies producing a moisture content FC behind it. Thus an amount of water equal to (FC—S) is removed from the sheet in each unit volume of soil. Thus a sheet of infiltrating water, starting its journey from the topmost layer of the soil in the root

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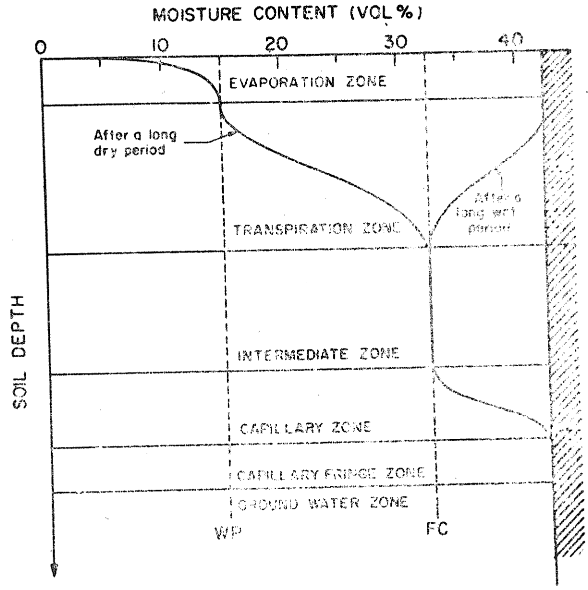

Fig. 1. Schematic representation of the moisture profile in the unsaturated zone. The curve applies to a more or less homogeneous soil.

zone, may or may not reach the intermediate zone and through it to the watertable, depending on the intensity of the rain producing it, moisture content of the root zone and its distance to the intermediate zone.

In rainy season there are long periods during which it does not rain. During such periods, in addition to the downward movement and dissipation of an infiltrating sheet formed during the previous rainfall, evapotranspiration loss from behind the sheet continues reducing the moisture content below the field capacity. So that the next infiltrating sheet must first satisfy the moisture deficit before proceeding further. This means that the infiltration of rain water first to the intermediate zone and then to the watertable takes place in pulses. This is easily seen from Anderson and Sevel (1974, Fig. 2). A front of gravitational water (*i.e.,* a pulse of infiltrating sheet) can be easily identified moving downwards in the successive moisture profiles taken, using a neutron moisture gauge, at monthly intervals in the same bore hole.

3. Multi-Cell model of the moisture transport through unsaturated zone

Zimmerman *et al.* (1967) proposed that the movement of soil moisture in the unsaturated zone is layer by layer like a piston flow with dispersion of each increment of recharge water due to molecular diffusion. This concept has gained support from Anderson (1970), Halvey (1970), Sukhija and Rama (1973), Datta *et al.* (1973) and Anderson and Sevel (1974). Generally, in a real hydrologic system, the dispersion and mixing occurring during the flow make whole process rather too complex for a full mathematical formulation. Predictions of a proper system response function is possible only under simplifying conditions like homogeneous nature of soil etc.

We describe a multi-cell model for moisture transport through the unsaturated soil zone. This model takes into account hydrologically important inhomogeneity of soil layers. The important assumptions of this model are: (*i*) Downward water transport through a given soil layer in the unsaturated zone does not take place unless the moisture content of the layer has first been brought to its field capacity, (*ii*) Recharge of rainwater from the root zone to the intermediate soil zone, and then to the watertable is in pulses of infiltrating sheets having water in excess of the field capacity of the soil, (*iii*) The soil profile can be sub-divided into several small homogeneous layers. While the absolute moisture content may be different from layer to layer, each layer transmitting water must have a minimum moisture corresponding to its field capacity, (*iv*) When a given infiltration sheet travels through a soil layer, it increases the moisture content of the layer above the field capacity (maximum upto saturation level). The water from the infiltrating sheet completely mixes with the water previously present in the soil layer. So that if the infiltrating sheet was carrying some amount of tracer, at concentration (C), its concentration (C_1) in the layer after it has mixed with the water in the layer, becomes equal to

