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Kinematic analysis of upper wind fields

G. R. MULKY

Regional Meteorological Centre, Bombay

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ABSTRACT. The paper explains a method of computing the velocity and direction of movement of troughlines, cyclonic centres etc associated with upper wind fields by evaluating vectorial changes of wind with time. The method is similar to the one first proposed for surface pressure fields by Petterssen, but has the advantages that it can be used for any level for which sufficient wind data are available. The displacement of a troughline in a pressure field is obtained by Petterssen in terms of pressure derivatives of the second order; in the present paper it is shown that the displacement of a trough line in a wind field is obtained in terms of wind velocity derivatives of the first order. The inaccuracy of upper wind data as compared to surface pressure data is somewhat offset by the lower order of the terms in the equations for the displacement of troughlines.

1. Introduction

A prognosis of the pressure field or the wind field, if it could be correctly done, would solve many of the problems in forecasting. In view of the complexity of dynamical methods, kinematic methods have been developed, chiefly by Petterssen (1940). Petterssen's analysis concerns pressure fields, and in his equations the displacements of characteristic lines and points associated with pressure fields are obtained in terms of pressure derivatives of the second order. As pressure at surface level is an accurately measurable element, Petterssen's analytical method gives good results when applied to surface pressure fields. In the upper air, the sparseness and comparative unreliability of radiosonde pressure data preclude the general use of Petterssen's equations. His method can be used indirectly to forecast the displacements of wind systems in the upper air by making use of the geostrophic relationship between wind and pressure. Such an indirect application will, however, be open to objection as the geostrophic approximations is not always valid, particularly in tropical latitudes. An attempt is made in this paper to develop a kinematic method of forecasting the displacements of wind systems in the upper air independent of pressure considerations.

2. Theoretical considerations

In the first instance it is necessary to define in mathematical terms the characteristic lines and points associated with a wind field. Let us consider the streamline of wind in a horizontal plane shown in Fig. 1. Ox and Oy are rectangular co-ordinate axes. Oy is a line of symmetry for the wind streams, and along Oy the component of the wind vanishes. This is a troughline in the wind field. Taking the velocity components as u and v along Ox and Oy respectively, we may define a troughline in the wind field by the condition,

$$v=0 \quad (\text{or } u=0) \tag{1}$$

and by the additional condition,

$$\frac{\partial v}{\partial x} > \qquad \left(\text{or } \frac{\partial u}{\partial y} < 0\right) \tag{2}$$

A cyclonic centre is the point of intersection of two trough lines, and we have the conditions,

$$v=u=0 \tag{3}$$

and

$$\frac{\partial v}{\partial x} > 0 \;; \quad \frac{\partial u}{\partial y} < 0 \tag{4}$$

Wedgelines, anticyclonic centres and cols may also be similarly defined.

In forecasting the displacements of a wind field, it would in general be sufficient to compute the probable displacements of the troughlines, cyclonic centres etc associated with the field. To find out these, it is necessary to calculate the time rate of change of any quantity with respect to a moving point. If the point is moving

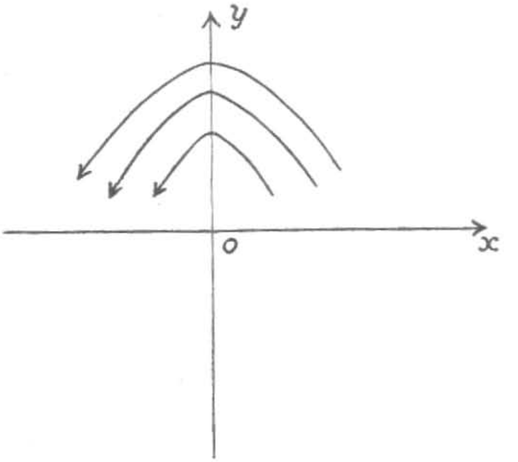

Fig. 1. Troughline in a wind field

in a horizontal plane with velocity components C_x and C_y along Ox and Oy respectively then the time rate of change of any function F for this point is expressed by the equation.

$$\frac{\delta F}{\delta t} = \frac{\partial F}{\partial t} + C_x \frac{\partial F}{\partial x} + C_y \frac{\partial F}{\partial y} \qquad (5)$$

