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Lunar Geomagnetic Tides in the low latitudes region

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ABSTRACT. The lunar geomagnetic tides at a few representative stations in the low latitude region have been synthesized to get a unified picture of the L field in the equatorial region. The general features in the low latitudes—like the equatorial anomaly, seasonal variations, geomagnetic control., latitudinal variation have been studied. The characteristics of the low latitude variations are compared with those of high latitude variations.

1. Introduction

Our knowledge of the lunar geomagnetic tides is very meagre, compared to the solar counter part. Partly, it is due to the fact that amplitude of the lunar geomagnetic tide is very small in comparison with the solar geomagnetic tide, being approximately one tenth of the solar, and the determination of the lunar tides involves processing of a large volume of data through a laborious method of computation, although theoretically the lunar daily variation in the geomagnetic field can be understood with more definiteness than the solar one. The lunar daily variation in the geomagnetic field is caused by the movement of ionised matter in the ionosphere (at a level which still remains to be determined) across the lines of the earth's main magnetic field. The movement of ionised air is caused by the gravitational attraction of the moon on the ionospheric layer. This, in substance, is the dynamo theory of the L field.

2. Importance of the study of L field in low latitude region

The geomagnetic variations in the low latitude region display some interesting features, the study of which has stimulated very fundamental theoretical work in geomagnetic and ionospheric research. For example, the opening of the Huancayo Magnetic Observatory soon revealed a surprising phenomenon hitherto unsuspected : namely the quiet day variation S_q in horizontal intensity is twice as great as that at any other tropical magnetic observatory existing at that time. Study of this anomalous variation led to very important contributions by Martyn, Hirono, Fejer, and Meida on the conductivity in the ionosphere in the geomagnetic equatorial region modified by Hall currents, Polarisation etc.

Later it appeared that at Huancayo the lunar daily variation L in H is even more abnormally large. In addition to this abnormality, the L shows peculiar seasonal variation. For example at Ibadan (geomagnetic Lat. $10°\cdot5$ N and geographic Lat. $7°\cdot5$ N) and Kodaikanal (geomagnetic Lat. $0°\cdot6$ and geographic Lat. $10°\cdot2$ N) the seasonal variation in L is exactly of the type noticed at Huancayo, *viz.*, larger amplitudes in the December solstice and smaller amplitudes in the June solstice. Therefore, to study the complete physical picture of the L field in the low latitude region, the lunar tides have been determined for a few representative stations for each of the three seasons. The co-ordinates of the stations are given in Table 1 (See Fig. 1).

The results obtained by earlier investigators for Huancayo and Ibadan are used and a synthesis of the L field in the low latitude region is made. The results for Kodaikanal, Alibag and Apia obtained by the author are being published separately. But they have been included here for the purpose of synthesis.

