550·342 : 534·647

Propagation of Love Waves in a layer overlying a heterogeneous half-space

RABINDRA KUMAR BHATTACHARYYA

Department of Mathematics, Brahmananda Keshabchandra College, Calcutta

(Received 3 May 1966)

ABSTRACT. Love wave propagation in a homogeneous layer overlying a solid half-space of varying density and rigidity is considered.

1. Introduction

The problem of propagation of Love waves through a crustal layer overlying a solid heterogeneous mantle has been studied by many authors (Ewing *et al.* 1957). Deresiewicz (1962) has recently given several plausible variations for heterogeneity of the sub-stratum and the corresponding dispersion equations in terms of tabulated functions. In the present paper, however, the heterogeneity consists in a special type of variation of rigidity (μ) and density (ρ) in the lower medium with depth. The period-velocity equation is represented in terms of Whittaker function and its first derivative. An attempt has been made to solve this equation numerically.

2. The problem and its solution

Let the interface be taken as the XOY plane and the positive direction of Z-axis point into the interior of the half-space. The specifications regarding the two media are as follows —

$$\mu=\mu_0, \quad \rho=\rho_0, \quad -H\leqslant z\leqslant 0 \tag{1}$$

$$\mu=\mu_0(1+az)^{3/2}, \quad \rho=\rho_0(1+az)^{1/2}, \quad 0\leqslant z\leqslant \infty \tag{2}$$

For Love waves —

$$u=w=0, \quad v=v(x, z)$$

and in the absence of body forces, the equation of motion for the y component is —

$$\rho\,\frac{\partial^2 v}{\partial t^2}=\frac{\partial}{\partial x}\left(\mu\,\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\,\frac{\partial v}{\partial z}\right) \tag{3}$$

In the upper medium, equation (3) assumes the form —

$$\frac{\partial^2 v}{\partial t^2}=\beta_0{}^2\nabla^2 v, \qquad \beta_0{}^2=\mu_0/\rho_0 \tag{4}$$

Its solution may, as usual, be taken as

$$v=A\exp i(\omega t+k\gamma z-kx)+B\exp i(\omega t-k\gamma z-kx) \tag{5}$$

where,

$$\gamma=\left(\frac{c^2}{\beta_0{}^2}-1\right)^{\frac{1}{2}}, \quad c=\text{phase velocity.}$$

In the lower medium, equation (3) reduces to

$$\rho\,\frac{\partial^2 v}{\partial t^2}=\mu\nabla^2 v+\frac{\partial\mu}{\partial z}\cdot\frac{\partial v}{\partial z} \tag{6}$$

Set $V=\mu^{\frac{1}{2}}.\ v$, and Eq. (6) becomes

$$\rho\,\frac{\partial^2 V}{\partial t^2}\,\mu\nabla^2 V+\left[\frac{1}{4\mu}\left(\frac{\partial\mu}{\partial z}\right)^2-\tfrac{1}{2}\,\frac{\partial^2\mu}{\partial z^2}\right]V$$

Substituting

$$V=Z(z)\ \exp.\,i\,(\omega t-kx)$$

one obtains

$$\frac{d^2Z}{dz^2}+\left[\frac{\omega^2}{\beta^2}-k^2+\frac{1}{4\mu^2}\left(\frac{d\mu}{dz}\right)^2-\frac{1}{2\mu}\cdot\frac{d^2\mu}{dz^2}\right]Z=0 \tag{7}$$

where,

$$\beta^2=\mu/\rho=\beta_0{}^2(1+az).$$

Equation (7), by virtue of (2) becomes —

$$\frac{d^2Z}{dz^2}+\left[\frac{\omega^2}{\beta_0{}^2}\cdot\frac{1}{1+az}-k^2+\frac{3a^2}{16}\cdot\frac{1}{(1+az)^2}\right]Z=0$$

By the substitution

$$2k\,(1+az)=a\xi$$

the above equation is finally transformed into

$$\frac{d^2Z}{d\xi^2}+\left[-\tfrac{1}{4}+\frac{\eta}{\xi}+\frac{\frac{1}{4}-(\frac{1}{4})^2}{\xi^2}\right]Z=0 \tag{8}$$