If the point under consideration is such that the value of the function remains constant during the motion of the point, then $\delta F/\delta t=0$, and we have,

$$-\frac{\partial F}{\partial t} = C_x \frac{\partial F}{\partial x} + C_y \frac{\partial F}{\partial y} \qquad (6)$$

We may now apply this general result to the case of the trough line in Fig. 1. We may limit ourselves to the displacement of the trough line in the direction of Ox. Throughout its displacement, all points on the troughline satisfy the condition $v=0$. Hence $\delta v/\delta t=0$. Substituting v for F in equation (6), we obtain,

$$C_x = -\frac{\partial v}{\partial t} \bigg/ \frac{\partial v}{\partial x} \qquad (7)$$

Similarly, if Ox coincides with another troughline, the velocity of this troughline is given by,

$$C_y = -\frac{\partial u}{\partial t} \bigg/ \frac{\partial u}{\partial y} \qquad (8)$$

The components of wind velocity vanishes at the cyclonic centre. The term $\delta v/\delta t$ (corresponding to $\delta F/\delta t$ in equation 5) will therefore be zero at the cyclonic centre wherever it moves.

A cyclonic centre, being defined as the point of intersection of two troughlines, will have velocity components C_x and C_y given by equations (7) and (8). The resultant velocity and direction of motion can be computed from the component values.

This derivation is analogous to that obtained by Petterssen for a pressure field except that wind components are used instead of pressure derivatives and only first order derivatives occur in the equations. As the lines and points in the wind field have been defined purely in relation to wind velocity components, the analysis is independent of pressure considerations, and can be used for a wind field at any level provided that the time and space derivatives can be computed with reasonable accuracy.

3. Practical considerations

Before discussing the practical methods of evaluating the terms in the equations, we may examine some of the implication of equations derived above. The following qualitative rules, which are closely analogous to those deduced by Petterssen for pressure fields, may be formulated.

(*i*) When the shear is uniform in all directions, or in other words the wind streams are circular, the direction of motion will entirely depend on the wind tendency term.

(*ii*) When the shear is large, the velocity of the system will be small or moderate; when the shear is small the velocity may vary within wide limits depending on the magnitude of the tendency term. In other words, sharp troughs and well-developed circulations move with small or moderate speeds while the speeds of shallow troughs and weak circulations may vary within wide limits.

(*iii*) When the shear is large in one direction, the system will have a tendency to move normal to that direction; in other words a system will tend to move in the direction of its longer axis of symmetry. This rule cannot be applied without reference to the wind tendency term as the tendency term *may* be aligned in such a way that there is no component of motion along the longer axis of symmetry, as is the case in Figs. 3 and 4.

The space derivatives, involving the shear terms $\partial v/\partial x$ and $\partial u/\partial y$ can be computed by plotting u and v components of the observed winds at the pibal stations in the neighbourhood of the troughlines under consideration and calculating their mean gradients. Wind data near the troughlines should be given preference as the winds far from the troughlines may be affected by other systems.

It is preferable to use winds on either side of the troughline as this may lead to a cancellation of systematic errors in the observations. In most wind fields the troughlines can be located without difficulty. If the field is circular or approximately so, all lines passing through the centre are troughlines, and the troughlines can therefore be drawn according to convenience, after locating the centre by dropping perpendiculars from the wind vectors in the innermost shells of the circulation.

The tendency terms $\partial u/\partial t$ and $\partial v/\partial t$ refer to instantaneous conditions. As it is not possible to obtain instantaneous values, an approximation has to be made by using finite differences in place of the differentials. The errors involved in this process of approximation will be larger, the larger the time interval used. In practice, however, it is not advantageous to use too small an interval of time as the errors of observation will be comparable to the tendencies themselves. The smallest interval of time that can be practically used in upper wind kinematic analysis is a 6-hour interval between 0130 and 0730 IST. As the diurnal variation of wind between these two hours may be neglected for levels at 3000 ft and above, upper wind tendency charts computed from the winds at these two hours may be useful in computing the displacements of wind systems. Further, the main forecasts issued in the India Meteorological Department are based on the morning charts. The upper wind tendency charts for a 6-hour interval may therefore be additional aids. Wind tendency charts for a 24-hour interval may also be used if necessary as the diurnal effects, if any, are eliminated.