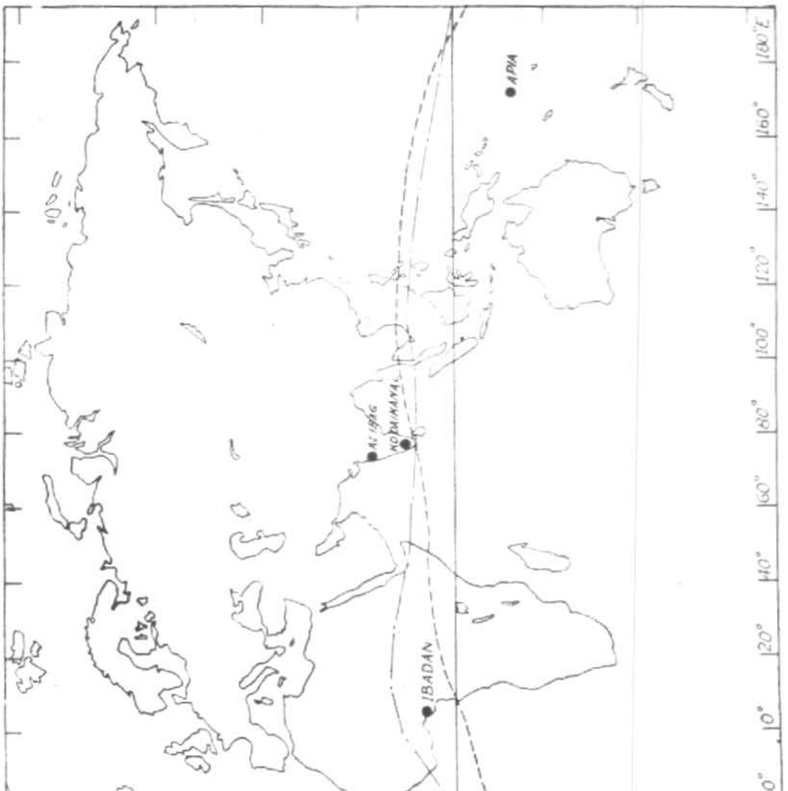

Fig. 1. Location of observatories with respect to the geomagnetic and geographic equators

3. Determination of lunar geomagnetic tides

In general the lunar daily variation can be expressed by the phase law

$$\Sigma L_n \sin (nt - 2\nu + \lambda_n)$$

Where t is the solar time, ν the phase of the moon and λ the phase angle, and n the order of the harmonic. As the solar time t is connected with the lunar time according to the relation $t = \tau + \nu$, the phase law can also be written as

$$\Sigma L_n \sin [n\tau + \lambda_n + (n-2)\nu]$$

$$\text{or } \Sigma L_n \sin [nt - 2\nu + \lambda_n]$$

Therefore in the study of the L field the three independent pairings (t, τ), (τ, ν) and (t, ν) are possible.

From the observed data, the analysis of a function of two variables can be done by regarding one variable as a parameter (kept constant) and studying the variation of the function with respect to other variable. In the case of $L(t, \nu)$, t may be regarded as constant and variation of L with respect to ν may be studied. This is called the "fixed hour" method or the so called "Van der Stokes" method. Bartels, Egedel, Bossolasco and others have used this method extensively in the calculation of the L geomagnetic tides at Huancayo, Ruda Skov etc.

The other method is to choose the pair (τ, ν). Regarding ν as constant the variation of L with respect to τ may be studied. This is the fixed age method. This method has been developed by Chapman and Miller (1940). Details of the method of computation have been described in an earlier paper (Raja Rao 1958). Following this method the lunar geomagnetic tides have been determined up to four harmonics and for the three seasons—December solstice, Equinox and June solstice, for the low latitude stations for the period 1950-54, the minimum part of the solar cycle. The amplitudes and phases of the first four harmonics are given in Tables 2(a) and 2(b).

Probable errors have been computed for the principle component of the geomagnetic tide, *viz.*, the second component, according to the method outlined by Tschu (1949) and later by Chapman (1952). The number of daily sequences used N, the amplitude L_2 (H) of the second component of the lunar geomagnetic tide together with its probable error (PE) are given in Table 3.

The amplitudes and phases of the main component of the lunar geomagnetic tide in the three seasons for all the low latitude stations, have been represented in the form of the twelve hour harmonic dial in Fig. 2 with amplitude and the time of occurrence of maximum amplitude in lunar hours reckoned from local transit of mean moon.

4. Discussion of results

General features—The second harmonic is the prominent one for all the stations. L_4, the fourth harmonic is small and it can be reasonably assumed that L_5, L_6 etc, are negligible. Similar observation has been made by Chapman (1957) in his analysis of the Greenwich magnetic data. He has pointed out that in general for any integer m, the components L_{m+2} and L_{m-2} will have comparable amplitudes. It is seen for all the stations, L_1 and L_3 are nearly of the same magnitude.