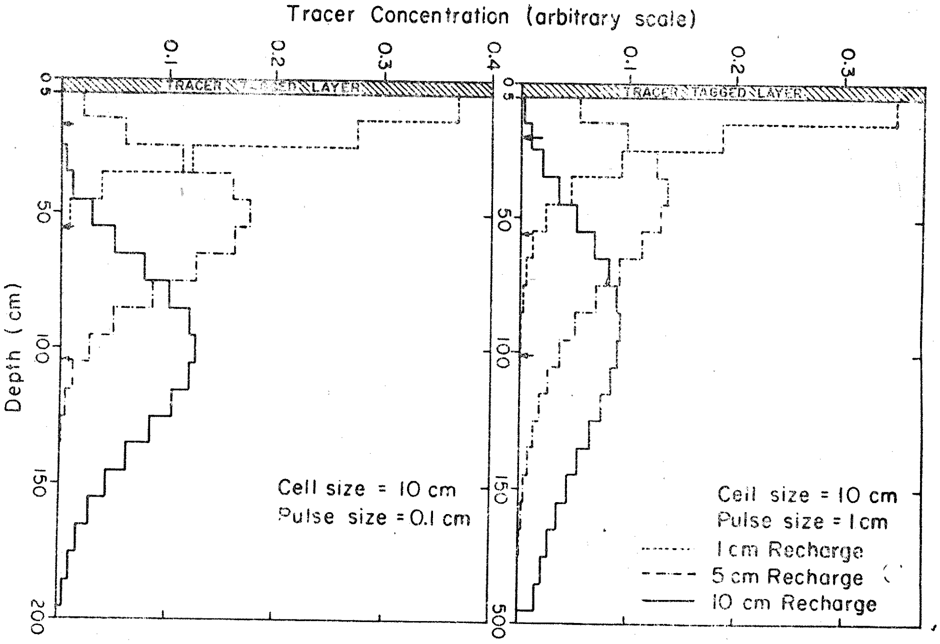

Fig. 2. Simulated tracer profiles for a given cell size (10 cm) for different amount of total recharge (1, 5 and 10 cm) with different sizes (0.1 and 1.0 cm) of recharge pulse.

Note : While the dispersion of tracer profiles is dependent both on the total rechaige and the pulse size chosen, the position of centre of gravity of profiles (shown by arrow marks) depends only on the amont of total recharge.

the total mass of the tracer in the system divided by the total volume of water in the system. As the infiltrating sheet advances to the next layer it leaves the moisture content of the layer behind it at its field capacity, with tracer concentration at C_1. We thus see that while the volume of the infiltrating sheet remains unchanged, the tracer concentration in it changes from C to C_1 due to mixing with the stationary water in the previous cell. The same process of mixing repeats as the infiltrating sheet advances from layer to layer.

If a given layer had moisture content below the field capacity, it not only changes the tracer concentration of the infiltrating water sheet as explained above but also retains a part of the water from the infiltrating sheet. The amount of water so retained is equal to the moisture deficit of the soil layer.

4. Mathematical formulation of the model

If a finite amount of fluid, carrying some amount of tracer, enters a system and another finite amount leaves the system, the mass balance of the tracer for such a system may be expressed as :

(Net accumulation) = (Input to the system) — (Output from the system) + (Generation inside the volume) — (Consumption inside the volume) (1)

This mass balance equation of the tracer can be applied to study the time evolution of the tracer for a given hydrologic system with known input conditions, on a discrete time basis. The general concept, therefore, is to apply the mass balance equation successively to each finite time increment (referred to as "iteration"). Since the Eqn. (1), describing the mass balance of the tracer is applied to each iteration, it is referred to as "recursive equation" (Przewlocki and Yurtsever 1974).

Let us assume that the fraction entering volume V during each iteration is denoted by 'BRF' (boundary recharge fraction — dimension, volume). This fraction carries some concentration of the tracer 'BRC' (boundary recharge concentration—dimension, mass/volume). T denotes mass of the tracer in the system

