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Data assimilation in a reduced gravity ocean model using ERS-1 scatterometer and TOPEX altimeter data

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सार — इस शोध पत्र में सहबद्ध दृष्टिकोण का उपयोग करते हुए ई. आर. एस. –1 स्कैटरोमीटर पवनों और महासागर मॉडल पर टोपेक्स तुंगतामापी से लिए गए समुद्र तल विभिन्नता आँकड़ों के साम्यान्वेषण के प्रभाव की चर्चा की गई है। इस हेतु जो मॉडल विकसित किया गया है वह उत्तर-पश्चिमी हिन्द महासागर के लिए विकसित किया गया रेखीय समानीत गुरूत्वीय मॉडल है। ई. आर. एस. –1 उपग्रह स्कैटरोमीटर के उपयोग से उपलब्ध हुए संवर्धनों और फ्लोरिडा स्टेट विश्वविद्यालय द्वारा दिए गए विश्लेषित पवन संवर्धनों का उपयोग करते हुए मॉडल के प्रभाव का अध्ययन किया गया है। विश्लेषित वायु प्रतिबल तथा ई. आर. एस. –1 स्कैटरोमीटर से उपलब्ध हुए वायु प्रतिबल क्षेत्रों के उपयोग से मॉडल पर पड़ने वाले प्रभावों का अध्ययन किया गया है। मॉडल से लिए गए समुद्र तल और तुंगतामापी प्रेक्षणों के बीच के अन्तर को परिव्यय फलन के रूप में परिभाषित किया गया है । मॉडल और प्रेक्षणों के बीच की अनुपयुक्तता को मॉडल की कृत्रिम त्रुटि के रूप में कम किया गया है। स्कैटरोमीटर पवन प्रणोदन का उपयोग करते हुए 30 दिन के आँकड़ों का साम्यान्वेषण किया गया है। यह देखा गया है कि साम्यान्वेषण-रहित समुद्रतल की तुलना में साम्यान्वेषित समुद्रतल के साथ स्कैटरोमीटर से व्युत्पन्न पवन प्रणोदन अच्छे परिणाम देते हैं।

ABSTRACT. This paper discusses impact of ERS-1 scatterometer winds and assimilation of sea level variability data derived from TOPEX altimeter on the ocean model using adjoint approach. The model developed for the purpose is linear reduced gravity model for the north-western Indian ocean. Experiments have been done with forcing provided using ERS-1 satellite scatterometer and analysed wind forcing provided by Florida State University (FSU). Impact on the model has been studied using the analysed wind stress as well as with ERS-1 scatterometer-derived wind stress fields. The cost function has been defined as difference between the model derived sea level and altimeter observations. This misfit between model and observations has been minimised with the model equations as constraints. Assimilation has been done for 30 days using scatterometer wind forcing. It has been observed that assimilated sea level with scatterometer-derived wind forcing gives much better results in comparison to unassimilated sea level.

Key words — Assimilation, Adjoint, Satellite, Altimeter, Scatterometer, Oceanography, Model.

1. Introduction

Oceanographic measurements using *in-situ* instruments are in general either single points, or a small cluster of points embedded in a vast domain providing data very infrequently in time.Even with the advent of spaceborne sensors, like altimeter and scatterometer, global ocean data are available only at the sea surface, not from the ocean interior. Ocean models and data assimilation techniques have to play greater role than their meteorological counterpart in successfully extrapolating these surface information into the ocean interior for achieving the ultimate goal of reconstructing the most realistic possible ocean circulation.

As the ocean models are forced by surface level wind fields, the accuracy of wind observations are quite important for the better prediction. Particularly in the case of tropical cyclones or other high wind cases, the *in-situ* measurements are very less. On these occasions, the satellite-borne scatterometer measurements are very much useful for the estimation of wind fields. Due to large and more reliable data base, their impact on ocean model is also supposed to be better than analysed wind fields, which sometimes provide smoother picture of actual observations due to less data availability in severe weather conditions like cyclonic conditions.

Sophisticated data assimilation techniques are required for successfully assimilating asynoptic satellite data into ocean models. The forecast capabilities of ocean models could also be immensely improved by assimilation of real time sea surface height data from satellite altimeter and sea surface wind vectors from satellite scatterometer.

Data assimilation is the process of optimally combining the measurements of one or more dynamical variables with the corresponding model-predicted values to obtain a better estimate of the state of the model leading to a better predictive capability. For the atmospheric model, the large number of data sets in space and time make initialisation possible, even climatology can provide good first guess for initial conditions. However, in oceanography, the data is not quite sufficient for initialisation. Over the years different assimilation schemes have been studied (Le Dimet and Talagrand 1986, Ghil and Rizzoli 1989, Panofsky 1949). These can be divided into basically three types of schemes:

(1) Polynomial Interpolation Schemes

(2) Optimal Interpolation Schemes

(3) Variational Analysis Schemes

In the first category, polynomial functions are fitted to the observations in the neighbourhood of analysis points. These types of schemes are quite simple in implementation but they do not take care of past analysis into consideration. In optimal interpolation techniques, previous analysis is being accounted as a major source of information by assigning a weight to the combination of misfit between observations and analysis fields. These types of techniques are widely used in models used for operational purpose which in some cases, smoothen out the analysed fields excessively.