If the cyclonic wind system is circular, the co-ordinate axes can be chosen arbitrarily in such a way that one of them, say Oy, will coincide with the direction of the wind tendency. As the tendency has no component normal to itself, $\partial u/\partial t = 0$ and hence $C_y = 0$. The cyclonic centre will then move along Ox with a velocity given by equation (7). For a circular cyclonic circulation we may thus enunciate the simple rule that its centre moves normal to the direction of the wind tendency in its immediate neighbourhood. Further, from the additional condition given for a troughline, *viz.*, $\partial v/\partial x > 0$, it will be seen that if the wind tendency is positive *i.e.*, southerly, the troughline will move west and *vice versa*. Similar result can be given for the troughline along Oy.

The wind tendencies can be easily evaluated by computing the vectorial change in winds between the hours of observation under consideration. Polar sheets will facilitate the computations The computations for a required level will not take more than 15 or 20 minutes ordinarily. When there is no pilot balloon station at the position of the centre, the value of the tendency at the centre will have to be obtained by extrapolation from neighbouring values. The wind tendencies plotted on the chart can be joined up into a pattern by drawing profiles in the same manner as streamlines in wind fields are drawn. The direction of the tendencies at points where no actual observations are available can be estimated for these profiles. These profiles are particularly useful in the sea area where observations of upper winds are not available. As the wind tendencies along the coast have magnitude as well as direction, the extrapolation of the tendency profiles in the sea area can be done with a greater degree of confidence than is the case with pressure tendencies.

4. Conclusion

Three illustrations of the method outlined above are given in the Appendix to the paper. (*vide* pp. 167-170). Two of them are for 6-hour intervals, while the third is for a 24-hour interval. All the illustrations have been worked out for a level of 3000 ft as the wind systems are associated with depressions, and the motion of the wind system at the top of the friction layer, if computed, would be approximately equal to that of the surface depression. This is an immediate need in forecasting. The analysis can obviously be carried out at other selected levels to see the motion of the system as a whole in three dimensions. The analysis, if carried out at upper levels in the region of 20,000 ft may give very useful results regarding the structure of moving cyclones and depressions.

Some of the advantages of the method outlined in the paper are summarised below—

(1) The terms in the equations for velocity of troughlines etc are of the first order and can be directly computed from wind data. Though upper wind observations are less accurate than surface pressure observations, the lower order of the terms in the equations somewhat compensates for the inaccuracy of the observations.

(2) Upper wind tendencies are vector quantities, and as such, lend themselves easily to graphical treatment, particularly while extrapolating in the sea area with coastal values.

(3) The method can be used for selected upper levels, thus giving a three-dimensional pictures of the motion of wind systems.

(4) The method can be applied to practically all wind systems in the Indian area as the fronts associated with low pressure systems are diffused and do not present sharp wind discontinuities which might render the analysis invalid.

Nevertheless, the method has some obvious limitations.

(*i*) When the analysis is carried out between two successive pilot balloon ascent hours, there is a chance of the vitiation of the results due to diurnal changes whose magnitudes are not known. This is particularly so for afternoon winds. It may however be stated that the diurnal change in winds between midnight and morning may be negligible at 3000 ft and above. In cases where the diurnal effect is expected to be considerable, 24-hour tendencies have to be worked out. The analysis in such cases may be subject to fairly large errors.

(*ii*) The analysis depends on observed pilot balloon winds which are subject to various errors.

(*iii*) On many occasions when the analysis is required for computing the displacement of a depression, wind data in the required area will be scanty due to low cloud cover or rain. It may, however, be pointed out that quite often wind data are available in the forward area, and it is these which are most useful for the computations.

5. Acknowledgement

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REFERENCE

Petterssen, S. 1940 *Weather Analysis and Forecasting,* pp. 378-425.

APPENDIX

1. Bay of Bengal depression in the first week of August 1953

The track of this depression is shown in Fig. 2. In its passage through Madhya Pradesh it sharply recurved from a westerly direction to a northnorthwesterly and weakened. Later, however, it again moved away towards Sind in a westnorthwesterly direction as a low pressure area. The wind tendency charts for the morning of 4 August 1953 were examined to see whether the sharp kink in the track of the depression was brought out by them.