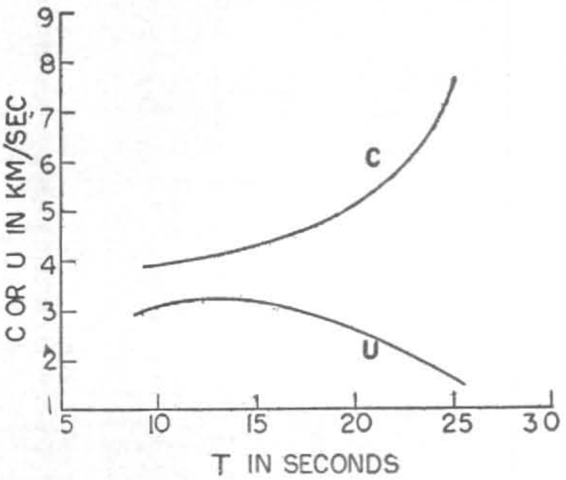

Fig. 1. Phase (c) and group (U) velocity curves

where, $$\eta = \frac{\omega^2}{2ak\beta_0^{\,2}} \qquad (9)$$

Equation (8) is Whittaker's confluent hypergeometric equation having solution of the form —

$$Z(\xi) = LW_{\eta,\frac{1}{4}}\,(\xi) + MW_{-\eta,\frac{1}{4}}\,(-\xi)$$

For surface waves,

$$\mathrm{Lt}_{z\to\infty} Z(\xi) = 0$$

Hence, we put $M = 0$

$$Z(\xi) = LW_{\eta,\frac{1}{4}}\,(\xi)$$

$$v = C.R(\xi).\ \exp.\ i\ (\omega t - kx) \qquad (10)$$

where,

$$R(\xi) = \frac{1}{\xi^{\frac{1}{4}}}\cdot\ W_{\eta,\frac{1}{4}}\,(\xi) \qquad (11)$$

The boundary conditions are —

$$\left.\begin{aligned} (p_{yz})_1 &= 0 && \text{at}\quad z = -H \\ (v)_1 &= (v)_2 && \text{at}\quad z = 0 \\ (p_{yz})_1 &= (p_{yz})_2 && \text{at}\quad z = 0 \end{aligned}\right\} \qquad (12)$$

By equations (5), (10) and (12),

$$A\ \exp(-ik\gamma H) - B\ \exp\ (ik\gamma H) = 0$$
$$A + B = C.R(\xi_0)$$
$$ik\gamma(A - B) = CR'(\xi_0) \qquad (13)$$

where,

$$R(\xi_0) = [\,R(\xi)\,]_{z=0}$$

$$R'(\xi_0) = \left[\frac{d}{dz}\,R(\xi)\right]_{z=0}$$

Eliminating A, B, C from (13), one obtains a relation, which when simplified, reduces to —

$$\frac{\gamma}{2}\tan\,k\gamma H = \frac{3a}{8k} - \frac{W'_{\eta,\frac{1}{4}}\,(2k/a)}{W_{\eta,\frac{1}{4}}\,(2k/a)}$$

This is period-velocity equation involving Whittaker function and its first derivative.

In equation (13) the prime refers to differentiation with respect to z; in equation (15), with respect of ξ.

3. Numerical calculations

The phase velocity (c) and the group velocity ($U = d\omega/dk$) curves, shown in Fig. 1, were drawn by the help of Tables of Coulomb Wave Functions (Whittaker Functions 1965). The values for H, a and β_0 were taken as 33 km, $\cdot 16 \times 10^{-6}$ and $3\cdot 6$ km/sec respectively.

4. Acknowledgement

I am grateful to Dr. S. C. Ganguly of Bengal Engineering College, Shibpur, Howrah, for his guidance.

REFERENCES

Deresiewicz, H.	1962	*Bull. seismol. Soc. Amer.*, **52**, 3, pp. 639-645.
Ewing, W. M., Jardetzky, W. S. and Press, F.	1957	*Elastic Waves in Layered Media*, pp. 341-349. Lamont Geol. Obs. contribution, 189.
Luk'yanov, A. V., Teplov, I. V. and Akimova, M. K.	1965	*Tables of Coulomb Wave Functions (Whittaker Function)*, Pergamon Press publ.