The most powerful is the technique based on adjoint equations used by Talagrand and Courtier (1987), Hoffman (1986), Thacker and Long (1988). In these investigations, model used are similar to ocean models and from success of these investigations, it can be inferred that variational techniques are best suited for oceanographic data assimilations. A detailed description of the variational data assimilation procedure has been provided by Smedstad and O'Brien (1991) with a simple wind driven model. In this method, observations distributed in time and space are used to constrain the entire phase-space trajectory of the model and this is the major difference form the first two data assimilation methods. However, the method is computationaly expensive to use, but due to use of all past observations simultaneously for fitting the trajectory, it appears to be much more logical to use than other sequential assimilation methods. In the present study, we have utilised this technique for a linear reduced gravity model. Since the wind fields are not available at the required time steps of the model. Earlier we have used the analysed wind fields provided by Prof. O'Brien of Florida State University. With the availability of actual satellite wind observations provided by scatterometer on-board ERS-1 satellite, we have also tried to analyse the impact of scatterometer-forced winds into the model and in the assimilation procedure. Sea level variations provided by TOPEX altimeter data have been assimilated into the model as observations and initial conditions have been recovered to see the capability of the technique for the prediction of the sea surface.

2. Data used

In this study, wind stress data of ERS-1 scatterometer and analysed field provided by FSU have been used. Scatterometer onboard ERS-1 measures backscatter through sea surface roughness using three antennas in different azimuth directions. These backscattering coefficient measurements have been utilised to derive wind speed and wind direction for the entire 500 km swath of satellite. Since the satellite coverage changes in each orbit and it covers the same area after 35 days, the data has been further averaged over the month to get monthly-averaged wind fields over the study period (Gohil and Pandey 1995). The sea level observations have been taken using TOPEX altimeter data, covering an area of 10 km in each swath and repeating the exact area after 10 days. This 10-days data has been averaged and various corrections have been done by Tapley *et al.* (1994). The processed $1^\circ \times 1^\circ$ grid-averaged data have been kindly provided by Prof. Shum of Texas University.

3. Methodology

Data assimilation is basically to reconstruct the present ocean state by fitting prediction model to all the observations gathered. The fitting is usually done in a least squares sense with the model equations acting as constraints. Specifically, one aims to minimise the distance between the model and the observations. It is usual to define a quadratic cost function J which measures the distance between the model solutions and the observations. Smedstad and O'Brien (1991), Basu *et al.* (1993) have described the approach in detail. Assimilation using simulated data of Kelvin waves and other simulation experiments have been done earlier by Basu *et al.*, (1997) and Raj Kumar *et al.* (1997). In the present study, the observations consist of measurements of interface depths. Minimisation has to be done with respect to a set of control variables. In this case the control variables are the model initial conditions and the problem is a constrained minimisation problem. The problem is solved by redefining it so that it becomes a problem of unconstrained minimisation. The approach for the same is based on Lagrange multiplier technique. For this, a Lagrangian is formed consisting of model equations and cost function. Since the cost function is an implicit function of the initial variables, minimisation amounts to setting the partial derivatives of the cost function with respect to the initial variables to zero. To find minimum of cost function, the partial derivatives of the Lagrangian are set to zero. The partial derivatives with respect to the Lagrange multiplier recovers the model equations and the partial derivatives with respect to the model variables constitute a set of adjoint equations. The solution of the adjoint equations at the initial time provides the partial derivatives of the cost function with respect to the initial variables. These partial derivatives are required for minimising the cost function using optimisation methods like conjugate-gradient or quasi-Newton. The algorithm thus proceeds in an iterative fashion. Each iteration consists of a forward run of the ocean model, a backward run of the adjoint model to provide the required partial derivatives for minimising the cost function. The minimum thus found provides the improved initial values and the integration of the model with these improved initial values results in a better forecast capability.

The model developed for this study is a linear reduced gravity model of the north-western Indian Ocean in spherical co-ordinates (Dubey *et al.* 1986). The study area is from 29°S to 23°N and from 40°E to 75°E. The reduced gravity simply describes the ocean consisting of two layers, one active upper layer and another passive lower layer. The density difference between the layers is $\Delta\rho$ and the mean density is ρ. The standard assumption of hydrostaticity has been invoked and reduced gravity is denoted by $g'(= g\Delta\rho / \rho)$. The motion of the upper layer represents the first baroclinic mode. Both the layers are homogeneous horizontally and vertically. The model equations in the transport form with spherical co-ordinates are written as :

$$\frac{\partial U}{\partial t} - fV = -\frac{c^2}{a\cos\theta}\frac{\partial h}{\partial \phi} + A\nabla^2 U \tag{1}$$

$$\frac{\partial V}{\partial t} + fU = -\frac{c^2}{a}\frac{\partial h}{\partial \theta} + A\nabla^2 V \tag{2}$$

$$\frac{\partial h}{\partial t} + \frac{1}{a\cos\theta}\frac{\partial U}{\partial \phi} + \frac{\partial}{\partial \theta}(V\cos\theta) = 0 \tag{3}$$

where $U = uh$ and $V = vh$ represent the eastward and northward components of upper layer transport, respectively, (u, v) are the depth-independent velocity components in the upper layer and h is the thickness of the upper layer, ϕ and θ denote longitude and latitude respectively, t is the time. c is the wave speed of the first baroclinic mode and the value chosen for c^2 is 6 m^2/s^2 which is typical for the Arabian Sea. f is the Coriolis parameter and a is the radius of the earth. The coefficient for eddy viscosity A has been chosen to be 1000 $m^2\ s^{-1}$. g' has been taken as 0.03 m/s^2 and ρ = 1023 kg/m^3. The grid employed is Arakawa C-grid and the spacing is 0.5°, uniform in both meridional and zonal directions. The time step is 30 minutes. The finite difference scheme is leap frog with a forward time step after every 99 steps to suppress the computational mode. The coastlines are treated as no-slip boundaries.