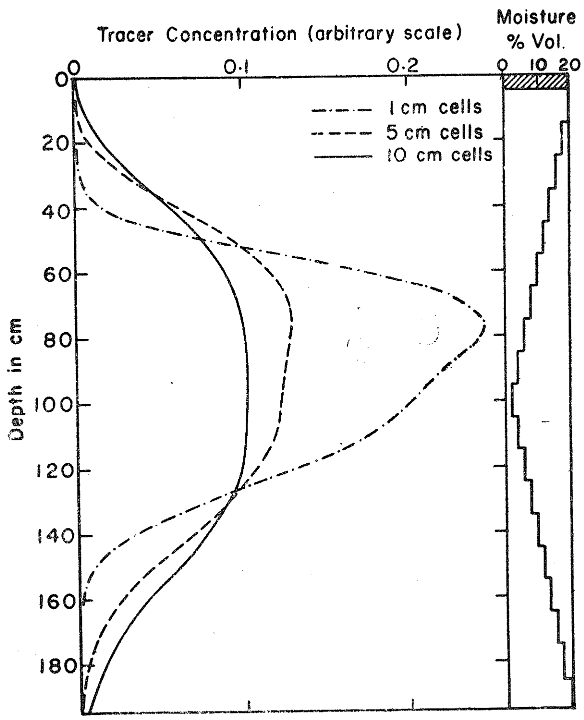

Fig. 3. Depth variation of tracer concentration for different cell sizes showing that dispersion increases with the increase in the cell size while centre of gravity of the tracer profile remains unchanged.

(Volume V) — Call it state at iteration T. Then using the principle of mass balance Eqn. (1), it is possible to derive the following recursive equation for non-expandable mixing cell describing change of state during unit time.

$$S_{(T+1)} = (S_T + \text{BRF} \times \text{BRC} - \text{BDF} \times \text{BDC})r \quad (2)$$

where, BDC = boundary discharge concentration (dimension—mass/volume) gives the concentration of tracer in the out flow, BDF, boundary discharge fraction (dimension-volume).

For a non-expandable cell BDC = S_T/V and BDF=BRF

$S_{(T+1)}$ = State of the cell at iteration $T+1$

r = Sink term. In case the tracer is radioactive it represents radioactive decay inside the system between two iterations.

Eqn. (2) describes the transformation of one nonexpandable mixing cell from one state to another during an arbitrary unit time interval. This unit time interval, may, correspond to any real time unit. In the case of interconnected cells, the total input to a given cell is the sum of an independent boundary recharge BRF (if any) at concentration BRC and the output BDF at concentration BDC of the previous cell.

It is easily seen that the Eqn. (2) implies a process of ideal mixing of the tracer carrying fluid with the fluid present in the cell under consideration. During the time between any two iterations, a certain amount of fluid with concentration characteristic of previous iteration comes and then mixes with the remaining fluid, creating inside the cell state characteristic for the next iteration (Przewlocki and Yurtsever 1974). It may be noted that the sequence of events in the non-expandable cell model is: recharge-discharge-mixing.

In contrast the sequence of events in an "expandable cell' is : recharge-expansion-mixing-discharge. The corresponding recursive equation has the same form as Eqn. (2), with the value of BDC being given by Eqn. (3).

$$\text{BDC} = (S_T + \text{BRF} \times \text{BRC})/(V + \text{BRF}) \quad (3)$$

It is obvious that non-expandable cell models can be used to simulate those hydrological situations, where recharge is small compared to the volumes of the mixing cells. This is generally the case in most ground water aquifers-volume of water stored in the aquifer being very large compared to annual replenishment. Expandable

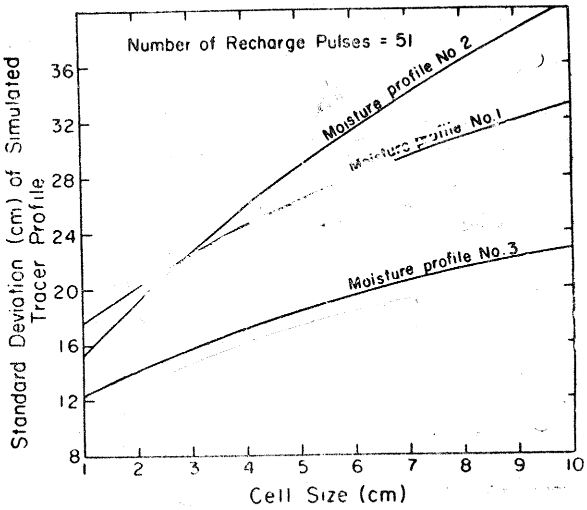

Fig. 4. Dispersion of simulated tracer profile with cell size for particular recharge volume. Moisture profiles are as shown in Fig. 6.

