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The variation of the angle of incidence of *P* waves with epicentral distance

R. K. DUBE

Central Seismological Observatory, Shillong

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ABSTRACT. Angles of incidence of *P* waves were obtained from the records of the short and long period seismographs of W.W.S.S.N. at Shillong. The study covers the range of epicentral distances from 20° to 100°. The observations indicate that the velocity of *P* waves at the earth's surface is approximately 6 km/sec which is the velocity of the upper layer (granitic layer) of the earth's crust.

1. Introduction

Nuttli and Whitmore (1961) made a study of the variation of the angle of incidence of *P* waves with epicentral distance and observed *P* wave velocity of 8·0 km/sec which is the value just below the crust fitted satisfactorily with the observed angles of incidence. It was, however, surprising to the authors that the observations could not be fitted with the *P* wave velocity near the surface of the crust as expected from theory. No satisfactory explanation was offered for the peculiar behaviour. The authors, however, suggested the use of data from seismographs in which daily calibration pulses are recorded so that the performances could be checked for each component for day-to-day recording. This condition is satisfied by the long and short period seismographs of W.W.S.S.N. in operation in Shillong since April 1963. The work has, therefore, been taken up again.

The present study was undertaken with data from the above seismograph system. The instrumental constants of the short period system are given below —

$$T_0 = 1\cdot0 \text{ sec}, T_g = 0\cdot75 \text{ sec}$$

Both seismometers and galvanometers are critically damped.

During the entire period, the long period seismographs did not operate with the same constants. The constants of the instruments were as follows —

$$\left.\begin{array}{l} T_0 = 30 \\ T_g = 100^s.0 \end{array}\right\} \text{from April 1963 to 23 April 1965.}$$

In the later part the seismometer period was changed to 15·0 sec (Both the seismometers and galvanometers were critically damped).

The present study is not, however, affected by the change in the instrumental constants. Because our requirement is that the components should be matched so that the constants and magnification of two horizontal and vertical components should have the same value. This will enable us to make direct use of the recorded trace amplitudes without any reference to actual response curves. In doing so we have assumed that the period of the initial *P* motion is same for two horizontals and the vertical.

2. Method

The apparent angle of incidence $\bar{i}$ is defined as the angle between the resultant *P* motion and the normal to the surface. This is given by —

$$\tan \bar{i} = U_h / U_z$$

where U_h and U_z are the horizontal and vertical components of the ground motion due to the incident *P* wave, respectively, U_h is obtained from the relation —

$$U_h = (U_N^2 + U_E^2)^{1/2}$$

where U_N is the ground motion recorded in the N-S component while U_E is the ground motion recorded in the E-W component. The initial motions of the *P* wave were measured from the three components and the value of $\bar{i}$ is computed for shocks of different epicentral distances. In selecting shocks proper care was taken to select those records for which the initial motions are very well recorded. The real angle of incidence i is related to the apparent angle of incidence $\bar{i}$ by the following relation (Bullen 1953) —

$$i = \sin^{-1}\left[\frac{a^2}{2b^2}(1-\cos\bar{i})\right]^{1/2}$$

The ratio a/b of *P* and *S* wave velocities were computed for Poisson's ratio σ for two assumed values, *viz.*,

$$\sigma = 0\cdot25 \text{ and } \sigma = 0\cdot286.$$

Assuming different values of *P* wave velocities a it is possible to calculate the i—$\triangle$ curve by using the relation (Bullen 1953) —

$$dT/d\triangle = (r/a_0) \sin i$$

where r is the radius of the earth. Nuttli and Whitmore (1961) have constructed a set of theoretical i—$\triangle$ curves for different assumed values of a_0 ranging from 6·0 to 9·0 km/sec. The solid curves in Figs. 1, 2, 3 and 4 based on the Jeffreys-Bullen travel-time tables, are taken from their paper.