The primary forcing of the model is windstress and τ^{ϕ} and τ^{θ} are the eastward and northward components of windstress.

The cost function J is defined as,

$$J = \int \frac{1}{2}(h - h_{obs})^2 \, d\sigma \tag{4}$$

where h_{obs} represents an observation of the upper layer thickness. The integration is taken over the entire spatio-temporal domain over which the model is integrated. The Lagrangian is formed as

$$L = \int \left[(\beta_u E_u + \beta_v E_v + \beta_h E_h) + \frac{1}{2}(h - h_{obs}) \right]^2 d\sigma \tag{5}$$

where, β_u, β_v and β_h are the Lagrange multipliers for U, V, and h respectively and E_u, E_v and E_h are the left hand sides of U, V and h equations when all the terms have been brought to the left hand side and the right hand side represents zero. The stationary points of the Lagrange function (which correspond to the minimum of the cost function) can be found by letting the first variation of L with respect to the variables $U, V, h, \beta_u, \beta_v$ and β_h vanish. The first variation of L with respect to β_u, β_v and β_h gives the original model Eqns. (1-3). Constraining the first variation with respect to U, V and h to zero leads to the adjoint equations:

$$-\frac{\partial \beta_u}{\partial t} + f\beta_v = \frac{c^2}{a\cos\theta}\frac{\partial \beta_h}{\partial \phi} + A\nabla^2 \beta_u \tag{6}$$

$$-\frac{\partial \beta_v}{\partial t} - f\beta_u = \frac{c^2}{a}\frac{\partial \beta_h}{\partial \theta} + A\,\nabla^2 \beta_v \tag{7}$$

$$-\frac{\partial \beta_h}{\partial t} - \frac{1}{a\cos\theta}\left[\frac{\partial \beta_u}{\partial \phi} + \frac{\partial}{\partial \theta}(\beta_v \cos\theta)\right] + (h - h_{obs}) = 0 \tag{8}$$

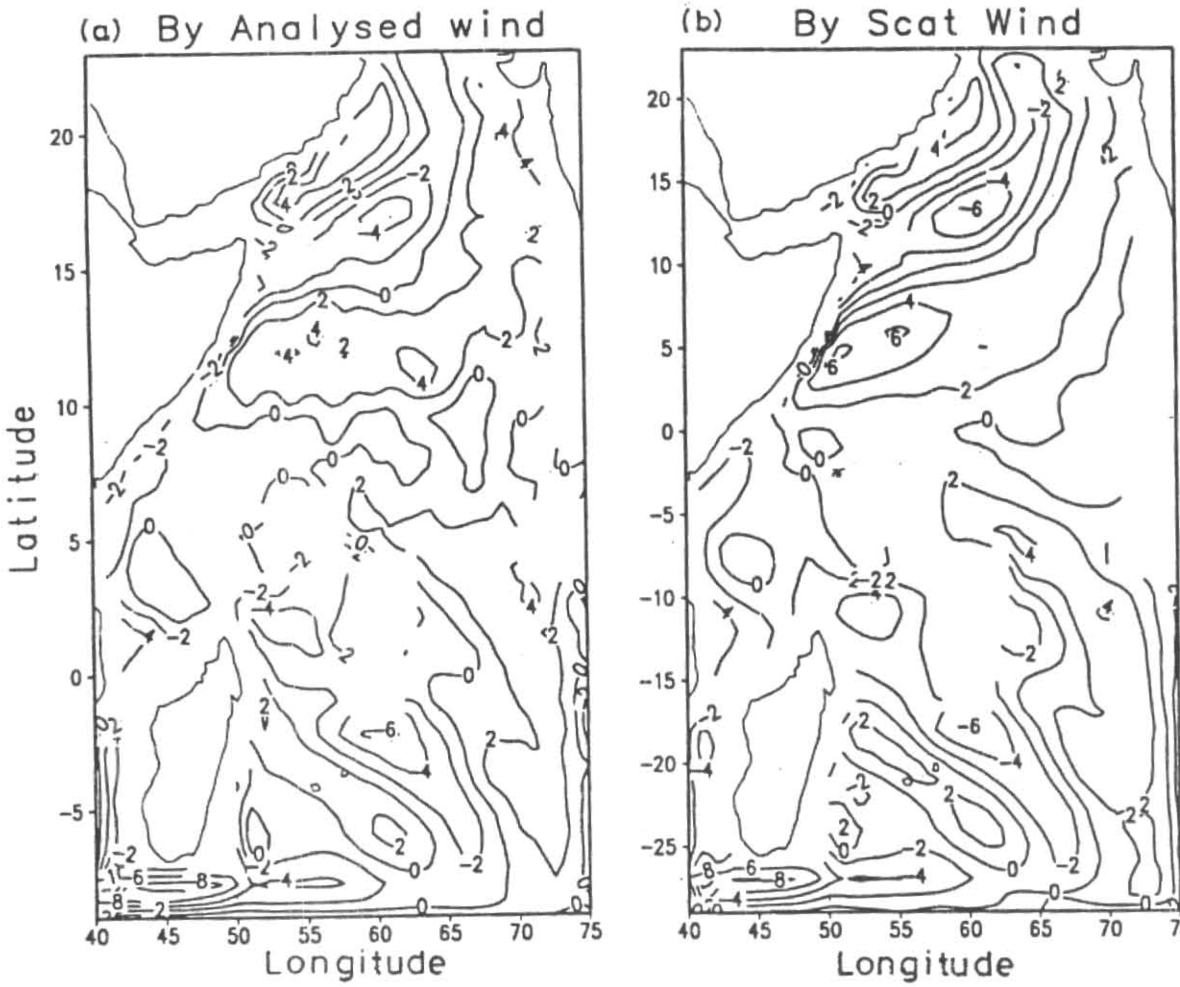
It can be easily seen that the adjoint equations are forced by data misfits represented by the last term in Eqn. (8). Comparing Eqns. (6-8) with model equations, one can see that they have similar form, except that adjoint equations correspond to evolution backward in time. The Lagrange variables therefore carry information about the misfit backwards to the initial time of the data assimilation period. As shown by Basu *et al.* (1993), the gradient of the cost function is contained in the values of the adjoint variables at the model initial time.