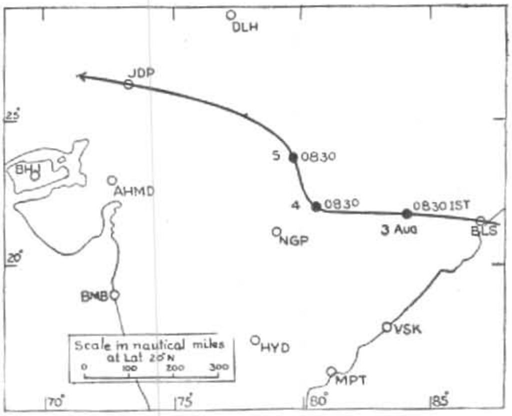

Fig. 2. Track of depression in the first week of August 1953

Fig. 3 shows the distribution of upper winds at 3000 ft at 0730 IST on 4 August 1953. The field in the region of the depression is elliptical with the longer axis (Ox) in the west-east direction. The cyclonic centre can be approximately located near Seoni and the shorter axis (Oy) of the elliptical field drawn in the south-north direction. Ox and Oy are troughlines in the wind field.

Fig. 4 shows the distribution of 6-hour wind tendencies at 3000 ft computed from the pilot balloon observations at 0130 and 0730 IST on the same day. Diurnal variation of wind has been neglected. If the wind tendencies are joined up into a pattern by drawing profiles in the manner of streamlines the direction of the wind tendency at the cyclonic centre can be extrapolated, and is westerly. The magnitude of the tendency is of the order of 5 knots. (The actual magnitude of the wind tendency at Jabalpur is 4 knots). As the tendency at the cyclonic centre has no component along the shorter axis of the wind field, $C_x=0$, and the cyclonic centre will move along the shorter axis Oy with a speed given by

$$-\frac{\partial u}{\partial t}\Big/\frac{\partial u}{\partial y}$$

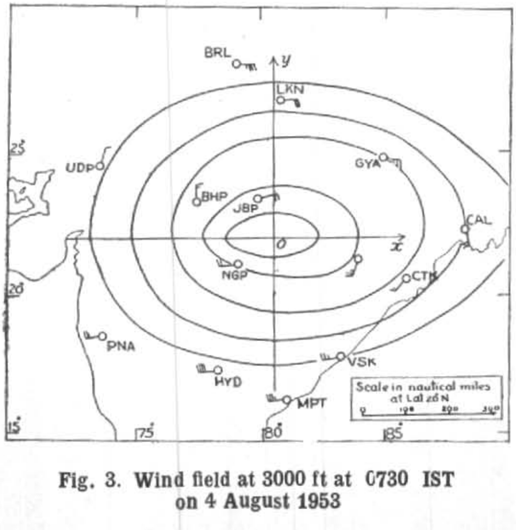

Fig. 3. Wind field at 3000 ft at 0730 IST on 4 August 1953

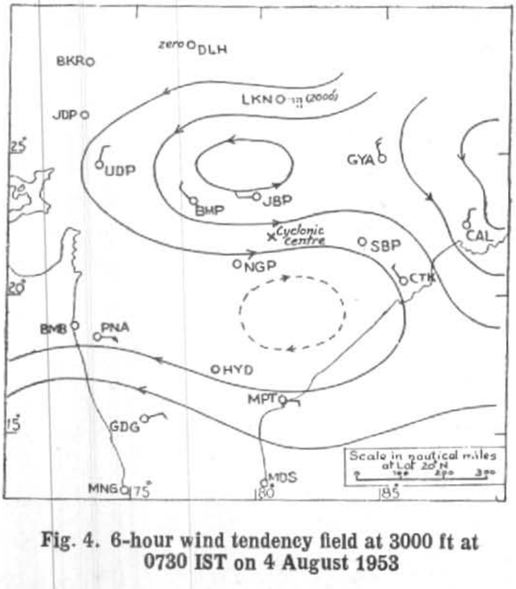

Fig. 4. 6-hour wind tendency field at 3000 ft at 0730 IST on 4 August 1953

Now $\partial u/\partial t = 5/6$ approximately. ($\triangle u = 5$; $\triangle t = 6$)

Using the actual winds observed at Jabalpur and Nagpur which are practically equidistant and on either side of the troughline Ox, the mean value of $\partial u/\partial y$ works out as $-2/7$ approximately, using knots as units of speed and nautical miles as units for distance.

Hence $C_y = +3$ knots approximately.