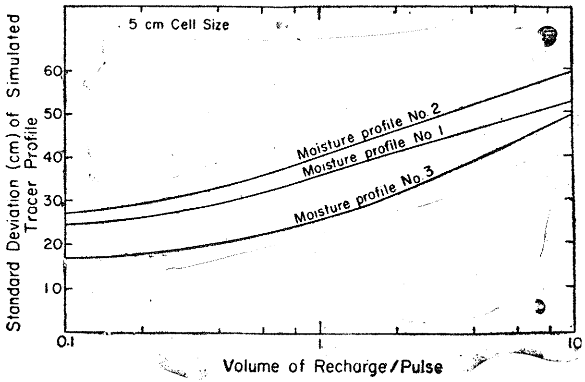

Fig. 5. Dispersion of simulated tracer profile with different volume of recharge pulse for a particular cell size. Moisture profiles are as shown in Fig. 6.

cells must, however, be used for modelling situations where periodic recharge volumes are comparable to the volume of water in each hypothetical mixing cell sub-dividing the system storage.

For many soils the saturation percentage is approximately twice the field capacity (Yaron *et al.* 1974) so that during recharge of water through unsaturated zone, the moisture content of the soil changes by as much as 100 per cent, the mixing cells must, therefore, be treated as expandable when applying the recursive equation approach to describe the movement of soil moisture.

5. Evaluation of the behaviour of various soil systems

By using the model of soil moisture transport described above, mathematical simulation experiments were carried out to study the effects of both soil properties and recharge pattern on the time evolution of the tracer profiles. These simulation experiments aim at providing more insight into the behaviour of soil systems and recharge pattern in shaping the tracer profiles.

Let us first consider the hypothetical case of a uniform soil with field capacity moisture content of 10 per cent by volume and saturation moisture content of 20 per cent by volume. Let us also assume that the first 5 cm of the soil profile is initially labelled with a non-radioactive tracer. We now compute the evolution of the tracer profiles after 10 cm (10 ml/cm^2) of recharge by using the recursive equation for interconnected expandable cells.

We divide the soil profile into hypothetical mixing cells (say 10 cm thick). We also subdivide the total recharge of 10 cm into equal size pulses of 1 cm or less (so that during passage of an infiltrating pulse through mixing cell, its moisture content is less than or at most equal to saturation value).

The computed tracer profiles for 1 cm size recharge pulse are shown in Fig. 2. It is noticed that the tracer profiles are similar to those obtained by using the displacement with dispersion model (Anderson and Sevel 1974). It may also be noted that volume of water in soil storage (at field capacity) above the centre of gravity of the tracer profile is always equal to the total value of recharge that has passed through the system. Tracer profiles for the similar cases but for 1 mm size recharge pulses are also shown in Fig. 2. It may be noted that while the centre of gravity of profiles remains at the same position for a given recharge volume, the dispersion of the profiles is less for the same amount of recharge but with smaller pulses. It is also noticed (Fig. 3) that dispersion of the tracer profile for a given total recharge and pulse size also depends on the cell size considered, being less for smaller size. Relation between the standard deviation of the computed tracer profiles and the hypothetical cell size on one hand and pulse size for a given total recharge on the other hand is given in Figs. 4 and 5.

It is seen from Fig. 6 that in case of non-uniform soils, tracer profiles will have varying shapes. But even in these cases:

TABLE 1

Estimated recharge using tritium tracing method and computed model parameters for various locations in the Indo-Gangetic plains