Equatorial anomaly—At Kodaikanal, the amplitude of the L variation is very large as at Huancayo and Ibadan. The anomalous large variation in L in the geomagnetic equatorial region is even more pronounced than the anomalous S variation in the same region. The latter has been explained on the basis of the electrojet theory, as due to enhanced ionospheric conductivity in the geomagnetic equatorial region (Martyn 1953, Hirono 1950, 1953). It is very likely that this enhancement of the electrical conductivity in the ionosphere is responsible for the large L variations also in this region for, both S and L variations in the geomagnetic field are explained on the basis of the dynamo theory wherein the principal factors are the earth's main field, the conductivity of the ionised region and the velocity potential

TABLE 1

Geomagnetic observatories in low latitude region

Station	Geographic		Geomagnetic		Dip
	Lat.	Long.	Lat.	Long.	
Kodaikanal	10°·2N	77°·5E	0°·6N	147°·1E	3°·5N
Alibag	18°·6N	72°·9E	9°·5N	143°·6E	24°·0N
Honolulu	21°·3N	158°·1W	21°·1N	266°·5E	39°·2N
San Juan	18°·4N	66°·1W	29°·9N	3°·2E	52°·8N
Apia	13°·8S	188°·2E	16°·0S	260°·2E	30°·0S

TABLE 2(a)

Amplitude in the lunar diurnal variation in the December solstice, June solstice and Equinox

Station	D. Season				E. Season				J. Season			
	L_1	L_2	L_3	L_4	L_1	L_2	L_3	L_4	L_1	L_2	L_3	L_4
Kodaikanal	2·20	3·32	0·93	0·30	1·08	2·28	1·50	0·67	0·95	1·43	1·18	0·46
Alibag	0·41	1·27	0·62	0·14	0·78	1·02	0·70	0·28	0·71	1·07	0·22	0·12
San Juan	0·52	1·10	0·47	0·12	0·61	0·81	0·50	0·18	0·70	1·39	0·64	0·18
Honolulu	0·65	1·08	0·51	0·17	0·73	1·01	0·39	0·22	0·65	1·22	0·23	0·18
Apia	0·70	1·21	0·61	0·30	0·73	0·98	0·56	0·02	0·57	1·08	0·48	0·20
Huancayo	7·1	8·0			4·3	5·9			2·6	2·4		
Ibadan	0·91	4·70	0·52	0·33	1·25	3·82	1·32	0·07	0·99	3·37	0·43	0·22

TABLE 2(b)

Phase angles of the first four harmonics of the lunar diurnal variation

Station	D. Season				E. Season				J. Season			
Kodaikanal	70°36′	39°04′	42°36′	83°18′	31°45′	154°24′	108°54′	59°46′	12°48′	64°24′	218°24′	119°02′
Alibag	22°34′	214°28′	4°38′	106°30′	17°08′	170°32′	17°22′	233°28′	17°36′	203°27′	13°27′	60°28′
San Juan	171°44′	150°58′	313°36′	206°38′	100°00′	139°28′	318°18′	179°54′	49°54′	148°57′	268°42′	188°00′
Honolulu	352°12′	104°30′	265°18′	44°48′	218°06′	341°24′	45°48′	78°00′	181°27′	33°20′	178°15′	280°00
Apia	95°25′	331°07′	350°39′	322°48′	143°15′	168°32′	127°55′	184°00′	104°59′	353°18′	118°27′	158°24
Ibadan	56°	121°	166°	83°	19°	129°	167°	44°	02°	129°	19°	65°