The temporal domain in which the cost function is minimised has been chosen as 30 days. The minimisation routine used for the convergence is quasi-Newton method (Gilbert and Lemarechal 1989).

4. Experiments

For this study, the monthly averaged data for 1993 in 1° by 1° boxes obtained from Florida State University and wind field data derived from ERS-1 scatterometer has been utilised. The data has been interpolated to the model grids using bicubic spline method. For the model time steps, linear interpolation was used. The model was first run from rest with the initial upper layer thickness of 200 m everywhere. A steady annual cycle was produced by running the model for six years with the same annual cycle wind provided by FSU. The current pattern simulated by the model resemble the generally observed patterns in the Arabian Sea. The data for January 15 has then been stored as initial state of the model.

An altimeter emits a short pulse of electromagnetic radiation and by measuring the time interval between its transmission and reception it finds the distance of the sea surface from itself. If the orbit height with respect to the reference ellipsoid is known, the sea surface height with respect to the reference ellipsoid can be found. We are however, more concerned with the dynamic topography which is the sea surface height measured with respect to the geoid. This is the quantity of interest from the point of view of oceanography and ocean modelling. However, the geoid is only imperfectly known and thus the dynamic topography measured by an altimeter is highly inaccurate. In practice one normally measures the variability with respect to some mean sea surface. TOPEX altimeter derived sea level variability data from Prof. C.K.Shum of the University of Texas has been procured for the study. The data are averaged over $1^\circ \times 1^\circ$ grid. Two types of time-averaged data sets were provided. The first one is monthly averaged and the second one is



Figs. 1(a&b). Sea level variability for the month of June using (a) analysed wind and (b) scatterometer (scat) wind

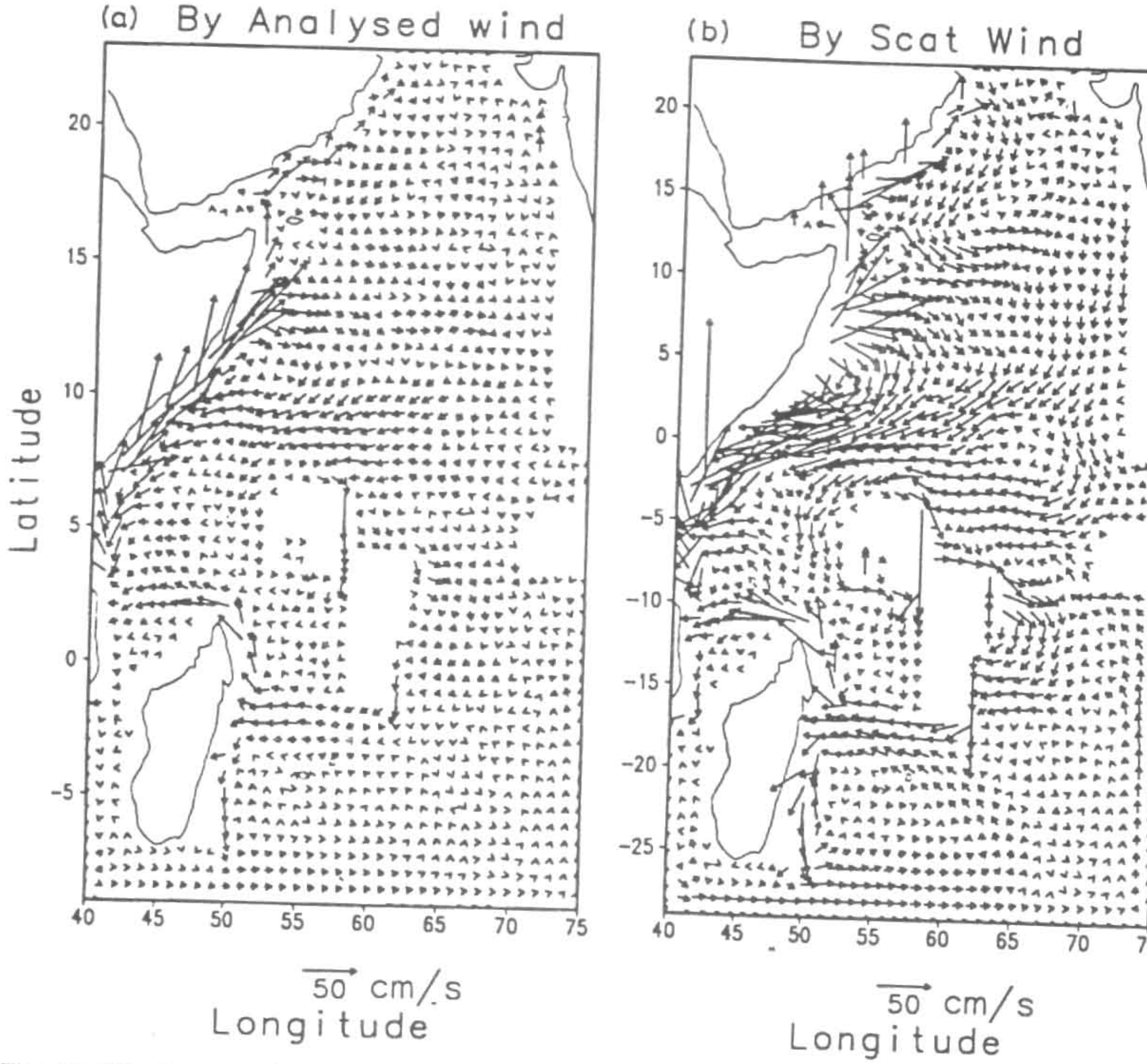
10-day averaged. For the present study 10 day averaged data were utilised. The data were interpolated to the model grid points using the same cubic spline routine which was used for interpolating the wind stress values.

