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The calculation of maximum elevation due to storm surge by using joint probability method

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सार — इस शोध पत्र में संयुक्त प्रायिकता विधि का प्रयोग करके, प्रत्यागमन वर्षों वाली तूफान महोर्मि की अधिकतम ऊँचाइयों का परिकलन किया है । गुआंगडोंग राज्य के तटीय क्षेत्र को प्रभावित करने वाले ऐतिहासिक टाइफूनों के विश्लेषण के आधार पर, मॉडल टाइफूनों के एक समुच्चय का निर्माण किया गया । टाइफूनों से संबंधित कुछ प्राचलों का चयन किया गया । प्रत्येक प्राचल के आँकड़ों को, उनके मानों के आधार पर, कुछ उप-समुच्चयों में वर्गीकृत किया गया । ऐसा ऐतिहासिक प्रेक्षणों के आधार पर किया गया । ऐतिहासिक रिकार्ड के आधार पर प्राचलों के प्रत्येक मान की प्रायिकता ज्ञात की गई । प्राचलों के मानों के समुच्चय वाले टाइफूनों से उत्पन्न तूफान महोर्मियों के अनुसार परिणामों और उनकी प्रायिकताओं का सांख्यिकी विश्लेषण किया गया । इस प्रकार एक संचित प्रायिकता वक्र बनाया गया और कुछ प्रत्यागमन वर्षों वाली अधिकतम ऊंचाईयों को ज्ञात किया गया ।

कुछ स्थानों का चयन किया गया । कुछ स्थानों पर ज्वार-भाटा केन्द्र हैं, जबकि अन्य पर कोई नहीं है । इन स्थानों पर कुछ प्रत्यागमन वर्षों वाली अधिकतम ऊँचाईयों का परिकलन किया गया और उनके परिणाम संतोषजनक पाए गए । इस विधि द्वारा सभी उपलब्ध मौसम और जल विज्ञानिक आँकड़ों को पूर्णतः प्रयुक्त किया जा सकता है । उन स्थानों के लिए जहाँ कोई भी ज्वार-भाटा केन्द्र नहीं है या एक साथ कई स्थानों के लिए अधिकतम ऊँचाईयां ज्ञात करने हेतु ये विधि अत्यन्त उपयोगी है ।

ABSTRACT. In the present paper the maximum storm surge elevations with certain return years were calculated by using a joint probability method. Based on the analyses of the typhoons which affected coastal zone of Guangdong Province in history, a group of model typhoons was established. A number of parameters, which described the typhoons, were selected. The data of each parameter was graded into a few sub-groups according to their values, and this was done in accordance with the historical observations. The probability of each value of the parameters was calculated based on the historical records. The probability of a typhoon with a group of values of parameters could be calculated. Simulation results of the storm surges caused by the above model typhoons with their probabilities were analysed statistically. Thus an accumulated probability curve and maximum elevations with certain return years were obtained.

A number of spots was selected. At some of the spots there are tidal stations and at the others there are none. The maximum elevations with certain return years at the spots were calculated and the results were found satisfactory. By using this method all the meteorological and hydrological data, which were available, can be fully utilised. This method is most suitable for calculating the maximum elevations at a place where there is no tidal station or at many places simultaneously.

Key words — Storm surge, Maximum elevation, Joint probabilistic method

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1. Introduction

Now-a-days there are four kinds of following methods (Chen 1982) of calculating the maximum storm surge elevations with certain return years :

(1) Based on the analysis of meteorological data, a typhoon with certain return years can be found and by using a numerical model the corresponding elevations can be computed. This method is suitable for calculating maximum elevations at a place where there is no tidal station. The results of this method are usually conservative and it is applicable, for example, to facilitate the study of a nuclear power station. The difficulty in using this method is that it is not easy to find out a typhoon with certain return years.

(2) For a place with a tidal station and storm surge elevation data, the maximum elevations during certain continuous years can be obtained. Then by using a statistical method, the maximum elevation can be calculated. The disadvantage of this method is that only one value in each year is selected and all other information is neglected. Of course this method can not be used at a place without a tidal station.

