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Path of a vortex in a semi-infinite region with a streaming motion along a plane wall boundary having a semi-circular bay or bulge

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ABSTRACT. An attempt has been made to study the motion of a vortex in a two-dimensional steady flow in presence of a plane wall with semi-circular bay or bulge. The technique of transformation has been used to study the problem. It has been found that for negative ratio of the strength of the vortex and uniform stream, the paths of the vortex make loops in a region closed to the curved wall with stationary point (points) and also in some regions the vortex moves in opposite direction of the uniform stream for both the cases of bay or bulge.

1. Introduction

It is a common observational fact that in a channel or a river, the vortices sometimes move with the water current and sometimes remain stationary. In certain cases the vortices move in a closed path or even move in the opposite direction of the current in some region.

An interesting and important aspect in this connection is the movement of the large-scale atmospheric vortices like tropical cyclones and low pressure systems. Such natural vortices are very complicated and so far no standard mathematical method is available to predict their movement taking into account of all relevant parameters.

Many authors including Thomson (1960) in their books have discussed motion of vortex in presence of plane wall, two perpendicular plane walls, and a vortex inside a circular cyclinder or outside it, in unbounded fluid. Lin (1943) has discussed the most general problem on the motion of vortices in two-dimensions. Singh (1954) studied the path of vortex in a channel with rectangular bend. Banerji (1930) has shown that on the rotating earth, a corner formed by two mountain ranges will have a tendency to develop closed stream lines in its neighbourhood and that the peculiarities in the configuration of the normal Indian monsoon isobars are shown to be due to a very large extent to the mountain ranges.

Increasing awareness of the importance of the topographical effects on the atmospheric flow have led in recent years to numerous studies and investigations all over the world. It has been recognised that mountain ranges do have strong interaction with and influence over typhoons, hurricanes, cyclonic storms and lows. In certain cases blocking and deflecting effects due to mountain barriers become very prominent. The phenomena associated with the interaction of atmospheric vortices with mountain barriers are quite complex. A detailed three dimensional analysis is more complex.

Very recently a series of laboratory experiments have been carried out by Pao (1976) to study the mechanical encounter of a vortex with two-dimensional elliptical barrier. He has demonstrated that many distinctive flow characteristics associated with the terrain effects as typhoons encountering the island of Taiwan can reasonably be simulated in the laboratory. Chang et al. (1975) have made some experimental simulations on the structure and tropographical influence of typhoon.

The high mountain ranges, like the great Himalayan ranges, can be regarded as a series of idealised barriers namely vertical walls. Their base lines are not everywhere straight, but have concave or convex curvature. To the northeast corner of Assam, the NEFA hills rise very steeply and stand like a distinct boundary wall and its horizontal cross-section has a prominent concave shape; the Northwest Frontier mountain ranges form another prominent curve over Pakistan. These mountain ranges have an appreciable influence on the movement of low pressure system.
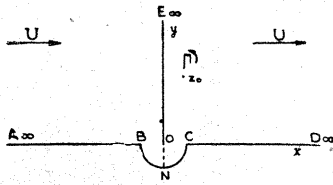
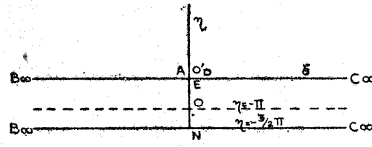The object of the present paper is to study

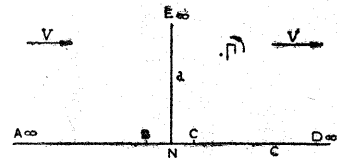Fig. 1. *z*-plane

Fig. 2. *ζ*-plane

Fig. 3. *s*-plane

analytically the dynamical aspects of the movement of vortex in two-dimensional perfect fluid with uniform stream when the boundary is not only straight or curved but combination of both.

The vertical wall has curved part, in the first case in the form of a semi-circular groove and in the second case semi-circular hump otherwise straight and extended to infinity on both sides. The uniform stream is parallel to the straight wall. The solutions for the motion of the fluid in presence of the vortex and the velocity of the vortex have been obtained in general. The path of the vortex has been calculated numerically from the equation in detail for the above two typical cases and the solutions for other symmetrical curved boundary have been extended.

In case of groove, the image system cannot be obtained easily in the original (initial) *z*-plane and hence the motion has been transformed to another plane, so that the fluid region is confined to the upper half of a plane with a straight rigid boundary. In general case also, the image system can not be easily obtained in original *z*-plane; though in the case of semi-circular hump, the image system can be obtained in *z*-plane ; however it has been found easier to study the motion in this case also in the transformed plane.

2. Transformation and formation of the problem

We take the plane of motion as $z(x+iy)$ plane with origin at the centre of the semi-circular bay. In Fig. 1, BNC represents the bay of radius a; $A_\infty B$ and CD_∞ represents the Bay of the plane wall ($y=0$).

The region above the plane wall and the bay is occupied by an inviscid fluid, which has at a large distance from the bay, a steady flow U, parrallel to x-axis. Let a vortex of strength Γ be situated at a point $z_0(=x_0+iy_0)$.