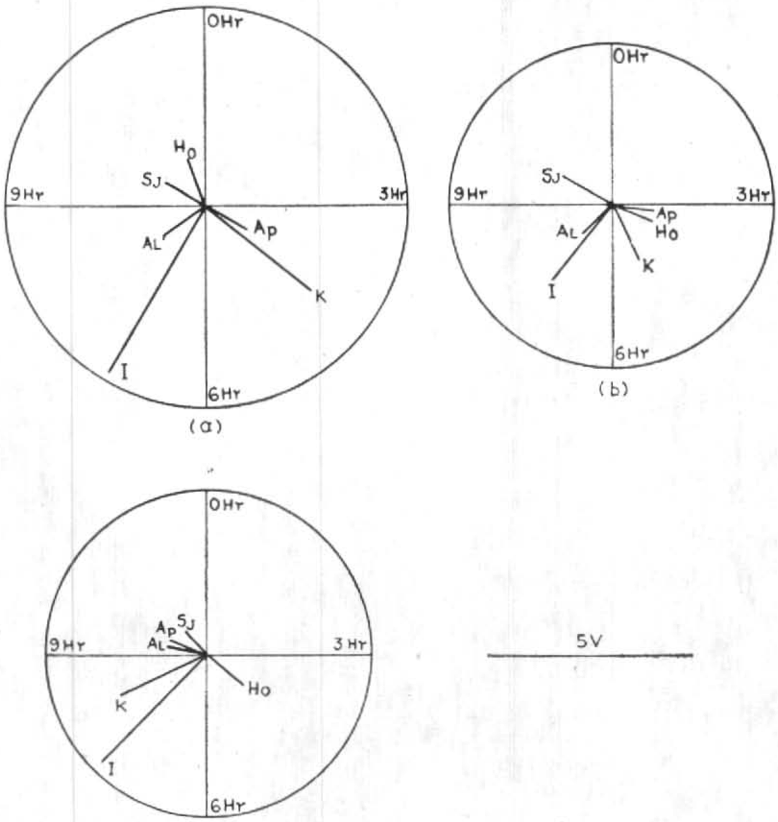

Fig. 2. Harmonic dials for the second component of lunar geomagnetic tides in the low latitude region, (a) December solstice, (b) June solstice, (c) Equinox

(AL—Alibag, Ap—Apia, Ho—Honolulu, I—Ibadan, K—Kodaikanal, Sj—San Juan)

TABLE 3

Number of daily sequences and probable errors for $L_2(H)$

Station	D. Season			E. Season			J. Season		
	N	$L_2(H)$	$P.E.$	N	$L_2(H)$	$P.E.$	N	$L_2(H)$	$P.E.$
Kodaikanal	520	3·32	0·90	497	2·28	0·85	578	1·43	0·55
Alibag	613	1·27	0·26	682	1·02	0·26	557	1·07	0·29
San Juan	196	1·10	0·38	228	0·81	0·29	217	1·39	0·42
Honolulu	407	1·08	0·38	480	1·01	0·34	457	1·22	0·41
Apia	481	1·21	0·36	574	0·98	0·28	531	1·08	0·34

$L_2(H)$ and P.E. are expressed in γ

due to gravitational attraction. The S and L fields differ in the last two factors, namely, the velocity potential and conductivity in so far as it pertains to the region where the currents responsible for these variations flow. As large conductivity exists in the ionosphere even outside the seat of the S_q currents it may safely be concluded that there is an equatorial electrojet even for the L currents. Onwumechilli and Alexander (1959) have suggested that the large L variations at Ibadan may be due to the proximity of Ibadan to the equatorial electrojet causing large S_q variations. It is unlikely that the latter has any effect on the large L variations unless the S and L currents are produced in the same layer in the ionosphere. It has been shown by the author in the course of study of geomagnetic variations near the geomagnetic equator that the S and L vary independently of each other (paper under publication). Bartels and Johnston (1940) have also arrived at similar conclusion from the study of magnetic variations at Huancayo. It is, therefore, likely that there is an electrojet causing large L variations near the geomagnetic equator—different from the one causing large S variations.