(3) The above method can be modified and used at a place without a tidal station. For a particular place, based on the typhoon data of the whole year, a typhoon case, which caused the maximum elevation at the place in the year, can be selected. The elevation induced by this typhoon can be simulated by using a numerical model. Thus the maximum elevations during certain continuous years can be obtained. Finally by using a statistical method the maximum elevations with certain return years can be calculated. Insufficiently utilising the information available is the shortcoming of this method.

(4) By using joint probability method, as said earlier, all the available meteorological and hydrological data can be fully utilized. Thus the reliability of the results is high. There was a successful case study of this method in United States of America (Heaps 1977 & Moore *et al*. 1996). The method has also been used by Chinese scientists (Wang 1983, 1985, 1986). Owing to
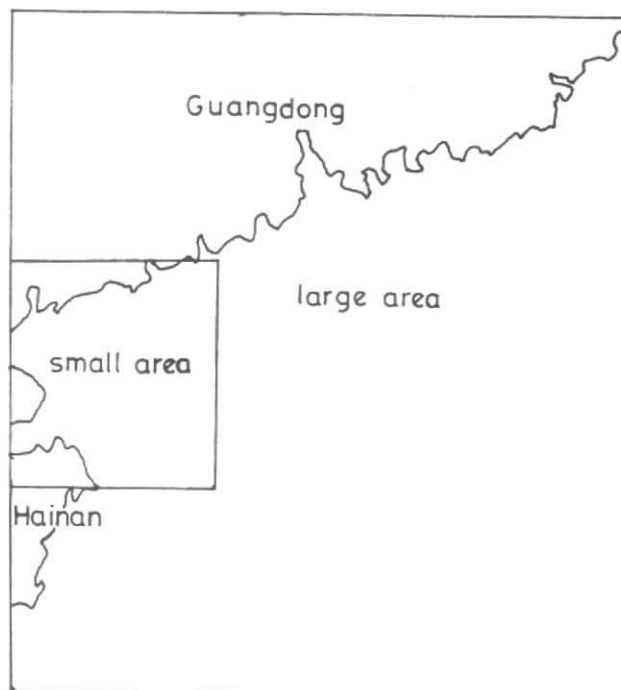

Fig. 1. Large and small areas and their relation

the small number of model typhoons the statistical analysis done by them only can be used to show the way how to use this method, but it has no practical significance. In the present paper, a group of 8000 model typhoons was obtained and a case study with practical meaning was conducted.

The analysis area of this paper is the western part of the sea area along the coast of Guangdong Province. There was a small area nested inside the large area. The western and northern boundaries of these two areas are the land boundaries of Guangdong and Hainan Provinces. Eastern and southern boundaries are open sea boundaries. The southern boundary of the large area is situated at $17°50'$N; eastern at $117°20'$E. The southern boundary of the small area is at $19°40'$N, eastern at $112°33'$E. The grid spacing in large area is 1/6 degree (longitude and latitude), in small area 1/3 of that of the large one. The grid system in large and small areas and their relation are shown in Fig. 1.

First of all the meteorological parameters of the typhoons which affected the analysis area in the 20 years from 1960 to 1979 were analysed and some typical values and their corresponding probabilities were obtained. By assuming that these parameters are

statistically independent and selecting a value for each parameter a model typhoon is composed. The probabilities of this typhoon are equal to the product of the probabilities of the values of all parameters. All these typhoons constitute a group of model typhoons. In the present paper 6 typhoon parameters were used. The values of 3 parameters were graded into 5 sub-groups and 5 typical values were used; for the other 3 parameters, there were 4 sub-groups and 4 typical values. Therefore, in the group of model typhoons there were $5^3 \times 4^3 = 8000$ typhoons. The maximum surge elevations caused by each of these typhoons were simulated by using a numerical model. Each of the elevations obtained has a probability, that is equal to the probability of its corresponding typhoon. For a given elevation ζ, the total of all the probabilities of the elevations, which were less than ζ, multiplied by annual occurrence of typhoons in this area, $R(\zeta)$ can be obtained. $R(\zeta)$ represents the annual occurrence of this elevation ζ. The reciprocal of $R(\zeta)$ is the return years of the elevation ζ.