	Location	Total† rainfall (mm)	Estimated recharge (mm)	Model parameters: Cell size (mm)	Model parameters: Recharge pulse size (mm)	Model parameters: No. of recharge pulses	No. of rainy days with rainfall 3 times recharge pulse size
	Haryana						
1	Palwal (H-2)	411	5.0	50	6.2	8	8
2	Jatusana (H-5)	461	22.2	30	4.1	54	14
3	Bawal (H-7)	511	10.3	40	2.8	37	19
4	Narnaul (H-8)	452	7.5	30	4.4	17	9
5	Karnal (H-15)	508	2.7	50	4.3	6	15
6	Bilaspur (H-18)	1351	7.4	60	9.3	8	17
7	Ambala (H-20)	649	4.1	20	1.3	31	26
8	Jhansa (H-22)	—	10.9	30	2.5	44	—
9	Hissar (H-26-C)	—	11.9	70	5.6	21	—
10	Hissar (H-26-B)	183	2.8	50	5.6	50	3
	Punjab						
11	Chamkaur Sahib (WR-1)	421	7.3	60	4.1	18	7
12	Dasuya (WR-13)	535	8.5	50	8.4	10	5
	Uttar Pradesh						
13	Soar (19-I-A)	915*	12.6	50	14.0	9	7
14	Joya (24-I-A)	—	17.9	60	4.2	43	—
15	Bilari (25-I-A)	875*	10.7	50	8.9	12	10
16	Chandausi (26-I-A)	—	8.4	90	16.8	5	—
17	Sambhal (27-I-A)	888*	15.9	50	17.7	9	3
18	Sikandra Rao (39)	703	9.5	40	4.3	22	13
19	Bulandshahr (43)	698	16.6	60	7.8	21	10
20	Anupshahr (42)	717	10.0	30	2.9	37	20
21	Gulaothi (45)	—	13.0	30	4.3	30	—
22	Dadri (47)	1139	17.6	50	7.6	23	14

†Based on data provided by the Director General of Observatories, India Meteorological Department.

Between the date of tracer injection and subsequent soil sampling :

Haryana — 20 Jun 73 to 4 Nov 73; Punjab — 14 Jun 72 to 27 Nov 72;

Uttar Pradesh — 3 Jun 71 to 30 Sep 71* and 13 Jun 71 to 17 Mar 72

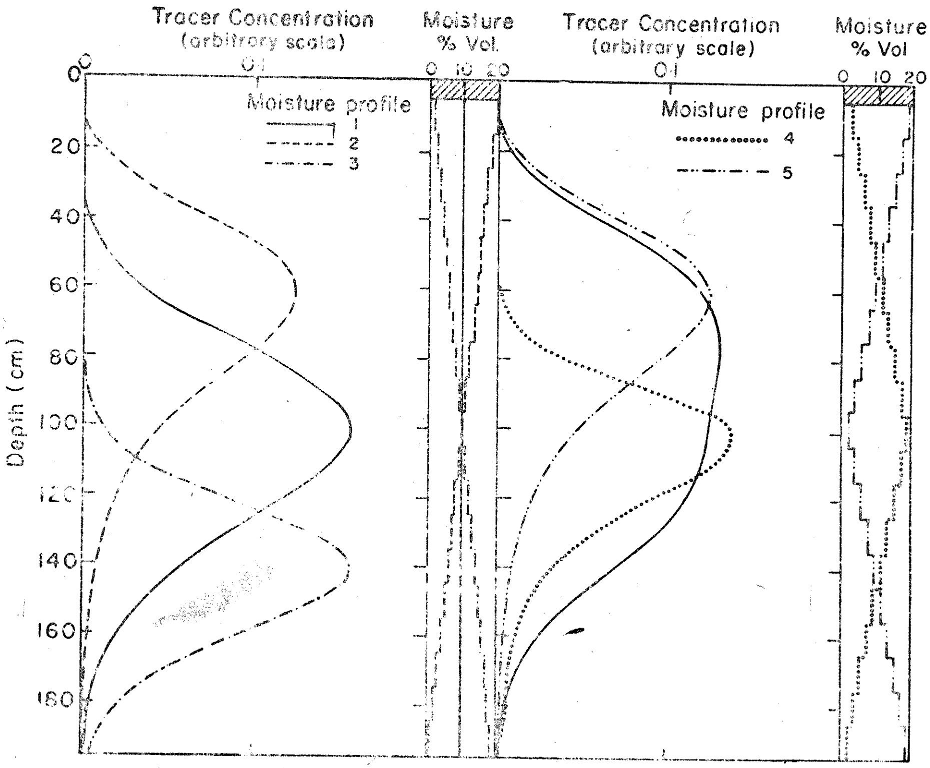

Fig. 6. Depth variation of tracer concentration for different types of moisture profiles for a given cell and pulse size.

(*i*) Volume of water in soil storage above the centre of gravity of the tracer profiles gives the total value of recharge that has passed through the system.

(*ii*) The shape (including dispersion) of the tracer profiles is characterised by the field capacity moisture profiles of the soil and also by the recharge pattern. Dispersion increases with increasing cell size and the size of recharge pulse.