To analyse the impact of scatterometer data on the model, it was run using analysed wind field upto May 1993. Thereafter, it was run for one month with analysed as well as scatterometer-derived wind field. The sea level and current values were stored for the month of June 1993. Similarly, run was made for two months and values for the month of July have been stored for analysis.

For the assimilation experiment, the model was run upto 4 May using analysed wind stress data and the upper layer thickness values have been generated. Since actual experiment was to start from 4 June 1993 now onwards the model was forced using scatterometer wind for one month to get upper layer thickness values for the day. The adjoint model was developed from the finite difference formulation of the reduced gravity model. The minimisation algorithm used is a variable storage quasi-Newton method suggested by Gilbert and Lemarechal (1989).

In the experiment, TOPEX altimeter-derived sea level values have been provided after 10, 20 and 30 days, *i.e.*, from 14 June to 4 July 1993 as observed values and assimilation has been done for 30 days to recover initial values of 4 June. The values between model time steps have been linearly interpolated.

The model calculates the interface depth, whereas altimeter data provides sea level variability. The variability has been converted to the interface depth



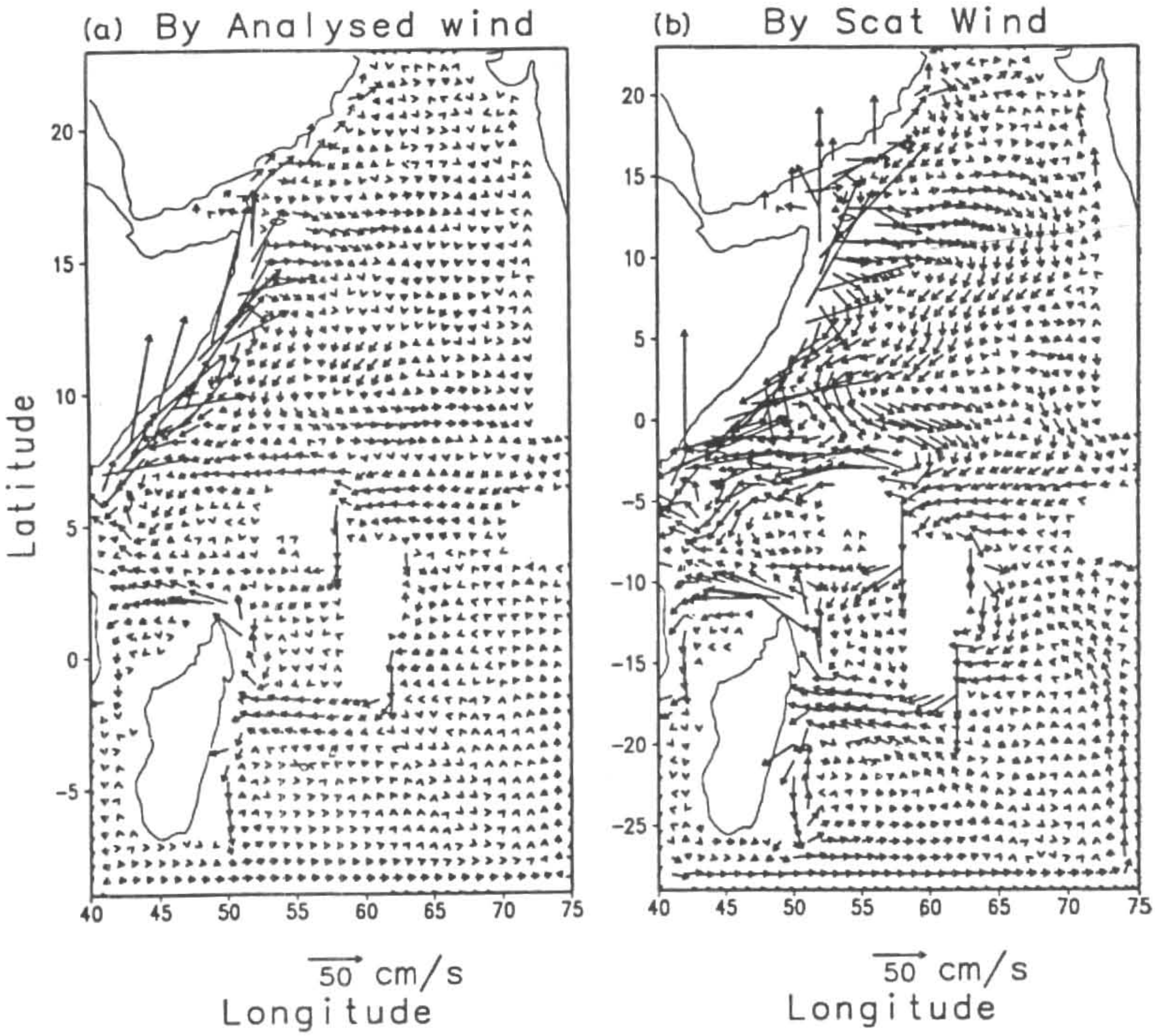
Figs. 2(a&b). Current variations for the month of June using (a) analysed wind and (b) scatterometer (scat) wind