2. Numerical model used in storm surge simulation

2.1. Hydrodynamical equations

Depth averaged equations used are given below :

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}[(\zeta + h)U] + \frac{\partial}{\partial y}[(\zeta + h)V] = 0 \tag{1}$$

$$\frac{\partial}{\partial t}[(\zeta + h)U] + \frac{\partial}{\partial x}[(\zeta + h)UU] + \frac{\partial}{\partial y}[(\zeta + h)UV]$$

$$-f(\zeta + h)V = -g(\zeta + h)\frac{\partial \zeta}{\partial x} + \frac{1}{\rho}(\tau_{x,\zeta} - \tau_{x,-h}) \tag{2}$$

$$\frac{\partial}{\partial t}[(\zeta + h)V] + \frac{\partial}{\partial x}[(\zeta + h)UV] + \frac{\partial}{\partial y}[(\zeta + h)V^2]$$

$$+f(\zeta + h)U = -g(\zeta + h)\frac{\partial \zeta}{\partial y} + \frac{1}{\rho}(\tau_{y,\zeta} - \tau_{y,-h}) \tag{3}$$

2.2. Boundary conditions

At the land boundaries the velocity components in the direction perpendicular to the coastline vanished. At the open sea boundaries radiation types of conditions were used for the large area. For the small area the water levels were copied from the large area's results. The hydrodynamical equations were integrated from an initial state of rest.

2.3. Model wind field

Jelensnianski's (1972) wind field was used in the present study.

$$V = V_m \left(\frac{T}{R_m}\right)^{1.5} \qquad (r \leq R_m) \tag{4}$$

$$V = V_m \exp\left(\frac{R_m - T}{\beta}\right)^{1.5} \qquad (r > R_m) \tag{5}$$

where V_m is maximum wind speed; R_m is radius of maximum wind speed; r is the distance between calculating point and typhoon centre and β is a reductive parameter in radial direction.

2.4. The result of the simulation of real typhoon surges

By using the above storm surge model, 10 storm surges, which affected the analysis area in 1968-77, were simulated. Elevations at 3 stations, namely Zhanjiang, Zhapo and Naozhou, were computed. The results were compared with the observations and they agree well. Some examples are given in Figs. 2-4. This indicates that the numerical model is reliable, and it can be used in the simulation of the model typhoons.

3. The selection of a group of model typhoons

3.1. Parameters describing typhoon's wind field

The following 6 parameters were used to describe typhoon's wind field :

Longitude of the spot where typhoon strikes the land X_B, maximum wind speed V_m, radius of maximum wind speed R_m, moving speed of typhoon centre V_f, angle between typhoon track and latitude line θ and wind speed reducing parameter in radial direction β.