Seasonal variations — In Kodaikanal, Alibag and Apia the amplitude of $L_2(H)$ is larger in the December solstice and smaller in the June solstice just as in Huancayo and Ibadan. San Juan and Honolulu show large L_2 variation in June and small variation in December like any other high latitude station such as Sitka. The large amplitude of L_2 in December and small amplitude in June observed at Kodaikanal, Alibag and Ibadan indicate that there is no symmetry of L field about the geomagnetic equator. The southern hemispheric current system in the ionosphere causing the lunar geomagnetic tides therefore, extends up to about 10°N geomagnetic latitude. The ratio of the amplitude in the December solstice to the amplitude in the June solstice gradually decreases as we go away from the geomagnetic equator, as shown in Table 4.

TABLE 4

Ratio of the amplitude of main lunar component in December to the same in June

Station	Geomagnetic Latitude	Dip.	$\frac{L_2\text{(December)}}{L_2\text{(June)}}$
Huancayo	$-0°\cdot6$	$20°\cdot5$	3·3
Kadaikanal	$+0°\cdot6$	$3°\cdot5$	2·3
Alibag	$+9°\cdot5$	25°	1·2
Ibadan	$+10°\cdot5$	−6°	1·4
Honolulu	$+21°\cdot1$	$39°\cdot2$	0·9
San Juan	$+29°\cdot9$	52°	0·8
Apia	−16°	−30°	1·1

This shows that there is some kind of geomagnetic control of the L variations.

Seasonal variation in the lunisolar components—The seasonal variation in the amplitudes and phases of the lunisolar components L_1, L_3, and L_4 are not quite systematic. The amplitudes of the first harmonic L_1 (H) increases from June to December solstice at Kodaikanal, Huancayo and Apia, the increase being more pronounced in Huancayo and Kodaikanal (nearly 2·5 times) due to the proximity of these stations to the geomagnetic equator. Thus the equatorial anomaly is displayed in $L_1(H)$ also, besides, $L_2(H)$, the main lunar component. At Alibag, Ibadan and San Juan there is decrease in amplitude from June to December solstice, just as in Sitka, a high latitude station. This is a further indication of the geomagnetic control of the L field.

The amplitude of $L_3(H)$ decreases from summer to winter solstice at Kodaikanal and San Juan only; at all other stations the amplitude increases.

The phase of $L_1(H)$ decreases from summer to winter at Alibag and Ibadan by 110° and 60° respectively. At all other stations there is a phase increase by about 120° in Sitka and San Juan, by 60° in Kodaikanal and by 10° in Apia. It appears that the decrease in the phase angle of $L_1(H)$ is confined to the geomagnetic latitude belt of 10° and elsewhere it is only an increase.

The phase of $L_3(H)$ decreases from summer to winter by about 20° at Apia, about 150° at Ibadan and by 176° at Kodaikanal. It, on the other hand, increases at Alibag by 20°, at San Juan by 45° and Sitka by 180°. This seasonal variation in the phase of $L_3(H)$ indicates some kind of relationship with the magnetic inclination which is negative at Apia and Ibadan, and small, though positive, at Kodaikanal. It is quite high at Alibag, San Juan and Sitka. Therefore, there is a decrease in the phase of $L_3(H)$ from summer to winter in low latitudes and an increase in higher latitudes.

The phase in $L_4(H)$ decreases from summer to winter at Kodaikanal, Alibag and Apia by about 35°, 45° and 170° respectively. At Ibadan there is an increase in the phase angle by about 150°. This type of phase change indicates that the seasonal variation in the phase of $L_4(H)$ appears to be a function of longitude, as there is increase from summer to winter only at Ibadan, it being a decrease at all other stations to the east.

The lunar and lunisolar components in the equinox—In the equinoctial season the amplitude of $L_1(H)$ is largest for all the seasons at Alibag, Apia and Ibadan and are intermediate between the summer and winter amplitudes at Kodaikanal, San Juan and Huancayo. The phase angles are highest for all the seasons at Apia and Ibadan, and intermediate between the summer and winter phases at Kodaikanal, Alibag and San Juan.