6. Practical example of simulation for the interpretation of tracer experiment data

Datta *et al.* (1973) have done several experiments in the Indo-Gangetic alluvial plains for estimating the vertical recharge to groundwater table by tagging a soil moisture layer with artificial tritium in the unsaturated zone and subsequently following the movement of tracer. In these experiments a moisture layer at a depth of 70 cm was tagged with tritiated water before the onset of monsoon rains. After the monsoon period, 10 cm cores of soil were taken with the help of 4.5 cm diameter hand auger, upto 3 m depth and depth profile of tritium concentrations were obtained. For details of experimental procedures, reference is made to Datta *et al.* (1973) and Datta (1975).

Under the assumption that the moisture content below the tracer injection depth (70 cm in the present case) has not significantly varied between the time of injection and sampling (*i.e.,* soil profile below the injection depth has remained at field capacity), the recharge has been estimated by computing the total amount of water in 1 cm^2 column of soil lying between the depth of injection and the centre of gravity of displaced (and dispersed) tracer profile.

In the following section, using the multi-cell model an atempt has been made to reproduce a few tracer profiles taken from Datta *et al.* (1973), Datta and Goel (1978) and Goel *et al.* (1978).

In all these cases the soil column is sub-divided into hypothetical cells (*say* 1 cm size) partly filled with the amount of water as estimated from the measurement of gravimetric moisture content and *in situ* density at the proper depth. The computed value of the total recharge (amount of water in storage between the point of injection and centre of gravity of the tracer profile) is now broken up into small equal sized pulses (1, 2, 3 . . . *n*). The model tracer profile is then computed and compared with actual field tracer profile. This exercise is continued by varying the cell size and the number of pulses dividing the total recharge (keeping the centre of gravity of the actual and model profiles at the

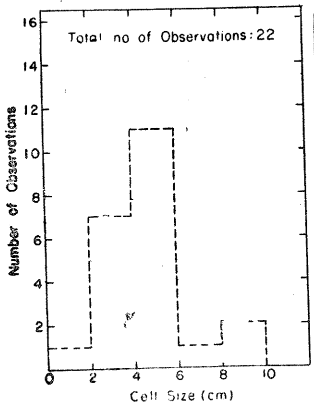

Fig. 7. Cell sizes estimated using the model for a number of experimental sites.

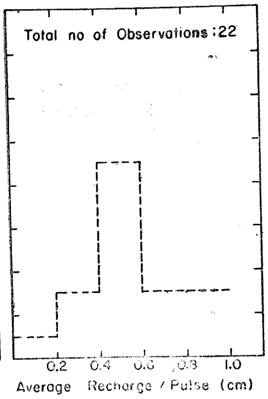

Fig. 8. Estimated sizes of recharge pulses using the model for a number of field stations.