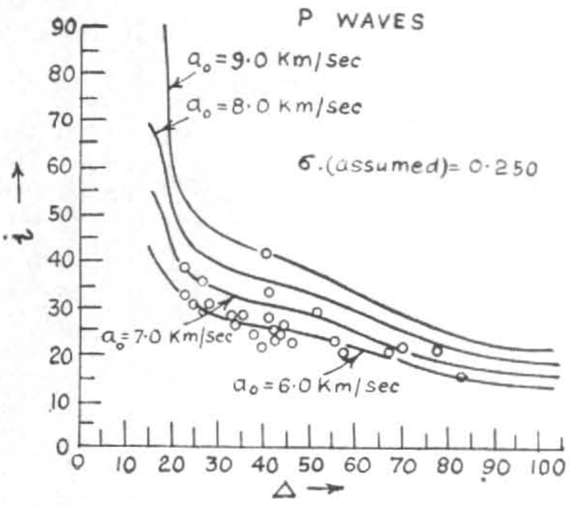

Fig. 1

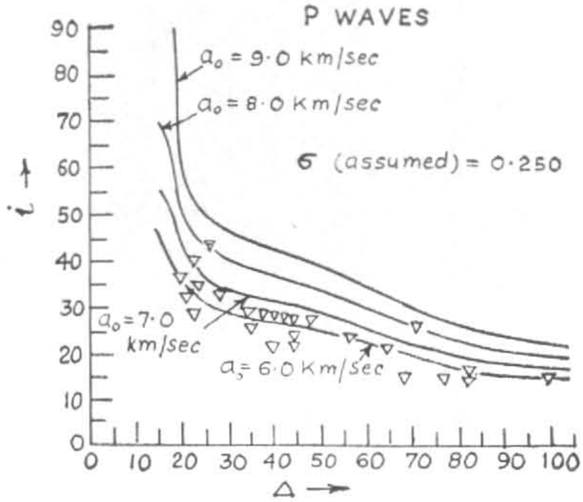

Fig. 2

Observed and theoretical values of i *vs* Δ; theoretical values based on Jeffreys-Bullen Tables

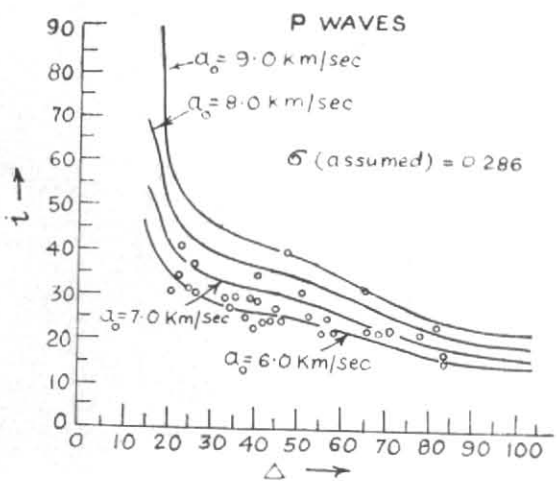

Fig. 3

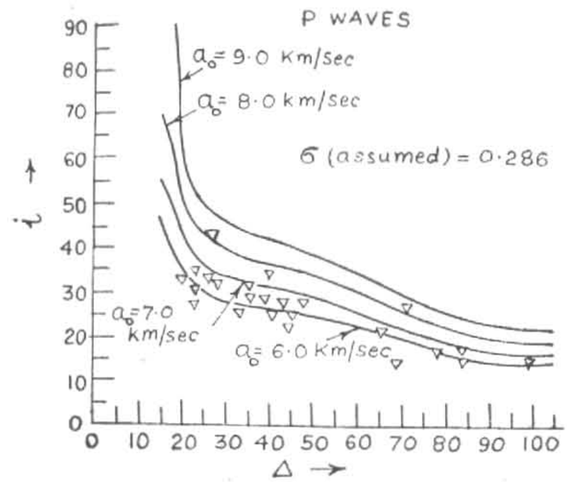

Fig. 4

Observed and theoretical values of i *vs* Δ; theoretical values based on Jeffreys-Bullen Tables