The $L_2(H)$ has amplitude intermediate between the summer and winter amplitudes at Huancayo, Kodaikanal and Ibadan, and they are the lowest for all the seasons at Alibag, San Juan and Apia. This feature shows some close association with the magnetic dip. At Apia, Alibag and San Juan, the dip value is very high and amplitudes are the lowest, in the equinoctial months. In this season, the phase is least in all the stations.

In $L_3(H)$ again the stations with smaller magnetic latitude, *viz.*, Kodaikanal, Ibadan and Alibag show largest value for amplitude. San Juan and Apia with larger dip values have amplitude intermediate between the summer and winter amplitudes. The phases do not disclose any definite relation to magnetic inclination. Kodaikanal and Apia have phases intermediate between those for summer and for winter, San Juan and Ibadan show largest and Alibag least value for the year.

$L_4(H)$ again displays some close connection with dip. Kodaikanal, Alibag and San Juan with positive dip values have largest amplitudes for the year. Apia and Ibadan with negative values of dip have the least values for amplitude. In phase also we find the similar feature. Apia and Ibadan have equinoctial phases midway between the summer and winter phases, while at Kodaikanal, Alibag and San Juan the equinoctial phases are largest for the year.

Variation with latitude—The annual values of the amplitude of $L_2(H)$ for various stations have been plotted against (*i*) Geographic latitude (*ii*) Geomagnetic latitude and (*iii*) against dip as shown in Fig. 3.

According to Chapman's dynamo theory the amplitude of the L variation is a function of geomagnetic latitude. Recent investigations of Baker and Martyn (1953), Hirono (1950, 1952) and Fejer (1953) have shown that there is an abnormally high conductivity in the ionosphere in the vicinity of the geomagnetic equator. Consequently, the dynamo theory will be able to explain the abnormally high amplitudes of $L(H)$ found near the geomagnetic equator. But the dependence on geographic latitude needs modification.

As the region of high conductivity extends only to a few degrees on either side of the geomagnatic equator, the occurrence of abnormally large $L(H)$ variations at Huancayo and Kodaikanal where geographic latitudes are 12°S and 10°·5 N, cannot be accounted for if geographic latitude dependence is considered. Secondly, Batavia and Apia where geographic latitudes are 6°·2 S and 13°·8 S