To obtain the image system of the vortex, we transform the region of interest into the upper half of the real axis in the $s(c+id)$- plane. The transformation is accomplished through two sub-transformations; in the first sub-transformation, the entire fluid region of the *z*-plane is transformed into an infinite strip between two parallel straight boundaries (Fig. 2) in $\zeta(=\xi+i\eta)$ plane by the relations:

$$z = a \coth \zeta/2 \qquad (1)$$

which is equivalent to

$$\frac{x}{a} = \frac{\sinh \xi}{\cosh \xi - \cos \eta} \text{ and } \frac{y}{a} = \frac{\sin \eta}{\cosh \xi - \cos \eta} \qquad (2)$$

The correspondence being $E_\infty, A_\infty, D_\infty \to \zeta = 0, i.e.,$ we have made E_∞, A and D_∞ coincident and made them correspond to $\zeta = 0$; $B \to \zeta = -\infty$, $C \to \zeta = \infty$, $0 \to \zeta = -i\pi$ and $N \to \zeta = -i\frac{3}{2}\pi$.

It may be noted that points on the real axis in *z*-plane between B and C is represented by $\eta = -\pi$, the semi-circular arc BNC by $\eta = -\frac{3}{2}\pi$ and the semi-circular region OBNC is transformed to the region $-\frac{3}{2}\pi \leqslant \eta \leqslant \pi$.

In the second sub-transformation, the infinite strip of breadth $\frac{3}{2}\pi$ in the ζ-plane is transformed into the upper half of the real axis of the s $(c+id)$-plane (Fig. 3) by using the theorem of Schwarz and Christoffel. Let us open out the upper edge of the strip at the origin 0, and shift it to infinity in *s*-plane and regard all points at negative infinity and positive infinity coincident separately and map the points B_∞, B_∞ on $s = -1$ (B) and C_∞, C_∞ on $s = 1$(C) so that N becomes the origin $s = 0$. the said theorem gives

$$d\zeta/ds = K'(s-1)^{-1}(s+1)^{-1}$$

whence $\qquad \zeta = \frac{K'}{2} \log \frac{s-1}{s+1} + \qquad (3)$

K and K' being constants. From the correspondance of the points, we get $K = -3$ and $K' = 0$

$$\therefore \zeta = \frac{3}{2} \log \frac{s+1}{s-1} \qquad (4)$$

or $\qquad s = \coth \zeta/3 \qquad (5)$

which gives the transformation relation between the ζ and *s*-plane. To obtain the relation the
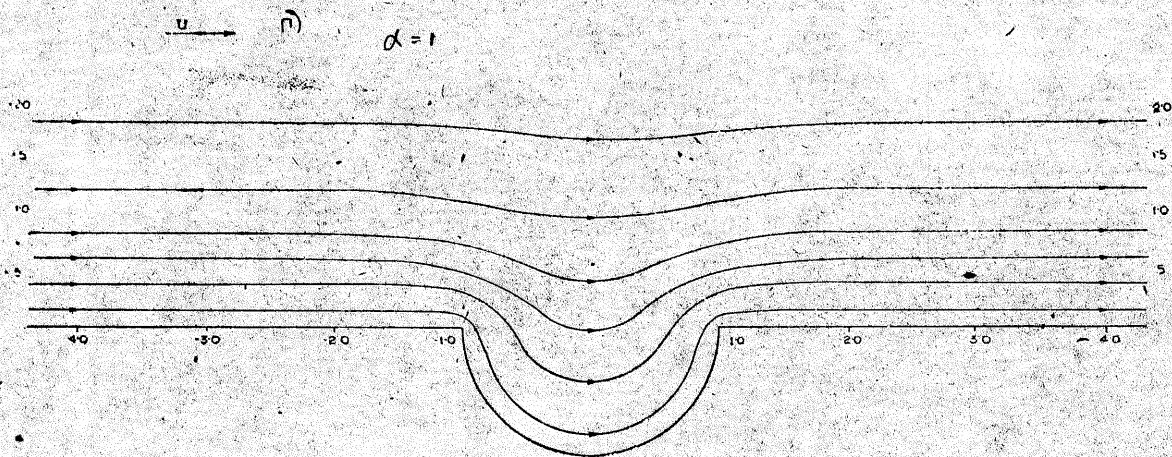
Fig. 4

original z-plane and final s-plane the relation (1) can be rewritten in the form :

$$\zeta = \log \frac{z+a}{z-a}$$

Hence the transformation relation between the z-plane and s-plane can be obtained as :

$$s = \frac{(z+a)^{2/3} + (z-a)^{2/3}}{(z+a)^{2/3} - (z-a)^{2/3}} \qquad (6)$$

3. Complex potentials and the equation of the path of the vortex

A uniform stream in z-plane may be taken to imply a source at A_∞ and an equal sink at D_∞. Thus in the s-plane we must also have a source and a sink at the corresponding points so that there is a uniform stream, V say, in the upper half plane, since the boundary has become straight in the s-plane. Hence the complex potential is $W_0 = -Vs$ and, therefore,

$$-\frac{dW_0}{dz} = V\frac{ds}{d\zeta} \times \frac{d\zeta}{dz} = \frac{2}{3a} V \frac{\operatorname{cosech}^2 \zeta/3}{\operatorname{cosech}^2 \zeta/2}$$