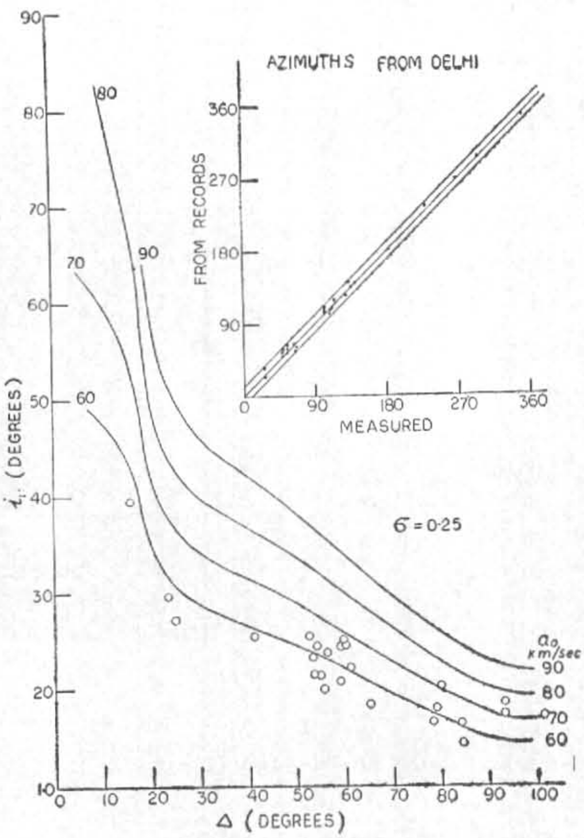

Fig. 5
Observed and theoretical values of i *vs* △; theoretical values based on Jeffreys-Bullen Tables

3. Observations and Conclusion

With a view to examine whether there is any systematic difference in the observed i — △ curve as far as they relate to the observations from short and long period seismograhs, these results are presented separately in Table 1. The data from short period instruments were plotted in Fig. 1 for $\sigma=0\cdot25$ and in Fig. 3 for $\sigma=0\cdot286$. The data from long period seismograms are presented in Figs. 2 and 4 for value of $\sigma=0\cdot25$ and $0\cdot286$ respectively. The epicentral distance △ was calculated from $P—O$ values, the time of origin O is taken from the P.D.E. cards of U.S.C.G.S. Although △ computed by this method will not be very accurate, it is felt that it will serve the purpose of the present study.

On examination of Figs. 1-4 it is noticed that the observed data scatter about the theoretical curve for $a_0=6\cdot0$ km/sec. This is in perfect accord with the theory that the real angle of incidence is dependent on the velocity of the wave in the upper crustal layer. The present writer is unable to account for the observations of Nuttli and Whitmore (1961) for higher value of $8\cdot0$ km/sec. In case the wave lengths are sufficiently large as compared to the thickness of the crustal layers, it is quite possible that the angles of incidences are not modified by the upper crustal layers having smaller velocities of wave propagation. Even with data from long period instruments (Figs. 2 and 4) the author could not find any noticeable effect. Moreover, it is almost an established fact that the seismic body waves do not suffer any dispersion.

The plotted data of Figs. 1-4 lie more or less centred about the curve $a_0=6\cdot0$ km/sec and they are scattered about the mean position. The author is unable to account for the same, although the observations were made with much care using luminous magnifiers having a graduated graticule in the eyepiece. One fact was, however, very clear that deviation or scattering is very much more for smaller ground motions when the movements are comparable with the microseismic noise level of the ground.

It is worth mentioning here that in a similar study from the long period matched component seismographs of W.W.S.S.N. of Delhi data for which are given in Table 2 and are plotted in Fig. 5, identical results have been obtained, which probably leads to more weightage for the results of of the present study.