variability by dividing it by density difference between the layers, *i.e.*, ULV = (SLV/ $\nabla\rho/\rho$). To compute upper layer thickness, one requires an absolute field of h. Thus a reference field of h should be added to the field of variability calculated from altimeter data. The reference field was chosen as the one year mean field of h simulated by the model. This is because the altimeter-observed sea level variability was referred to a mean sea surface which was obtained by calculating the two-year mean of TOPEX data. It was thus reasonable to take the reference as the annual mean simulated by the model.

This is because the model has been able to produce the annual cycle of currents and sea surface heights reasonably well. It is thus to be expected that the annual mean h simulated by the model can act as a good reference value. Thereafter model- derived initial values and assimilated initial values have been separately utilised to run the model for 40 days and the sea level values at every 10 days have been compared with TOPEX altimeter derived sea level values.

5. Results and discussions

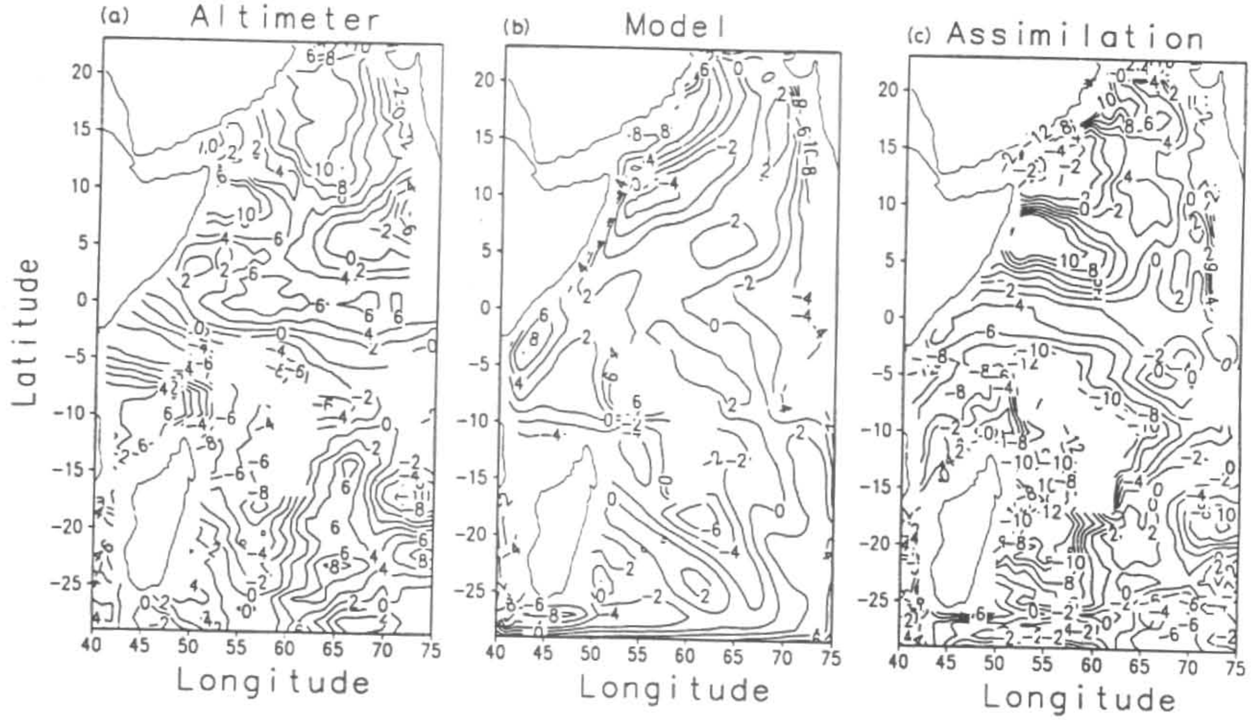
The sea level variability predicted for June 1993 using the analysed wind field and scatterometer-derived winds have been shown in Figs. 1(a&b). The sea level derived by the two have not much difference as can be seen from the figure, however the effect of wind forcing can be clearly seen in the current patterns of



Figs. 3(a&b). Current variations for the month of July using (a) analysed wind and (b) scatterometer (scat) wind

June 1993 Figs. 2(a&b). The overall currents predicted using scatterometer winds are higher. Around equator, there is a strong flow of current towards the west. Similarly near Somali region, currents are higher, whereas in the case of analysed wind fields, currents are quite low. Similar patterns are seen in the month of July also Figs. 3(a&b). In July, currents predicted using scatterometer winds, show very well-behaved clockwise pattern around 12°N.

The effect of assimilation can be judged by comparing the assimilated results with actual altimeter-derived values only. On comparison with TOPEX-derived sea level variability, the correlation between unassimilated values and TOPEX is found to be almost negligible with rms difference of 6.0 cm. However, correlation of assimilated initial values increases to 0.67 with rms of 5.9 cm. The model has been run with these initial values for 10 days and the correlation of TOPEX values with unassimilated values is still negligible, however running the model with assimilated initial values gives much better correlation of 0.79, with rms difference of 3.28. In order to test the validity of the model for forecasting purpose, we tried to run the model upto 40 days and the results of sea level prediction by model and by altimeter have been compared Table 1. It has been seen that in general model is unable to predict sea level with good accuracy, however after assimilation of satellite-derived sea level data, the prediction is found to improve drastically.