The data of 80 typhoons, affecting the analysis area during the 20 years from 1960 to 1979, were collected from typhoon almanac and for each typhoon the values of the 6 abovementioned parameters were decided. By using the numerical model and taking the 6 parameters to be constant, 10 typhoon surges were simulated and the maximum elevations computed were in good agreement with the observations. Some examples are given in Figs. 2-4. Therefore, using the simple model, i.e., taking the parameters to be constant,
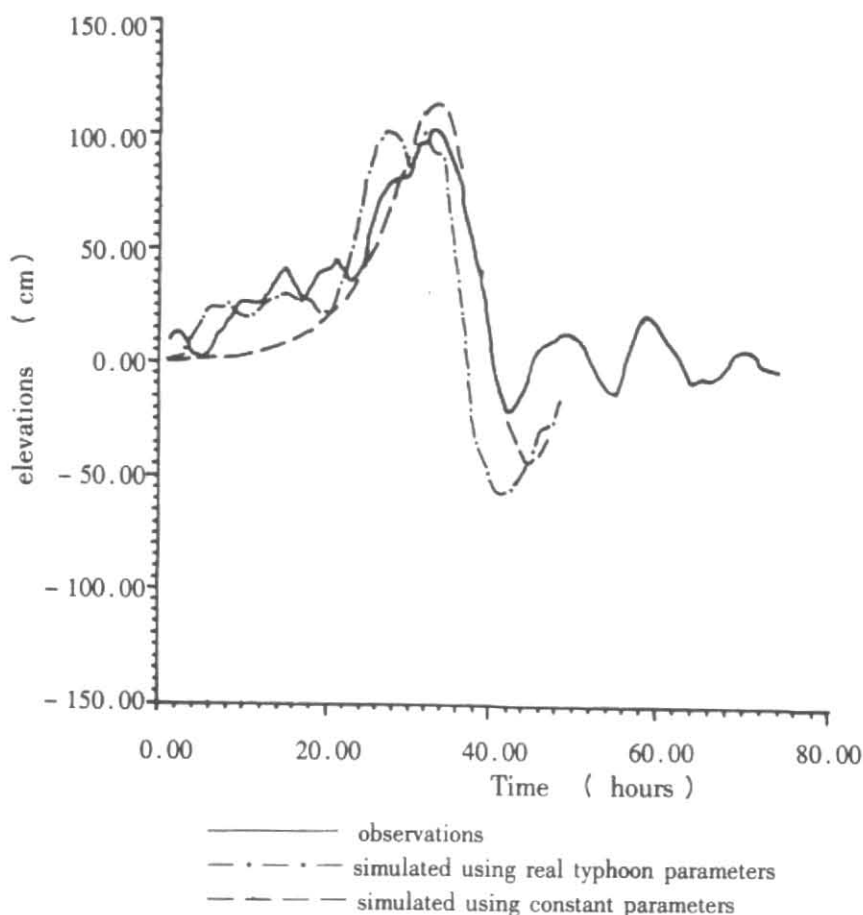
Fig. 2. Elevations caused by typhoon 7109 at Naozhou

does not cause large errors in the simulation of the maximum elevation. Thus the maximum elevations obtained by using the simple model can be used to calculate the maximum storm surge elevation with certain return years.

With regard to the selection of the value of the parameters the following points should be noted:

(1) The maximum wind speed V_m was taken from the "almanac of typhoon's wave and elevation" at a period of time just before the maximum elevation occurred.

(2) The principle of selecting the radius of maximum wind speed R_m is to obtain the best agreement between the model wind field and the observed one.

(3) The value of the moving speed of typhoon centre V_f was chosen during the day just before the maximum elevation occurred.

(4) Wind speed reducing parameter in radial direction β was calculated from the following formula :

$$\beta = \frac{R_m - R_1}{\ln(V_1/V_m)} \qquad (6)$$

where R_1 is the radius of Beaufort 6 scale wind speed, V_1 is equal to 10.6m/sec.