But at infinity $\zeta = 0$ and $-\dfrac{dW_0}{dz} = U$

Hence $\qquad U = \dfrac{3}{2a} V$

Thus $\qquad W_0 = -\dfrac{2a}{3} Us \qquad (7)$

The vortex at z_0, in the z-plane, will remain at the corresponding point, s_0, in the s-plane without change of its strength. Therefore in the s-plane,

the complex potential W at any point s, for the motion of the fluid in presence of uniform stream and a vortex at s_0, is given by

$$W = -\frac{2a}{3} Us + i\Gamma \log(s-s_0) - i\Gamma \log(s - \overline{s_0}) \qquad (8)$$

where $\overline{s_0}$ is the complex conjugate of s_0 and Γ is the strength of the vortex which remains constant in every plane.

Hence the stream function χ_1, for the motion of the vortex itself in s-plane is given by :

$$\chi_1 = -\frac{2}{3} aUd - \frac{\Gamma}{2} \log \overline{l_0} \qquad (9)$$

or in current co-ordinates :

$$\chi_1 = -\frac{2}{3} aUd - \frac{\Gamma}{2} \log d \qquad (10)$$

Now, making use of Routh's theorem, the path of the vortex in the z-plane is given by $\chi = $ constant, where,

$$\chi = -\frac{2}{3} aUd - \frac{\Gamma}{2} \log d + \frac{\Gamma}{2} \log \left| \frac{ds}{dz} \right| \qquad (11)$$

in which all the terms in the r.h.s. are to be expressed in terms of the co-ordinates in the z-plane. To obtain the result in non-dimensional form the length will be referred to a, the radius of the semi-circular hump.

Applying the transformation relation (6) and using bipolar co-ordinates the stream function χ, in the original z-plane in non-dimensional form has been obtained as :

$$\chi = -\frac{4}{3}aU \; \frac{R_1^{2/3} R_2^{2/3} \sin\frac{2}{3}(\phi_2 - \phi_1)}{R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)} - \frac{\Gamma}{2} \log\left[\frac{3a^2}{4} R_1 R_2 \sin\frac{2}{3}(\phi_2 - \phi_1)\right]$$

or putting $\lambda = \exp.\left(\frac{-2\chi}{\Gamma} - \log\frac{3a^2}{4}\right)$

$$\lambda = R_1 R_2 \sin\frac{2}{3}(\phi_2 - \phi_1) \exp.a \; \frac{R_1^{2/3} R_2^{2/3} \sin\frac{2}{3}(\phi_2 - \phi_1)}{R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)}$$

where, $a = \frac{8aU}{3\Gamma}$, $z = are^{i\theta}$, $z + a = aR_1 e^{i\phi_1}$ and $z - a = aR_2 e^{i\phi_2}$

The Eqn. (12) gives the entire family of paths of the vortex in the orignial z-plane.

4. Velocity components and stationary points

Let u and v be the velocity components of the vortex in the direction of x and y respectively, therefore,

$$u = \frac{\partial \chi}{\partial y} = \left[\frac{2}{3}aU + \frac{\Gamma}{2d}\right] \text{real } \frac{ds}{dz} - \frac{\Gamma}{2} \frac{\partial}{\partial y} \log\left|\frac{ds}{dz}\right| \tag{13}$$

$$v = \frac{\partial \chi}{\partial x} = -\left[\frac{2}{3}aU + \frac{\Gamma}{2d}\right] \text{Imaginary } \frac{ds}{dz} + \frac{\Gamma}{2} \frac{\partial}{\partial x} \log\left|\frac{ds}{dz}\right| \tag{14}$$

where $\frac{ds}{dz} = \frac{8a}{3} \times \frac{1}{(z^2 - a^2)^{1/3}[(z+a)^{2/3} - (z-a)^{2/3}]^2}$

Therefore, the velocity components in bipolar co-ordinates are given by :

$$u = \frac{16aU}{9} \times \frac{R_1^{4/3} \cos\frac{1}{3}(5\phi_1 + \phi_2) + R_2^{4/3} \cos\frac{1}{3}(\phi_1 + 5\phi_2) - 2R_1^{2/3} R_2^{2/3} \cos(\phi_1 + \phi_2)}{(R_1 R_2)^{1/3}[R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)]^2}$$

$$+ \frac{2}{3}\Gamma \; \frac{R_1^{4/3} \cos\frac{1}{3}(5\phi_1 + \phi_2) + R_2^{4/3} \cos\frac{1}{3}(\phi_1 + 5\phi_2) - 2R_1^{2/3} R_2^{2/3} \cos(\phi_1 + \phi_2)}{R_1 R_2 \sin\frac{2}{3}(\phi_2 - \phi_1)[R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)]}$$

$$+ \frac{\Gamma}{3R_1^2 R_2^2}\left[3r(a+r^2)\sin\theta - \frac{2(R_1^{4/3} - R_2^{4/3}) r^2 \sin 2\theta + 4(a^2 - r^2 \cos 2\theta) R_1^{2/3} R_2^{2/3} \sin\frac{2}{3}(\phi_2 - \phi_1)}{R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)}\right] \tag{15}$$