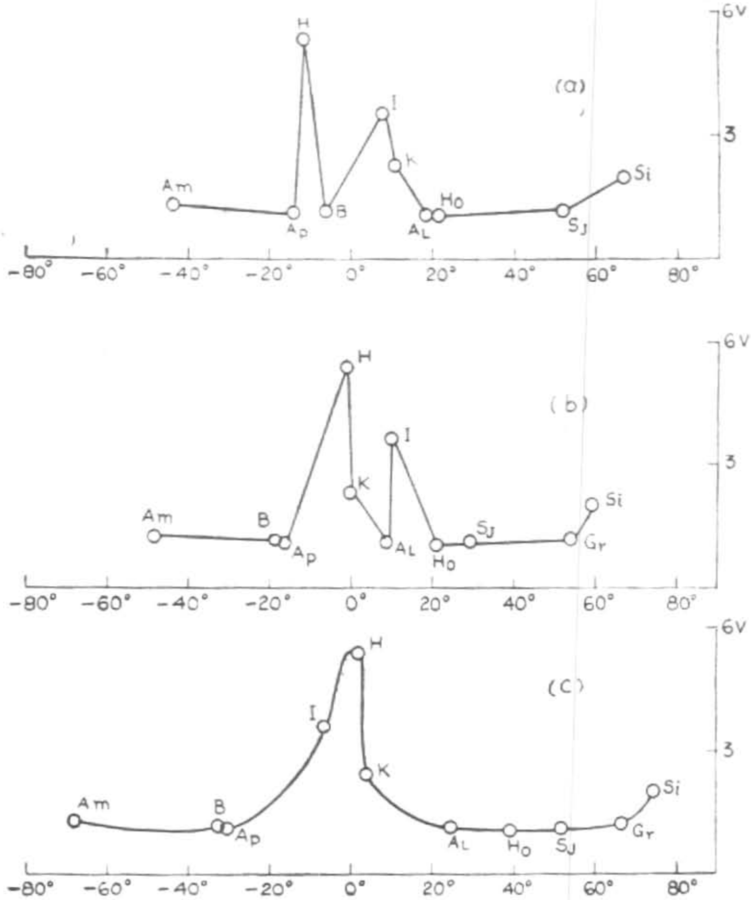

Fig. 3. Variation of the main components of lunar geomagnetic tide with (a) Geographic latitude, (b) Geomagnetic latitude, (c) Dip

(AL—Alibag, Ap—Apia, Am—Ambarkey, B—Batavia, H—Huancayo, Ho—Honolulu, I—Ibadan, K—Kodaikanal, Sj—San Juan, Gr— Greenwich, Si—Sitka)

respectively do not show any abnormal L variation. This is so because their geomagnetic latitudes are 17°·6 S and 16°·0 S respectively and their magnetic latitudes (inclination) are 32°S and 30°S respectively, and these two places are quite far away from the geomagnetic or the magnetic equator. Therefore, the observations in the low latitude region indicate that there is some type of geomagnetic control—either through geomagnetic latitude or through magnetic latitude—of the L field. This feature is not conspicuous in high latitudes because there the difference between the geographic on the one hand and geomagnetic or magnetic latitude on the other hand, is small. These characteristics have been clearly brought out in Figs. 3(a) to 3(c).

A choice between the geomagnetic and the magnetic latitude, is somewhat difficult. The magnetic latitude appears to be more suitable, for between Ibadan and Alibag—with geomagnetic latitude 10°·5 N and 9°·5 N respectively— Alibag is nearer the geomagnetic equator than Ibadan. But Ibadan shows abnormally large lunar geomagnetic tides, while Alibag shows normality. This fact is satisfactorily explained if we consider magnetic dip which is 6°S for

Ibadan and 25°N for Alibag. Hence the proximity of Ibadan to the magnetic equator explains the large amplitude of geomagnetic tide here. Moreover, as it has been shown in a separate communication, the geomangetic control of the solar geomagnetic tides can be explained better through the magnetic latitude than through the geomagnetic latitude. As the same theory —the dynamo theory—is applicable for the lunar and solar diurnal variations, it would be more appropriate if the dip is taken as the agency through which the geomagnetic control lunar variations, is exercised.

5. Conclusion

The large amplitudes of lunar geomagnetic tides at Huancayo, Ibadan and Kodaikanal indicate that there must be an equatorial electrojet for the L field also. The L field shows clear seasonal variation— larger amplitudes of $L_2(H)$ occurring in the December solstice and smaller one in the June solstice up to about 10°N geomagnetic latitude. This indicates that the southern hemispheric currents in the ionosphere responsible for the L magnetic variations extend up to 10°N geomagnetic latitude in the northern winter. The seasonal variations in the amplitudes and phases of $L_1(H)$, $L_3(H)$ and L_4 (H) in the low latitude region is of a different type from the variations at higher latitudes. In the equinox stations near the geomagnetic equator like Kodaikanal, Huancayo, Ibadan and Alibag show generally amplitudes intermediate between the summer and winter amplitudes in the first two harmonics. Other stations show equinoctial amplitudes highest for all the seasons. The plots of annual means of amplitudes for the main lunar components against geographic, geomagnetic and magnetic latitudes indicate geomagnetic control of L field. Dependence on magnetic latitudes appears to be clearer than the dependence on the geomagnetic latitudes.

There is clearly no dependence on geographic latitude. It, therefore, seems necessary that the dynamo equations have to be modified in order to allow for the variation with magnetic latitude, of the amplitudes of the semi-diurnal lunar geomagnetic tides.

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