3.2. Values of typhoon's parameters

Values of the typhoon's parameters were arranged in the ascending order. The probability distribution curves were drawn. The values of the 6 parameters were graded into sub-groups according to the sensitivity with which they affect the elevations. Landing longitude X_B, maximum wind speed V_m and radius of maximum wind speed R_m were graded into 5 sub-groups. This means that $I = J = K = 5$. For the other 3 parameters there were 4 sub-groups for each, *i.e.*, $L = M =$
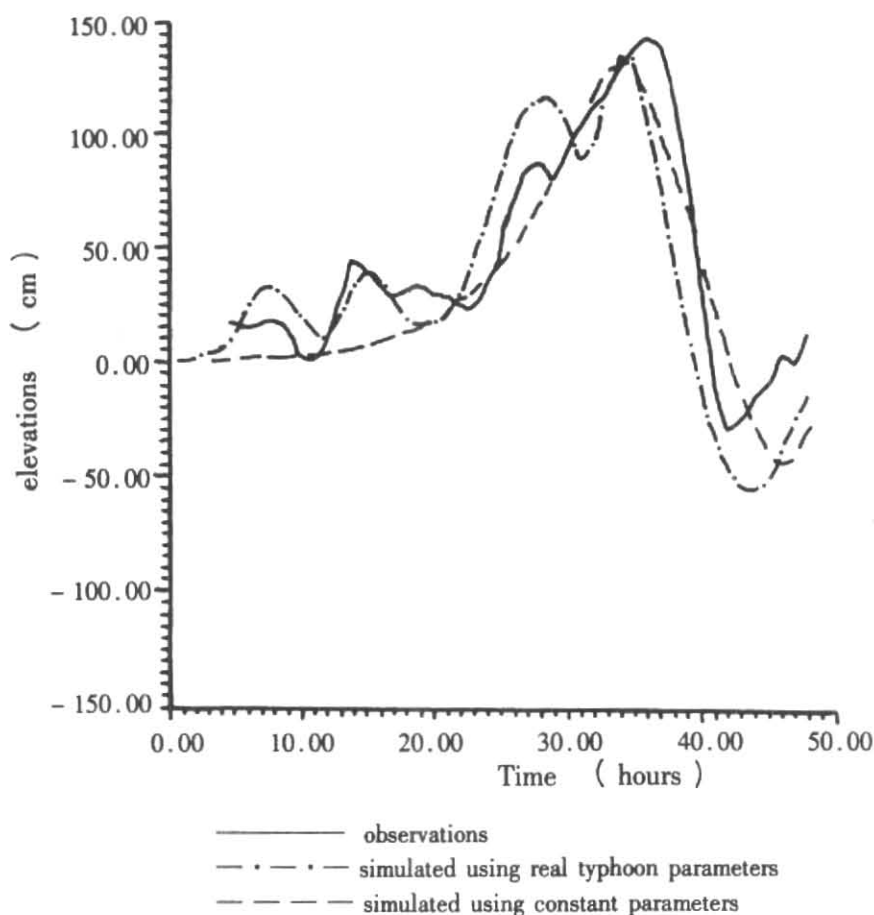
Fig. 3. Elevations caused by typhoon 7109 at Zhanjiang

$N = 4$. When the values of the parameters are graded it should be ensured that the numbers of the parameter values in each sub-group are roughly the same. The expected value (mean value or EX) is the representative of this group and in the present paper it is called typical value. The probability of a given typical value (a sub-group) can be calculated in the following way.

First, the ratio between the area corresponding to the value S_i and the total area under the probability distribution curve S, that is $f_i = S_i/S$, can be calculated, then the probability Δi was calculated by the following equation :

$$\Delta i = (f_i/\Delta x_i) \Big/ \sum_{i=1}^{n} (f_i/\Delta x_i) \qquad (7)$$

where Δi is the probability of a given typical value

(sub-group), Δx_i is the width of this group, n is the number of sub-groups of a parameter (equal to 5 or 4). The typical values of the parameters and their probabilities are given in Table 1. Sequence number typical value probability is given by :

$$\sum_{i=1}^{5} \Delta (X_{Bi}) = 1; \quad \sum_{j=1}^{5} \Delta (V_{mj}) = 1; \quad \sum_{k=1}^{5} \Delta (R_{mk}) = 1$$

$$\sum_{l=1}^{4} \Delta (V_{fl}) = 1; \quad \sum_{m}^{4} \Delta (\theta_m) = 1; \quad \sum_{n=1}^{4} \Delta (\beta_n) = 1$$