$$v = \frac{16aU}{9} \times \frac{R_1^{4/3} \sin\frac{1}{3}(5\phi_1 + \phi_2) + R_2^{4/3} \sin\frac{1}{3}(\phi_1 + 5\phi_2) - 2R_1^{2/3} R_2^{2/3} \sin(\phi_1 + \phi_2)}{(R_1 R_2)^{1/3}[R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)]^2}$$

$$+ \frac{2}{3}\Gamma \; \frac{R_1^{4/3} \sin\frac{1}{3}(5\phi_1 + \phi_2) + R_2^{4/3} \sin\frac{1}{3}(\phi_1 + 5\phi_2) - 2R_1^{2/3} R_2^{2/3} \sin(\phi_1 + \phi_2)}{R_1 R_2 \sin\frac{2}{3}(\phi_2 - \phi_1)[R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)]}$$

$$+ \frac{\Gamma}{3R_1^2 R_2^2}\left[3r(a^2 - r^2)\cos\theta + \frac{2(r^2 \cos 2\theta - a^2)(R_1^{4/3} - R_2^{4/3}) + 4r^2 \sin 2\theta \, R_1^{2/3} R_2^{2/3} \sin\frac{2}{3}(\phi_2 - \phi_1)}{R_1^{4/3} + R_2^{4/3} - 2R_1^{2/3} R_2^{2/3} \cos\frac{2}{3}(\phi_2 - \phi_1)}\right] \tag{16}$$
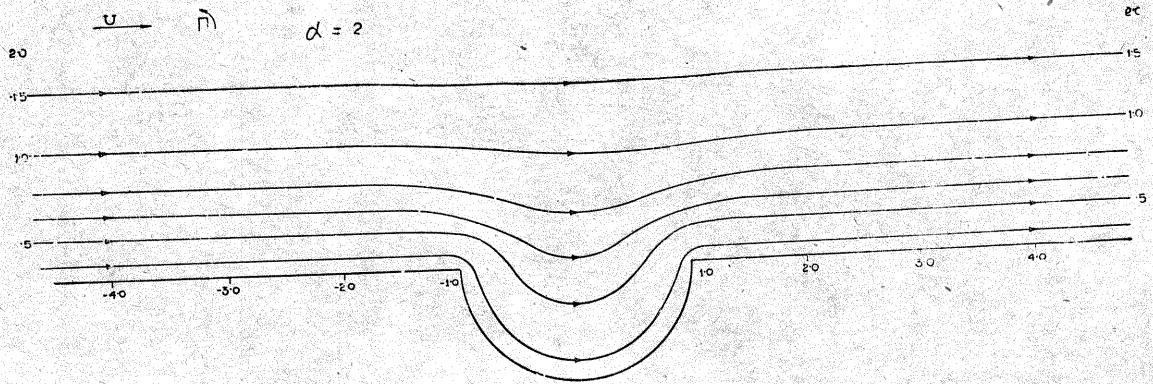
Fig. 5

At a large distance from the origin, *i.e.*, when $z \to \infty$

$$\frac{ds}{dz} \to \frac{3}{2a}, \quad c \to \frac{3}{2a} x \quad \text{and} \quad d \to \frac{3}{2a} y$$

Therefore from Eqns. (13) and (14) :

$$u \to U + \Gamma/2y \quad \text{and} \quad v \to 0$$

i.e., the path of the vortex becomes parallel to the x-axis and no stationary point can exist when both U and Γ have same sign. At a large distance, u can become zero when U and Γ have opposite sign and at a distance $y = -\Gamma/2U$ form the straight boundary wall the velocity of the vortex is nil.

On the y-axis, $R_1 = R_2$, $\theta = \frac{\pi}{2}$, $r = y$

and $\phi_1 + \phi_2 = \pi$

Let $\phi_2 - \phi_1 = A$,

$$\therefore \quad \phi_2 = \frac{\pi}{2} + \frac{A}{2} \quad \text{and} \quad \phi_1 = \frac{\pi}{2} - \frac{A}{2}$$

It can be shown from Eqn. (16) that, v vanishes on y-axis for any finite values of U and Γ, whether positive or negative. At any point on y-axis, u can also become zero, when the value of A, satisfies the equation :

$$a = -2 \tan A/3 - 6y \sin^2 A/3 + 2 \sin 2A/3$$

where $\quad a = 8aU/3\Gamma$ (17)

since, as $A \to \frac{3\pi}{2}$, $y \to -1$ and $a \to -\infty$

and as $A \to 0$, $y \to \infty$ and $a \to 0$

i.e., in the range $0 \leqslant A \leqslant 3\pi/2$ r.h.s. of Eqn. (17) is negative, therefore, the stationary point can exist on y-axis only for negative values of a, *i.e.*, when U and Γ have opposite sign.