Figs. 4(a-c). Initial Sea level variability: (a) Altimeter, (b) Model and (c) Assimilation

TABLE 1

Statistical Parameters

Experiment	Model		Assimilation	
	CC	RMS Error	CC	RMS Error
Initial	3.E-02	6.00	0.67	5.97
10 day	7.E-02	5.74	0.79	3.28
20 day	3.E-02	6.16	0.81	2.93
30 day	0.13	6.28	0.75	3.53
40 day	6.E-02	5.78	0.47	4.95

Figs. 4(a-c) shows the sea level variability at the initial time step by altimeter, by model and after assimilation. It can be clearly seen that, there is no similarity between Figs. 4(a&b), however 30 days assimilation [Fig. 4(c)] shows somewhat similar structure as seen in altimeter. Figs. 5(a-c) shows the variability after 10 days. In this, Figs. 5(a&c) are almost similar to each other and it can also be inferred from the correlation between the two. Assimilation for 20 days also shows a few structures similar to altimeter one, specially in southern region and near the Somali coastal region. The correlation between the altimeter and assimilated value also increases significantly in comparison with unassimilated values Figs. 6(a-c) & Figs. 7(a-c) show the results after 30 and 40 days of model run. It has been observed that, there is not much change in the variability from 30 to 40 days of run for unassimilated model values, effect of altimeter assimilation is seen upto 30 days, however afterwards it is being overpowered by actual wind forcing as can be seen by the correlation. The results clearly show the impact of altimeter data assimilation on the model. With the availability of more sea level data, the predictability can be much better.