TABLE 1

P waves arrival time (GMT) *h m s*	Origin time *O* (GMT) *h m s*	Epicentral distance from Shillong (in deg)	Apparent angle of incidence (SP) Short period seismograph	Apparent angle of incidence (LP) Long period seismograph	Real angle of incidence for σ=0·25 SP	Real angle of incidence for σ=0·25 LP	Real angle of incidence for σ=0·286 SP	Real angle of incidence for σ=0·286 LP
23 42 31·0	23 38 00·5	21·3	32·9	36·0	29·4	32·4	31·2	34·4
08 16 38·5	08 11 33·7	22·7	42·6	37·1	39·0	33·5	41·6	35·6
06 17 02·5	06 12 00·4	22·9	—	42·0	—	38·4	—	40·9
15 24 06·5	15 19 03·2	23·0	—	33·3	—	29·8	—	31·6
18 19 25·5	18 14 23·0	23·0	—	31·0	—	27·6	—	29·2
23 28 11·0	23 22 53·0	24·6	33·7	—	30·1	—	32·0	—
22 21 20·0	22 15 42·5	26·6	33·0	36·0	29·5	32·4	31·3	34·4
17 25 07·0	17 19 26·0	27·0	39·0	45·0	35·3	41·5	37·6	44·4
16 37 08·0	16 31 22·0	28·2	—	34·5	—	30·2	—	32·8
00 10 24·0	00 04 34·7	28·5	30·4	—	27·0	—	28·6	—
23 27 44·0	23 21 10·6	32·9	32·0	29·0	28·5	25·7	30·2	27·2
17 50 04·5	17 43 21·4	34·1	30·3	35·0	26·9	31·4	28·5	33·3
09 49 39·0	09 42 28·2	35·5	32·0	32·0	28·5	28·5	30·2	30·2
16 28 8·5	16 20 51·4	38·1	28·0	31·5	24·8	28·0	26·3	29·7
09 04 08·0	08 56 46·0	39·7	31·8	—	28·3	—	30·0	—
23 49 36·0	23 41 58·8	39·9	25·0	37·0	22·0	33·4	23·3	35·4
00 17 04·0	00 09 34·4	40·1	—	28·9	—	25·6	—	27·1
20 48 22·0	20 41 16·0	41·0	31·0	—	27·6	—	29·2	—
12 33 49·5	12 26 06·2	41·3	37·0	—	23·4	—	35·4	—
11 16 11·0	11 08 16·2	42·9	27·0	31·0	23·8	27·6	25·3	29·2
00 19 17·0	00 11 12·1	43·3	28·6	—	25·3	—	26·8	—
16 54 25·0	16 46 15·5	43·8	27·0	27·0	23·8	23·8	25·3	25·3
10 55 46·0	10 47 37·6	44·3	29·6	26·0	26·3	22·9	27·8	24·3
02 25 53·0	02 17 38·5	46·0	26·0	—	22·9	—	24·3	—
11 47 08·0	11 38 28·0	48·3	—	31·0	—	27·6	—	29·2
16 35 20·5	16 26 21·2	51·6	33·1	—	29·6	—	31·6	—
14 39 48·5	14 30 29·1	56·3	27·0	25·0	23·8	22·0	25·3	23·3
09 57 19·5	09 47 30·7	58·6	24·0	—	21·1	—	22·3	—
08 26 16·5	08 15 39·3	64·7	—	24·3	—	21·4	—	22·6
13 41 22·0	13 30 28·0	65·0	34·0	—	30·4	—	32·3	—
13 18 48·0	13 07 50·0	68·0	24·0	17·0	21·1	14·9	22·3	15·7
23 55 57·5	23 44 46·2	70·2	24·4	—	21·5	—	22·7	—
03 10 17·5	02 59 02·0	70·9	—	29·8	—	26·4	—	28·0
17 02 30·0	16 50 29·0	77·9	24·2	19·0	21·3	16·6	22·5	17·6
23 12 41·0	23 00 24·0	81·8	25·7	—	22·7	—	24·0	—
15 01 44·0	14 49 23·0	83·0	17·2	—	15·0	—	15·9	—
13 48 55·0	13 36 30·7	83·2	18·3	19·8	16·0	17·3	16·9	18·3
00 52 41·0	00 41 11·0	83·4	—	18·0	—	15·7	—	16·6
15 42 27·0	15 28 43·0	100·0	—	18 0	—	15·7	—	16·6