5. Technique for drawing the curves

In the Eqn. (12) giving the path of the vortex, there are two parameters, *viz.*, α and λ for certain assigned values of α (which depends upon the strength and sign of the uniform stream and vortex), λ will have different constant values along different paths of the vortex. Giving a specific values to α, the values of λ have been calculated at a number of points of different radius vectors in the z-plane. Then by the method of interpolation, the points having a particular value of λ, have been joined, which gives a path of the vortex. In this way different paths at suitable intervals have been drawn. For positive values of α, λ is zero at the boundary and gradually increases with increasing y. But for negative values of α and at any constant value of x, λ increases from zero at the boundary, attains a maximum value and again decrease to zero at infinite value of y. A critical curve (of partial maxima) for negative α, joining different maximum values of λ on different x, has been drawn in each case by dashes. Using computer a few curves in each case for four typical values of α, *viz.*, $\alpha = \pm 1, \pm 2$ have been calculated numerically as shown in Fig. 4($\alpha = 1$), Fig. 5 ($\alpha = 2$), Fig. 6 ($\alpha = -1$) and Fig. 7($\alpha = -2$).

6. Discussion of the results

In all cases the paths are symmetrical about the imaginary axis. For the positive values of α (Figs. 4 & 5) the paths are open and simple, as if, the vortex floats with the stream and no stationary point exists. In the vicinity of the curve wall the paths are curved but away from it, gradually it becomes straight. The influence of the curved wall on the path with distance diminishes with the increasing value of α.

For negative values of α, the path show some distinctive features. In this case the stationary points always do exist on the imaginary axis
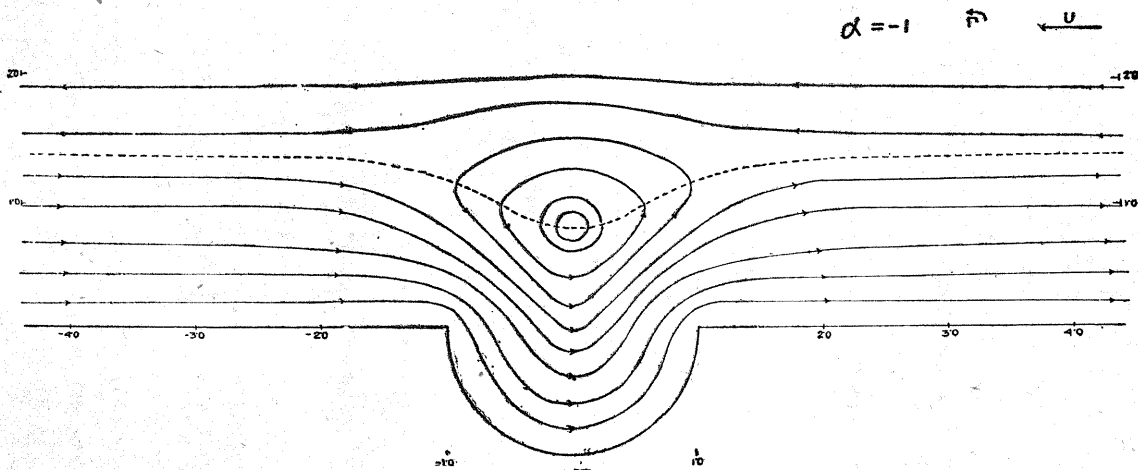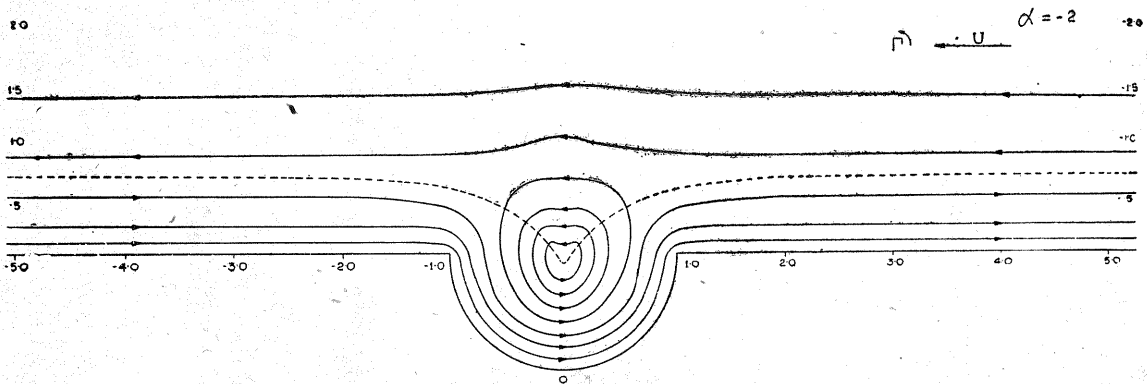
Fig. 6

Fig. 7

and its position on it changes from $y=-1$ to $y=\infty$ as α varies from $-\infty$ to 0 respectively. At the stationary point, the value of λ is maximum and the paths make loops around it. The stationary point is the vortex point of the path of the vortex. As shown in Figs. 6 and 7, the critical curves of partial maxima of λ, shown by dashes, always pass through the stationary point and at a large distance from the curved boundary, it become very nearly parallel to the straight boundary wall. The velocity of the vortex along the tangent of the curve is zero. As the stagnation point shifts away from the boundary with the increasing value of α, this critical curve, gradually becomes straight and loops become more flat. At large distance where the effect of the curved boundary is not significant, these curves become parallel to the straight boundary and vortex moves in opposite direction above and below it.