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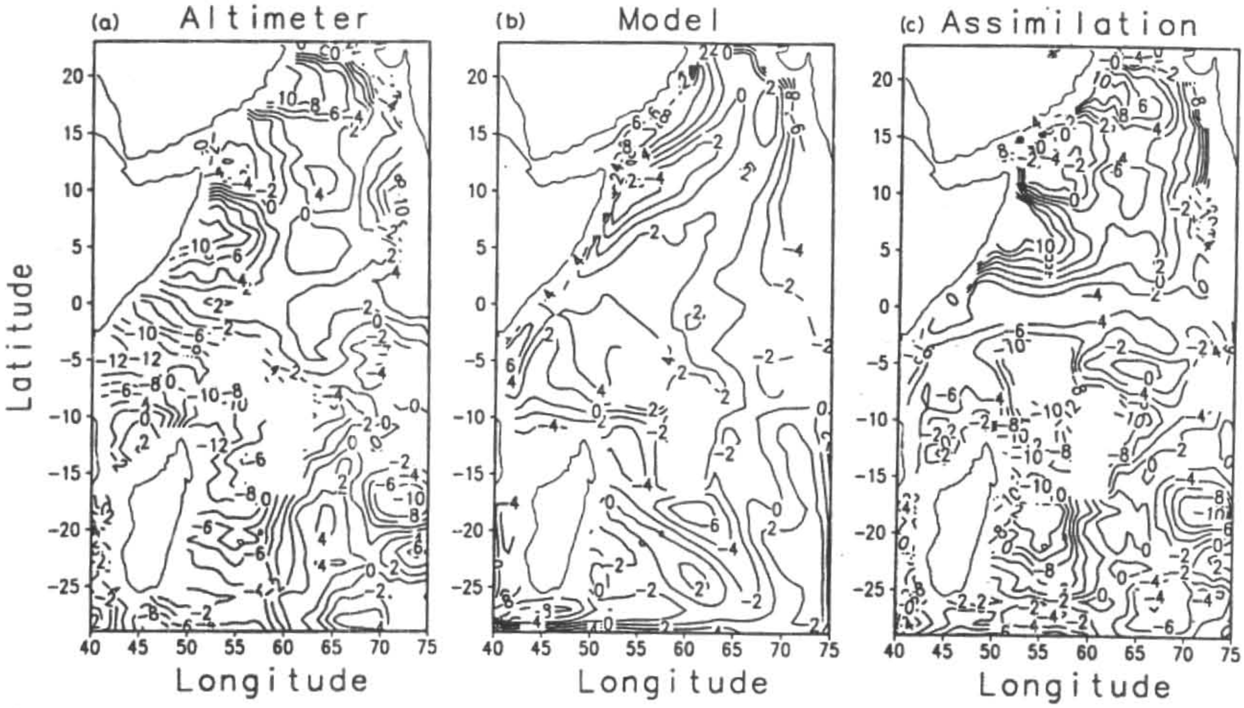
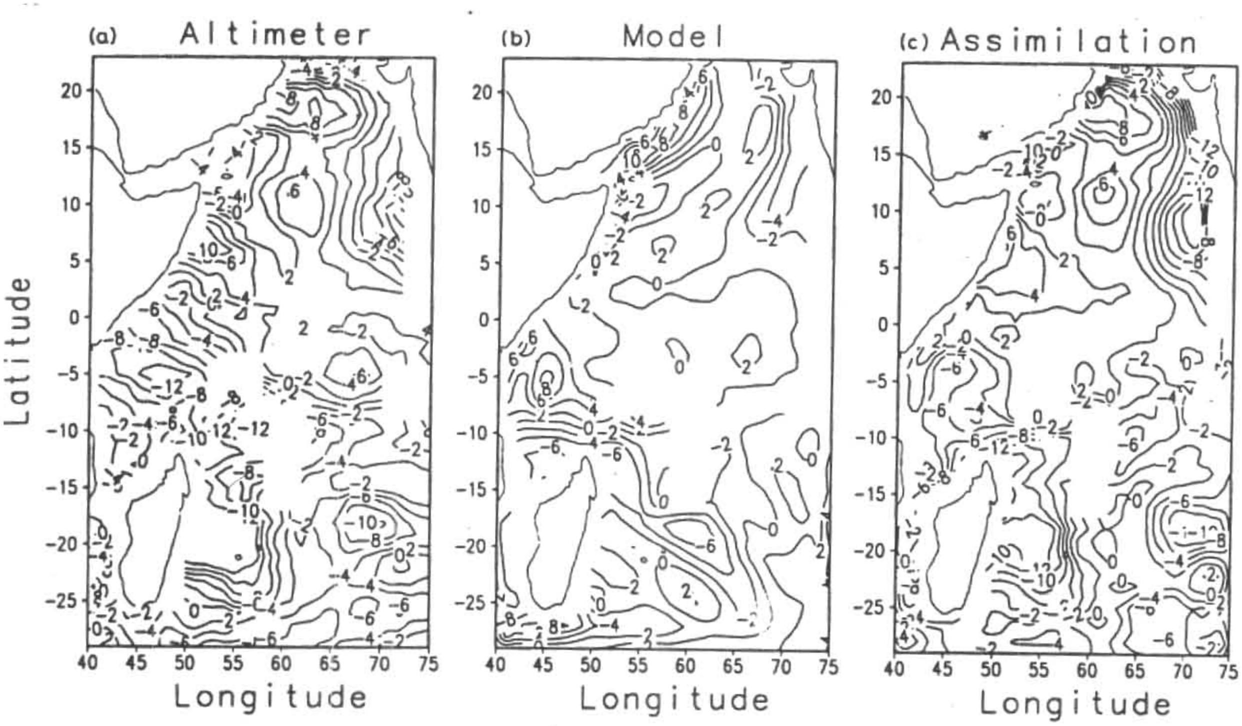
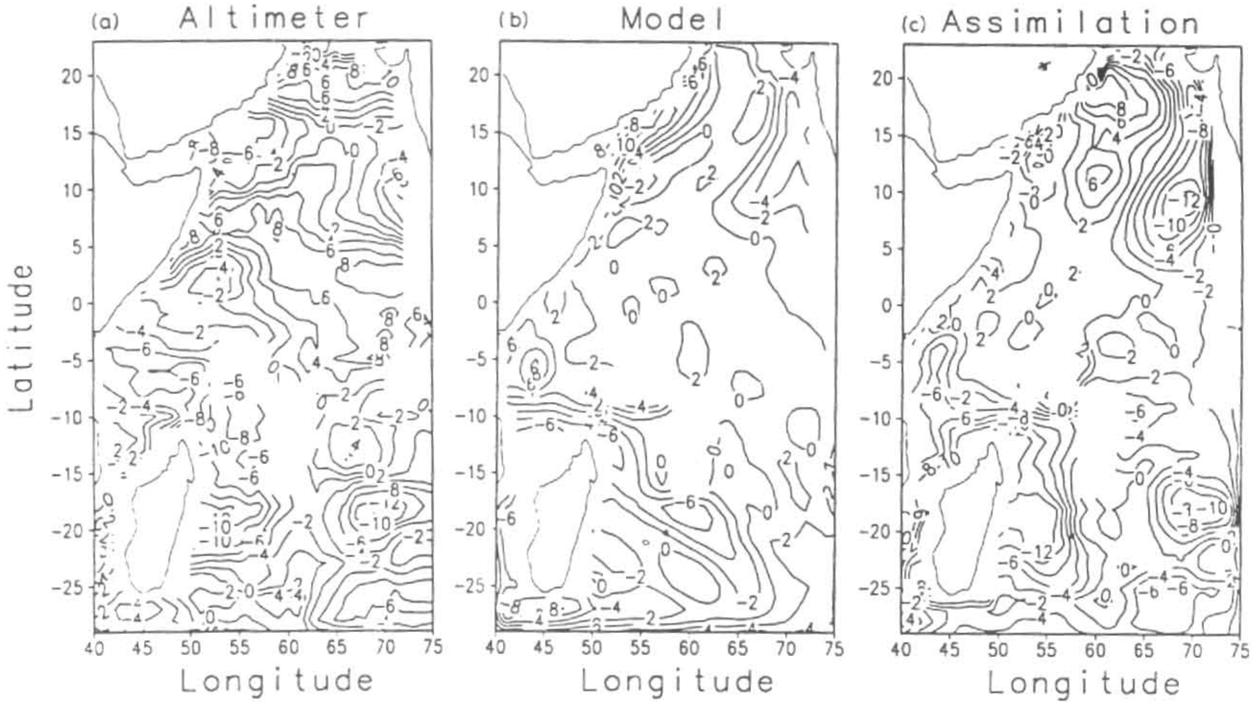


Figs. 5(a-c). Sea level variability after 10 days: (a) Altimeter, (b) Model and (c) Assimilation



Figs. 6(a-c). Sea level variability after 30 days: (a) Altimeter, (b) Model and (c) Assimilation



Figs. 7(a-c). Sea level variability after 40 days: (a) Altimeter, (b) Model and (c) Assimilation

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