TABLE 2

S. No.	Date	Origin time (GMT)			Epicentre		Depth (km)	Magnitude	△ (measured in degrees)	i_0 (deg)	i_0 (deg)	Azimuth	
		h	*m*	*s*	Lat.	Long.						Measured	Computed
1	1-5-1963	10	03	20·0	19·0S	169·0E	140	6·2	100·8	19·0	16·6	107	104
2	8-5-1963	10	22	11·2	36·6N	141·0E	53	6·1	53·4	26·5	23·1	63	65·5
3	12-5-1963	20	08	43	57·4N	153·9W	—	5·9	83·7	18·3	16·1	25	37
4	22-5-1963	13	56	43·0	48·6N	154·7E	22	6·3	61	25·5	22·0	48	54·5
5	22-5-1963	15	42	48·6	4·3N	127·9E	33	—	53·8	27·6	24·4	106	106·7
6	22-5-1963	21	52	02·5	8·2S	115·7E	33	5·4	52·5	28·3	25·1	128	126
7	29-5-1963	08	35	02·9	27·2N	59·3E	33	5·4	16·0	43·2	39·5	268	270
8	4-6-1963	21	04	42·3	1·2S	127·3E	31	5·2	56	26·7	23·6	114	119·7
9	28-6-1963	21	55	38·8	46·5N	153·2E	33	6·1	60	27·5	24·3	48	53
10	10-7-1963	05	22	57·1	46·3N	152·9E	33	5·6	59·5	27·9	24·6	48	57
11	4-9-1963	13	32	12·3	71·4N	73·3W	33	5·9	78	18·8	16·3	351	349
12	9-9-1963	02	45	45·5	4·4S	152·7E	34	5·6	80	22·8	20·0	101	107·3
13	15-9-1963	00	46	54·1	10·3S	165·6E	43	6·3	93	19·6	17·2	101	106·1
14	17-9-1963	19	20	08	10·1S	163·3E	17	6·1	93	20·8	18·2	101	107
15	18-9-1963	16	58	12·5	40·9N	29·2E	33	5·2	41	28·3	25·2	298	299·7
16	23-9-1963	09	01	56·8	16·6S	28·8E	33	5·8	65	20·7	18·2	228	238
17	20-10-1963	09	10	43·9	44·4N	150·2E	40	5·5	58·8	24·1	20·6	53	60
18	20-10-1963	11	52	20·7	44·7N	150·2E	45	5·1	59	27·5	24·3	53	57·3
19	5-2-1964	11	30	15·7	36·5N	141·0E	46	5·4	53·4	21·3	18·7	63	59
20	6-2-1964	13	07	25·2	55·7N	155·8W	33	5·6	84·5	16·2	14·2	26	24·6
21	14-2-1964	16	29	45·0	5·1S	151·7E	55	6·0	79	20·4	17·8	102	113
22	29-2-1964	15	20	12·8	34·8N	141·7E	34	5·1	54	24·1	21·2	65	63·5
23	29-2-1964	23	49	40·8	8·5S	112·7E	73	5·8	55·5	22·6	19·9	131	143
24	15-9-1964	15	29	32·2	8·9N	93·1E	37	6·2	24·4	31·1	27·1	138	141
25	16-9-1964	01	26	26·9	10·9N	93·1	47	5·7	23·0	33·2	29·6	135	137

The observations were carried out in Shillong where the thickness of sedimentary layer is very insignificant and the observations are in good accord with the velocity of 6·0 km/sec which is approximately the velocity of *P* waves in the granitic layer. The same is true for Delhi also. In places where the thickness of the sedimentary layer is appreciable, it might be worthwhile to make a similar study and to examine whether the real angle of incidence is modified by the sedimentary section having velocities much less than the velocity of the granitic layer underlying the sedimentary top.

It is difficult to draw any conclusion on the 20° discontinuity in the travel time from the study of the $i-\Delta$ curve because of the steepness of $i-\Delta$ curve near that range of distance. The study has, however, verified the theoretical result that the real angle of incidence is dependent on the velocity of the upper crustal layer. No conclusion, however, could be drawn regarding the appropriate value of Poisson's ratio in the upper crustal layer because there is very little difference in the observations when we use $\sigma = 0 \cdot 25$ (Figs. 1 and 2) and $\sigma = 0 \cdot 286$ (Figs. 3 and 4).

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