7. The path of the vortex in case the wall has a semi-circular hump

In this case of z-plane (Fig. 8 the transformation relation (6) becomes:

$$s = (z^2+a^2)/2az \qquad (18)$$

To obtain the results in non-dimensional form in the original z-plane, as in earlier case the length will be referred to a, the radius of the semi-circular hump.

The relation (18) is equivalent to:

$$c = \frac{(r^2+1)\cos\theta}{2r}, \quad d = \frac{(r^2-1)}{2r}\sin\theta \qquad (19)$$

where, $\quad z = r\,e^{i\theta}$

The complex potential W, at any point, s for the motion of the fluid in presence of uniform stream and a vortex at s_0 is given by :

$$W = -2aUs + i\Gamma\log(s-s_0) - i\Gamma\log(s-s_0) \qquad (20)$$

Hence the stream function χ, for the path of the vortex in the z-plane in non-dimensional form in presence of uniform stream at infinity is given by :

$$\chi = \frac{-Ua(r^2-1)}{r}\sin\theta - \frac{\Gamma}{2}\log\frac{ar(r^2-1)\sin\theta}{(r^4-2r^2\cos\theta+1)^{1/2}}$$
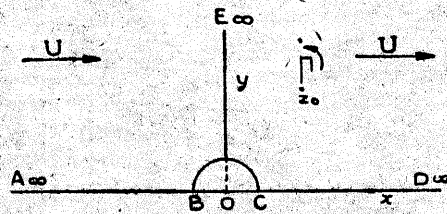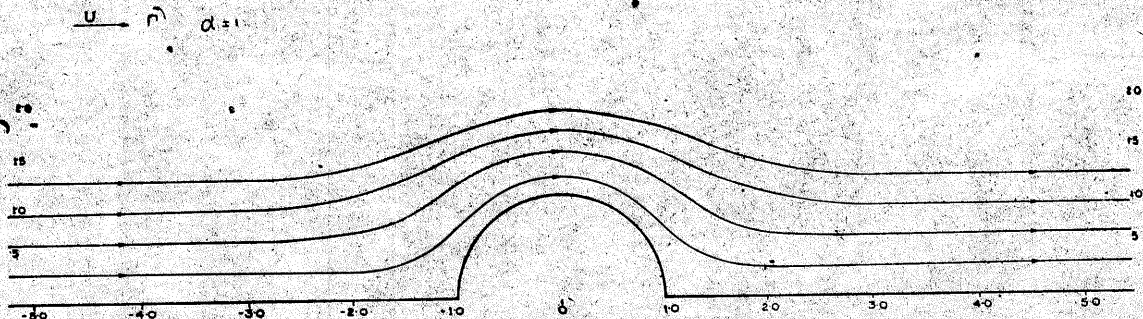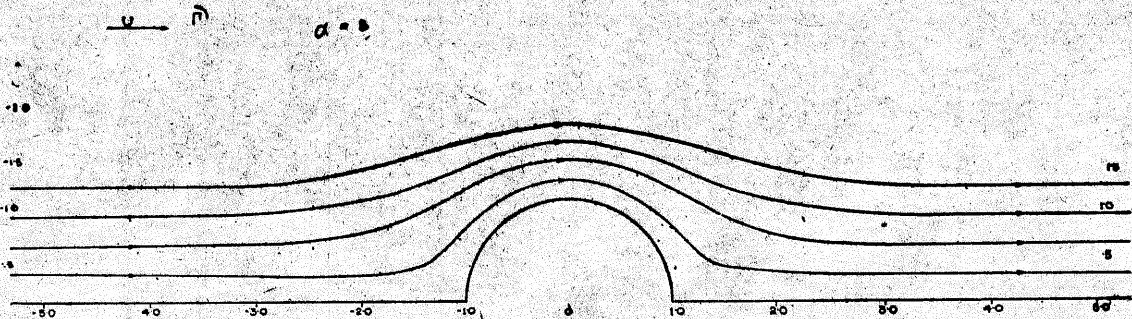
Fig. 8. z-plane

Fig. 9

Fig. 10

or putting $\lambda = \exp.\,(-2\chi/\Gamma - \log a)$

$$\lambda = \frac{r(r^2-1)\sin\theta}{(r^4-2r^2\cos 2\theta+1)^{1/2}}\exp.\,\frac{a'(r-1)}{r}\sin\theta \tag{21}$$

where $\quad a' = 2aU/\Gamma$

As in the first case, the Eqn. (21) gives the entire family of the paths of the vortex in original z-plane.

The velocity components q_r and q_θ of the vortex, in the radial and cross radial directions respectively, derived from stream function are :

$$q_r = -\frac{\partial\chi}{r\partial\theta} = \frac{\Gamma}{2}\cos\theta\,\frac{[r^5-2r^3+r+a'\{(r^6-3r^4+3r^2-1)\sin\theta+4(r^4-r^2)\sin^3\theta)]}{r^2(r^4-2r^2\cos 2\theta+1)\sin\theta} \tag{22}$$