For each of the parameters a typical value is selected. Thus, a model typhoon was constructed. Altogether $N_t = 5^3 \times 4^3 = 8000$ model typhoons were obtained and they formed a group of model typhoons. Because the parameters are independent statistically,
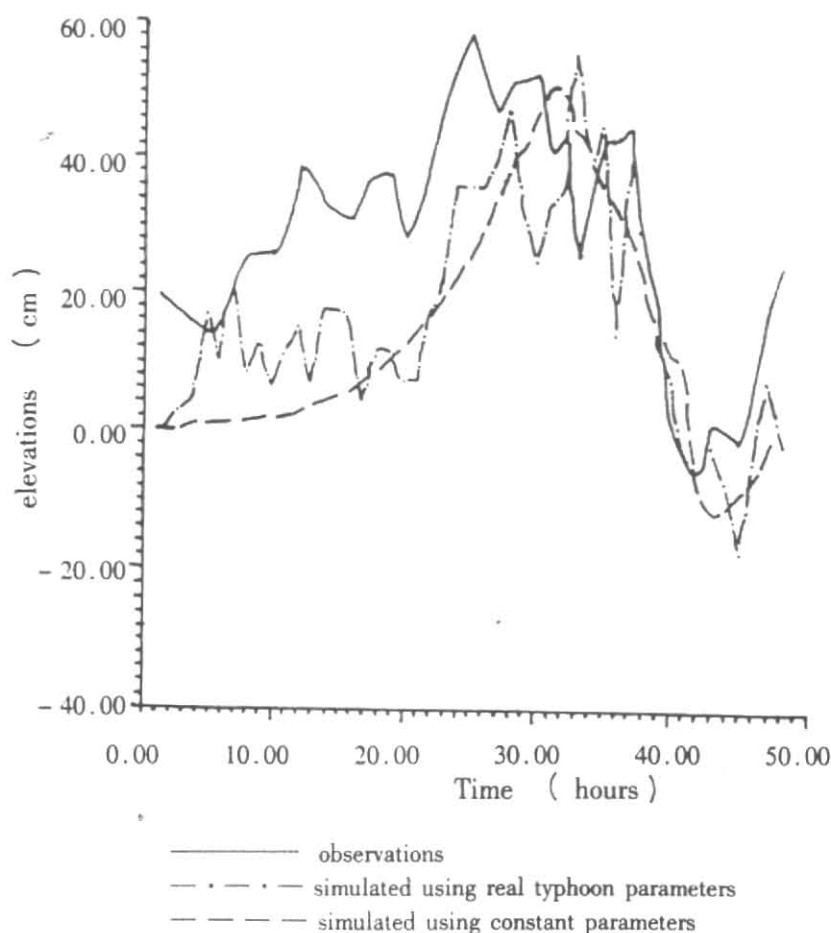
Fig. 4. Elevation caused by typhoon 7109 at Zhapo

the model typhoon with the typical values of $X_{Bi}, V_{mj}, R_{mk}, V_{fl}, \theta_m$ and β_n has the probabilities of $\Delta(X_{Bi}) \cdot \Delta(V_{mj}) \cdot \Delta(R_{mk}) \cdot \Delta(V_{fl}) \cdot \Delta(\theta_m) \cdot \Delta(\beta_n)$.

4. The simulation of the model typhoons

In the present paper 8 spots were selected and they are Naoshou (N_z), Zhanjiang (Z_j), Zhapo (Z_p), X_1(111°26′E, 20°23′N), X_2(110°53.29′E, 21°23′N), X_3(111°E, 21°17′N), X_4(111°13′E, 21°30′N) and X_5(111°40′E, 21°30′N). Among them at the first 3 there are tidal stations, at the other 5 there are none. The maximum elevations caused by the 8000 model typhoons at these 8 spots were obtained by using numerical simulations. Each computed maximum elevation has a probability, *i.e.*, the probability of the model typhoon, which induces this elevation. For each spot the 8000 maximum elevations were analysed

statistically. These elevations were arranged in the ascending order. They were graded into sub-groups with a width of 10 cm. The value of a sub-group was represented by the mean value of this sub-group. The probability of a given maximum elevation equals to the sum of the probabilities of all computed maximum elevations, which fall into the sub-group represented by the given maximum elevation. Finally, for a spot, an accumulated probability curve was drawn.