The cyclonic centre will therefore move in a northerly direction with a speed of about 3 knots. Both the direction and the speed of movement tally fairly well with what was actually observed, as may be seen from Fig. 2. The low speed is probably due to the fact that the system was recurring sharply. The speed obtained in the computation cannot have much accuracy rating as the magnitude of the tendency is small and hence the percentage of error possible in the process of extrapolation from neighbouring values may be high.

2. *Masulipatam cyclone of October 1949*

The track of this cyclone is shown in Fig. 5. The process of recurvature started immediately after the cyclone struck the coast. Fig. 6 shows the wind field at 3000 ft at 0730 IST on 29 October 1949. Data near the cyclonic centre are entirely absent, but the associated circulation appears to be very extensive as can be seen from the streamlines even at great distances from the cyclonic centre. The axes Ox and Oy have been drawn from the symmetry of the stream lines.

The distribution of the 5-hour wind tendencies at 0730 IST on the same day for 3000 ft is shown in Fig. 7. The tendency at the cyclonic centre is westnorthwesterly of the order of 5 knots. The cyclonic centre will therefore move along Oy with a velocity which can be computed as 10 knots approximately. The shear term has been calculated as roughly $-1/10$ by taking as the mean from the winds at Bhopal, Gaya, Cuttack, Masulipatam and Hyderabad which, though by no means close to the troughline, are well distributed with respect to it.

In this case also, the accuracy rating cannot be high as the shears have been computed from observations far removed from the troughline, and the magnitude of the tendency is of a small order.