$$q_\theta = \frac{\partial\chi}{\partial r} = -\frac{\Gamma}{2}\,\frac{r^7-3r^5+3r^3-r+8r^5\sin^2\theta+a'(r^8-2r^6+2r^2-1)\sin\theta+a'4r^4(r^4-1)\sin^2\theta}{r(r^2-1)(r^4-2r^2\cos 2\theta+1)} \tag{23}$$
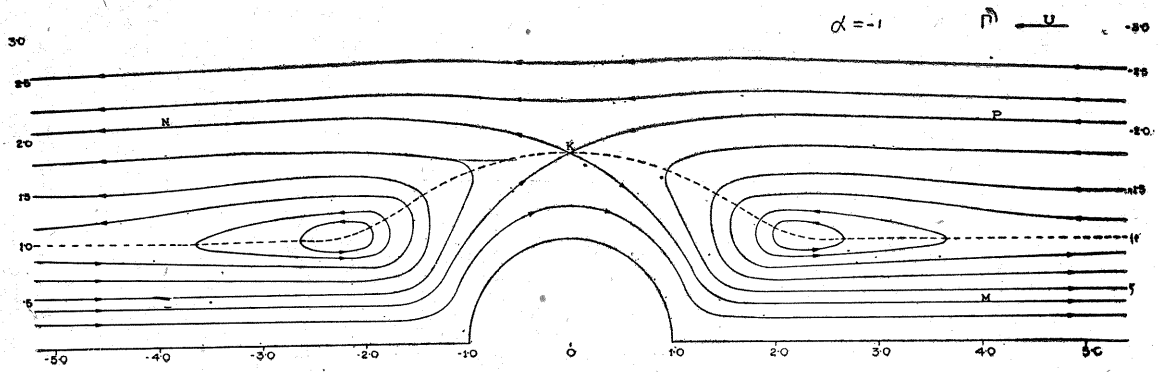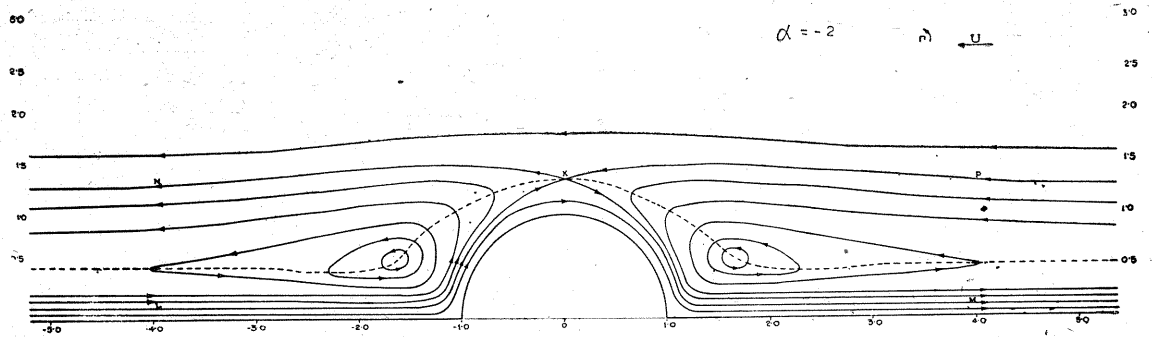
Fig. 11

Fig. 12

(a) *Stationary points*

At stationary points both q_r and q_θ must be zero.

The radial component of velocity q_r has two factors in its numerator, so in order q_r to be zero either $\cos \theta = 0$ or terms within [ ] is zero.

Case (i) : When $\theta = \dfrac{\pi}{2}$, $\cos \theta = 0$ and $q_r = 0$

Putting $q_\theta = 0$ from Eqn. (23) we get :

$$\alpha' = -\frac{r^5 + 4r^3 - r}{r^6 + r^4 - r^2 - 1} \qquad (24)$$

Since for, $1 \leqslant r$, r.h.s. of Eqn. (24) is always, negative so on the imaginary axis q_θ also can be zero only for negative values of α' :

as $\qquad r \to 1,\ \alpha' \to -\infty$
and $\qquad r \to \infty,\ \alpha' \to 0$

Therefore, the stationary point on the imaginary axis moves from $r = 1$ to $r = \infty$ as, α' increases from $-\infty$ to 0.

Case (ii) : $\cos \theta \neq 0$

In this case the second term in the numerator of q_r and numerator of q_θ to be zero; which on simplification reduce to the following two equations in r and θ :

$$4\alpha' r^2 (r^2 - 1)\sin^3 \theta + \alpha'(r^2 - 1)^3 \sin \theta + r(r^2 - 1)^2 = 0 \qquad (25i)$$

$$4\alpha' r^2 (r^2 - 1)(r^2 + 1)\sin^3 \theta + 8r^5 \sin^2\theta +$$
$$\alpha'(r^2 - 1)^3 (r^2 + 1)\sin \theta + r(r^2 - 1)^3 = 0 \qquad (25ii)$$

Though these two equations are sufficient to get the values of r and θ in terms of α' for the stationary points, yet is is difficult to solve it.