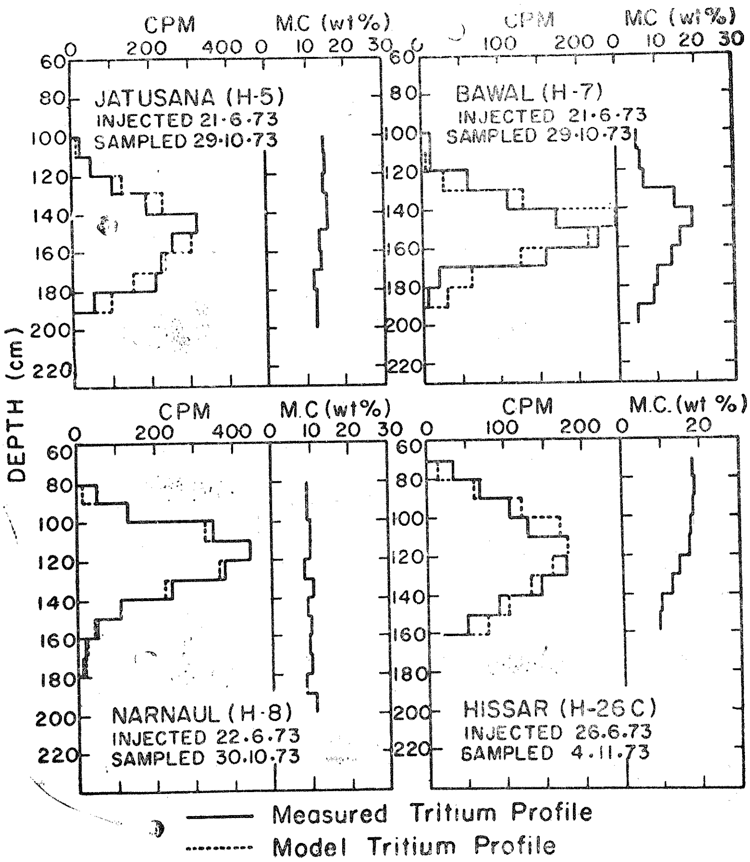

Fig. 9. A few typical simulated profiles superposed on the observed Tritium tracer profiles

same place) till the variance of the model tracer profile and the actual tracer profile are closest.

Twenty two profiles from the Indo-Gangetic alluvial plains have been studied in this manner. It can be seen (Figs. 7 and 8) that most of the tracer profiles are reproducible by assuming cells to be 4-8 cm thick and taking each recharge pulse to be of the size 0.4 to 0.8 cm. Some of the tracer profiles (actual and model) are given in Fig. 9.

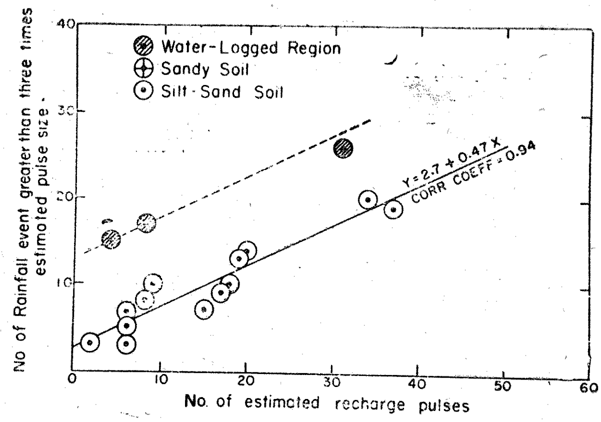

Fig. 10. Plot of estimated number of recharge pulses (using the model) *versus* the number of days with rainfall greater than three times the estimated pulse size, during the period of experiments.

It seems reasonable to assume that the effective size of the mixing cells and field capacity moisture content characterise the hydrological properties of the unsaturated soil profile at a given place. So that on regional scale, for a given soil type, the effective size of the mixing cell can be assumed to vary within narrow limits. This is precisely what we observe in the twenty two cases described above where by simulation we find that the mean mixing cell size is 5 cm.

Similarly, it is known that all the rainfall events do not contribute to recharge and also large rainfall events do not necessarily mean larger recharge (as these lead to larger run off). It is, therefore, reasonable to expect that average pulse size on a regional scale will vary within narrow limits and the value will be characteristic of the hydrometeorological and soil conditions in the area. From the simulation studies we find that average recharge pulse size in Indo-Gangetic alluvial plains at the sites studied is 0.5 cm.

It is clear that if the hypothetical cell size and recharge pulse size are characteristic of a region and if the model is fairly realistic it may be possible to see some relationship between the number of estimated recharge pulses and the number of rainy days at a given site. In Fig. 10 we have plotted the number of recharge pulses (x-axis) against the number of rainy days (between the time of tracer injection and sampling) with rainfall > 3 times the estimated size of recharge pulse. From these figures it can be clearly seen that regression line with a correlation coefficient of 0.94 can be passed through the data points. The data also suggest that for coarser and finer soil types the regression line may have the same slope but with a different intercept.

7. Conclusion

A simple conceptual model of water transport in the unsaturated soil zone has been presented. In the model we subdivide the soil layers into interconnected expandable mixing cells, the water transport from one layer to next takes place only when the moisture content of the layer under consideration has been increased to beyond its field capacity value. The recharge of water is visualized in pulses of infiltrating sheet. By applying the mass balance equation at discrete time intervals to the hypothetical mixing cells the process of dispersion and mixing taking place during water transport in the unsaturated zone is mathematically approximated.

By making use of this model, it has been possible to closely simulate the observed data of tritium tracing experiments in Indo-Gangetic plains of north India. A very close correlation is seen to exist between the rainfall data and the number of recharge pulses estimated as a result of simulation of field data using this model.

Acknowledgement

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