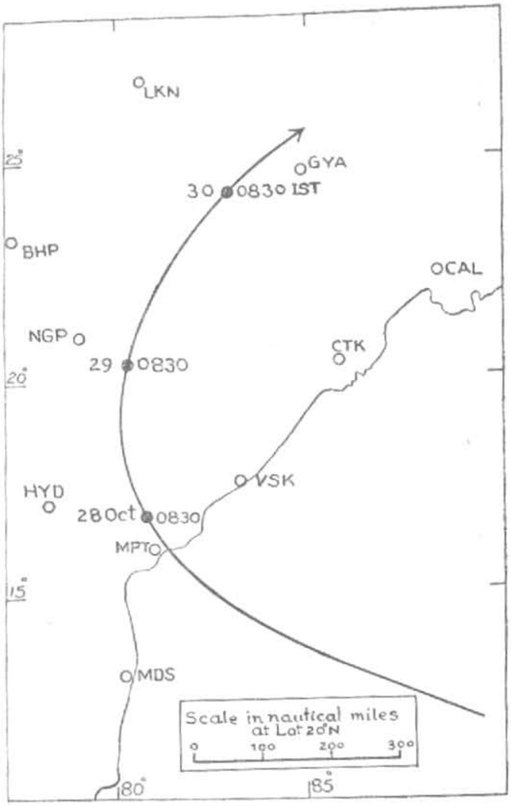

Fig. 5. Track of the Masulipatam cyclone of October 1949

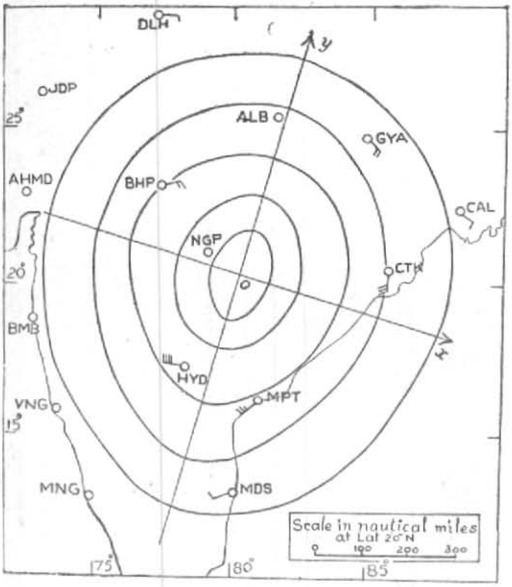

Fig. 6. Wind field at 3000 ft at 0730 IST on 29 October 1949

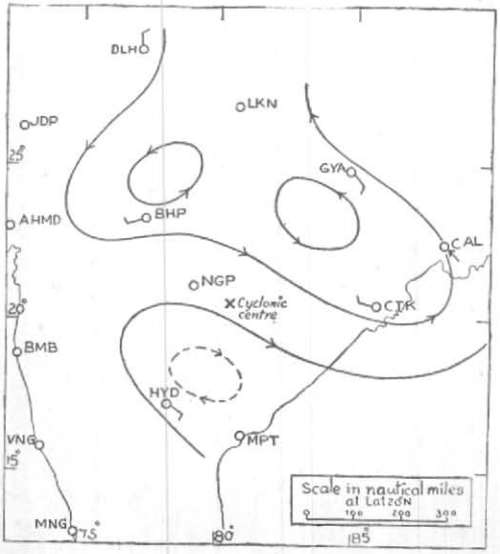

Fig. 7. 5-hour wind tendency field at 3000 ft at 0730 IST on 29 October 1949

3. *Depression in the last week of July 1951*

The track of the depression and the wind field at 3000 ft at 0730 IST on 27 July 1951 are shown in Figs. 8 and 9 respectively. A marked troughline in the wind field can be located in the northwest-southeast direction, and another troughline drawn normal to it by symmetry from the cyclonic centre which can be located by dropping perpendiculars from the wind vectors at the nearby stations.

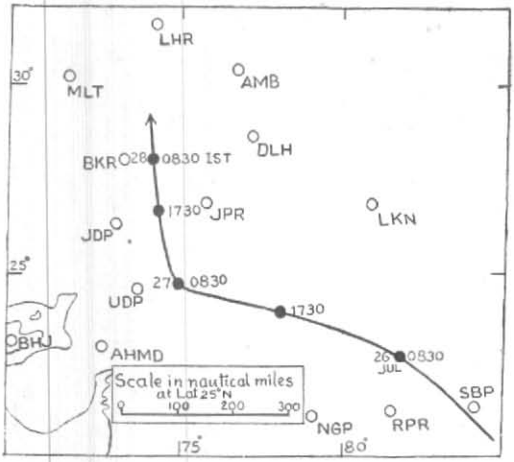

Fig. 8. Track of the depression in the last week of July 1951

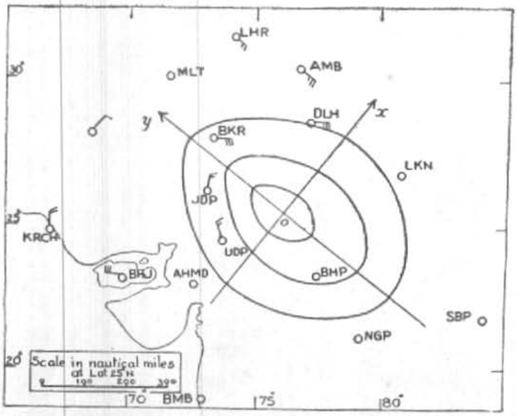

Fig. 9. Wind field at 3000 ft at 0730 IST on 27 July 1951

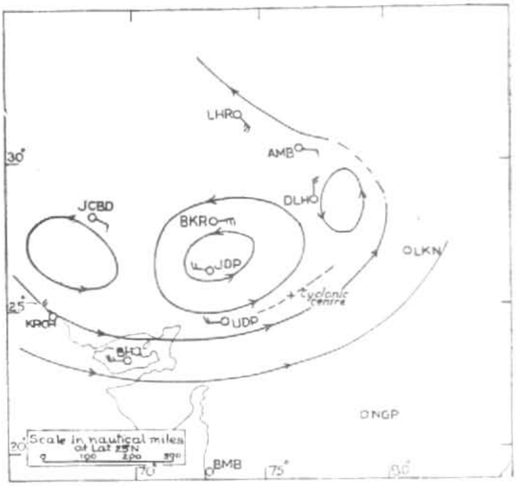

Fig. 10. 24-hour wind tendency field at 3000 ft at 0730 IST on 27 July 1951

Fig. 10 shows the distribution of tendencies computed for an interval of 24 hours. The diurnal effects, if any, are therefore automatically eliminated. The direction and the speed of the system, computed by the method described in the previous cases, comes out as NW to NNW, 6 knots. It will be seen that there is substantial agreement between this inference and what was actually observed next day as regards direction. The result suggests that the use of a long tendency interval need not necessarily give less reliable values. The tendency term is of large magnitude, and the error in determining it is of a small order when compared to itself. However, the validity of equating the finite difference to a differential may be open to objection, as the interval is large.