Multiplying Eqn. (25i) by $(r^2 + 1)$ and subtracting it from Eqn. (25ii), we get the following relation between r and θ, α' being eliminated :

$$\sin \theta = \pm \tfrac{1}{2}\left(1 - \frac{1}{r^2}\right) \qquad (26)$$

These are equations of two symmetrical curves about imaginary axis and the second set of stagnation points must lie on these curves for different values of α'. Using Eqn. (26) and eliminating $\sin \theta$, from either of the two Eqns. of (25), we get a relation between r and α', as below which will locate the stagnation point on the stagnation curves of Eqn. (26) for particular value of α' :

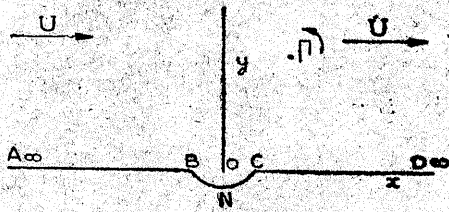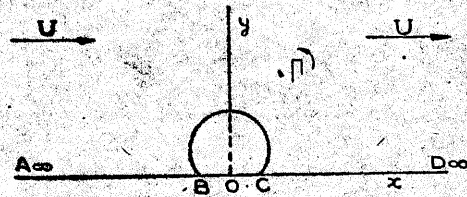$$\alpha' = -\frac{2r^5}{(r^2 - 1)^2 (r^2 + 1)} \qquad (27)$$

Fig. 13

Fig. 14

since, for $1 < r$, r.h.s. of Eqn. (27) is always negative and when $r \to 1$, $\alpha' \to -\infty$ and when $r \to \infty$, $\alpha' \to 0$. Therefore, the second set of stagnation points can exist, only for negative value of α' and move on the curves of Eqn. (26) from $r = 1$ to $r = \infty$ as α' increases from $-\infty$ to 0.

(b) Discussion of the results

The same method as in the case of bay has been used to draw the paths of the vortex. Taking four typical values of α', viz., $\alpha' = \pm 1$, ± 2, Fig. 9 ($\alpha' = 1$), Fig. 10 ($\alpha' = 2$), Fig. 11 ($\alpha' = -1$) and Fig. 12 ($\alpha' = -2$) have been drawn from numerical results.

For both negative and positive values of α', the paths are symmetrical about the imaginary axis.

For the positive value of α' (Figs. 9 and 10) the paths are open and simple, as if the vortex floats with the stream. No stationary point exists in this case. In the vicinity of the curved wall, the paths are curved but away from it, gradually it becomes straight. The influence of the curved wall on the path with distance diminishes with increasing values of α'. For negative α' (Figs. 11 and 12), there are three sets of stationary points, one of the imaginary axis and the other two on the curves $\sin\theta = \frac{1}{2}(1 - 1/r^2)$. A critical curve of partial maxima of λ has been drawn with dashes. The velocity of the vortex along the tangent of this critical curve is zero and it always passes through the stagnation points. Above and below to the critical curve, the movement of the vortex are in opposite direction. At a large distance to the left or right of the hump, this curve becomes nearly parallel to the straight wall.

The critical paths (in Figs. 11 and 12,) LK and PK meet from opposite direction and KM and KW move away in the opposite directions from the stationary point K on the imaginary axis. The critical paths divide the whole region into four distinct sectors such that the vortex in one cannot enter into other. Above PKN and below LKM, the vortex moves in opposite directions from infinity to infinity, while within the sectors PKN and LKN, it makes loop around the second set of symmetrically situated stationary points. With decreasing numerical values of $-\alpha'$, the loops becomes flat.

The stationary points on the imaginary axis is a saddle point or col and other two are the vortex point of the path of the vortex.

8. Generalisation of the problem

Since, in the transformed ζ-plane (Fig. 2) any particular value of η in $-2\pi < \eta < 0$ corresponds to the arc of the circle $x^2 + (y + a\cot\eta)^2 = a^2 \operatorname{cosec}^2\eta$ in the z-plane passing through B and C, the centre being on y-axis. The arc represents a bay or hump on the straight wall in a z-plane according as $-\pi < \eta$ or $\eta < -\pi$. Any infinite strip in ζ-plane from $\eta = 0$ to $\eta =$ any value $> -2\pi$, represents semi-infinite region with bay or hump in z-plane (Fig. 13).

Any infinite strip in the ζ-plane can be transformed to the upper half of the s-plane by using the Eqn. (3), in which the values of the constant K will be different for different width of the strip and the value of K will lie in the range $-4 \leqslant K < 0$.

When the bay or hump are not semi-circular but a part of a circle passing through two fixed points as given by the equation in the first para of this section it can still be tackled by transforming the fluid regime to a half plane as in the cases discussed.

With the notation already used, the final transformation for general case (Fig. 14) is given by:

$$s = c + id = \frac{(z+a)^{-\pi/\eta} + (z-a)^{-\pi/\eta}}{(z+a)^{-\pi/\eta} - (z-a)^{-\pi/\eta}} \qquad (28)$$

where $-2\pi \leqslant \eta < 0$

The value of η can be found out from the equation of the bay or hump.

9. Conclusions

The stagnation points determined from both the graphical and analytical methods have been found to agree with each other.

The problems in presence of more than one vortex can also be solved by the method used in this paper.

Since, this paper deals with the motion of vortex in perfect fluid and in two dimensional uniform stream it has its own limitation when applied to natural atmospheric vortices, in which case the effect of friction and the variation of strength of the vortex with height are to be considered and also the stream in which it is embedded is not uniform. However, the orographical effect on the motion of vortex as found out in this paper may be helpful in practical cases.

Acknowledgements

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