5. The calculation of the maximum elevation with certain return years

Based on the above statistical analysis the occurrence probability of a given elevation $R(\zeta)$ can be calculated as follows :

$$R(\zeta) = N_s \sum_{i=1}^{\alpha} \delta(i) \qquad (8)$$

TABLE 1

Typical values and their probabilities of the parameters of the typhoons

S. No.	$X_B(°)$ Typic. value	$X_B(°)$ Proba. value	V_m (m/sec) Typic. value	V_m (m/sec) Proba. value	$R_m(\times 10^4)$ Typic. value	$R_m(\times 10^4)$ Proba. value	V_f (m/sec) Typic. value	V_f (m/sec) Proba. value	$\beta(\times 10^4)$ Typic. value	$\beta(\times 10^4)$ Proba. value	$\theta(°)$ Typic. value	$\theta(°)$ Proba. value
1	107.4	0.16	18.5	0.08	4.6	0.08	2.5	0.12	10.0	0.18	0.70	0.15
2	108.6	0.33	27.5	0.23	7.27	0.20	4.03	0.36	17.3	0.37	17.8	0.52
3	109.4	0.31	35.0	0.31	9.1	0.35	5.25	0.38	23.7	0.36	33.2	0.26
4	110.6	0.15	36.4	0.32	10.4	0.31	6.8	0.14	34.0	0.09	31.2	0.07
5	113.5	0.05	43.0	0.06	14.5	0.06						

TABLE 2

Maximum elevations, their occurrence probabilities and return years at the spots

Return years	200	100	50	25	10	5
Occurrence probabilities	0.005	0.01	0.02	0.04	0.1	0.2
Max elevations at Nz	248	230	210	188	174	165
Max elevations at Z_j	272	240	225	204	194	183
Max elevations at Z_p	153	138	130	128	122	110
Max elevations at X_1	150	149	130	122	117	110
Max elevations at X_2	241	220	200	190	170	161
Max elevations at X_3	204	189	170	158	148	141
Max elevations at X_4	197	180	163	152	148	140
Max elevations at X_5	150	135	128	119	115	110

where α is the number of the sub-group with a representative value less than or equal to ζ; $\delta(i)$ is the probability of the sub-group; N_s is the averaged annual occurrence of typhoons in this area. 20 years' data were collected and there were 80 typhoons. Therefore, $N_s = 4$.

The return period of a given ζ is equal to

$$1/R (\zeta) = 1/[N_s \cdot \Sigma \, \delta(i)] \qquad (9)$$

Thus the relation between elevation and its return period was obtained. By using this relation we can calculate the maximum elevation when the return years are known and *vice versa*. The calculated maximum elevations with certain return years for the 8 spots are given in Table 2.

TABLE 3

A comparison of maximum elevations with certain return years and occurrence probabilities at Naozhou station

Method		Return years	5	10	25	50	100
		Occurrence probability	0.2	0.1	0.04	0.02	0.01
Joint probability method	Max elevations		155	174	188	210	230
S-method	Max elevations		145	170	204	228	253

6. Conclusions

(*i*) At Naozhou, there is a tidal station and we have the data of maximum elevations during 20 continuous years. By using the method given in "Technical standard for harbour engineering" (hereafter it is called S-method) the maximum elevations with certain return years can also be obtained. Table 3 is a comparison between the results obtained by using the joint probability method and S-method.

It can be seen from Table 3 that there are no large differences between the results of these 2 methods. For the return years shorter than 50 these results are roughly the same and differences between them are less than 15%. For the return years of 100 the maximum elevation computed with the joint probability method is lower than that of the S-method. When the S-method was used, much less information was taken into account than when joint probability method was used. Therefore, the results of the joint probability method were much more reliable.

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When using the joint probability method, it is quite difficult to select the parameters and grade the values into sub-groups. In the present paper we have done much work in this respect. The parameters we used and their typical values could ensure that the simulation of both real surges and the surges induced by model typhoon are good enough to make the results accurate and reliable. The work reported in this paper shows that the joint probability method can be used in the coastal engineering to calculate the maximum elevations with certain return years. When using this method all meteorological and hydrological information available can be fully utilised and maximum elevations, at places where no tidal station exists, can be calculated.

Acknowledgements

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