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DYNAMICS OF THUNDERSTORMS

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PART 1 Review of the present explanations for pressure changes in thunderstorms.

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Abstract.—It has generally been believed that the pressure rise is the only type of pressure change which can occur with the passage of a thunderstorm and that this rise is invariably accompanied with a fall in Dry Bulb temperature. Evidence has been brought forth to show that this is not always the case and that in addition to instances of thunderstorms with pressure rise, there are also cases of thunderstorms accompanied by no pressure change or even a pressure fall. It is also observed that these pressure variations are not always associated with a temperature fall, but are in some cases, also accompanied by a rise in Dry Bulb or no appreciable change in Dry Bulb.

A review of the present explanations of pressure changes in thunderstorms is given and it is shown how these are inadequate to explain all the observed facts.

1. *Introduction.*

Of all the phenomena of the atmosphere there is none that excels the thunderstorm in beauty and majesty as it builds up from small detached Cumuli into the towering and awe-inspiring Cumulo-Nimbus ; none also that surpasses it as an example of the fury and violence which nature exhibits when she seeks to restore a state of balance which due to some reason, has been temporarily upset. Although to those on the ground there is very little to fear from these thunderstorms except for the damage due to lightning or high wind, to those who are in the air, however, they are a terror principally because of the lightning and the violent vertical movements and the squall associated with them. In view of the serious danger to aviation realised particularly during the World War II, they have attracted considerable attention of meteorologists and a number of papers discusssing their mechanism and the characteristics associated with them from a qualitative as well as quantitative point of view have appeared in many journals.

2. It is well known that amongst the changes in the meteorological elements associated with the passage of thunderstorms, the surface pressure change is the most important, as it represents the cumulative effect of all the processes at the various levels. As a satisfactory interpretation of this feature is likely to go a long way to throw light on the dynamics and structure of the thunderstorm, the problem has attracted considerable attention of meteorological workers. The names of Shaw, Humphreys, Sucksdorf, Levine and Buell may specially be mentioned in this connection.

3. Since in middle latitudes, most of the thunderstorms are the cold front thunderstorms associated with a pressure rise and a Dry Bulb fall, most meteorologists believed that this was the only type of pressure change which could occur with the passage of a thunderstorm, and as such all their efforts were directed to formulate such physical processes as would explain this tendency of pressure rise. A detailed examination of the autographic records on thunderstorm days in tropics however revealed not only cases with pressure rise but also cases with no pressure changes and with pressure fall. It was also noted that although in the majority of the cases these were associated with D. B. fall, there were also cases with no temperature change or even a rise of D. B. It appeared therefore advisable to examine the whole question of pressure and temperature variation in detail with a view to find out whether the explanations offered so far could be extended to include also cases with no pressure change or pressure fall.

II—Classification of Thunderstorms.

4. Thunderstorms represent one of the relatively small thermo-dynamic agencies by which one form of energy is converted into another in the atmosphere. They belong to that group of meteorological phenomenon which is most complex and concerning whose physical nature there is the least understanding and least agreement. Their fast changing and varied aspects usually convey an impression of chaos in the first instance, yet when studied systematically, each one of these aspects is found to be governed by natural laws leading to a more or less satisfactory and convenient classification. A study of synoptic weather maps on days of thunderstorms shows that they can be most conveniently classified into two main divisions :—

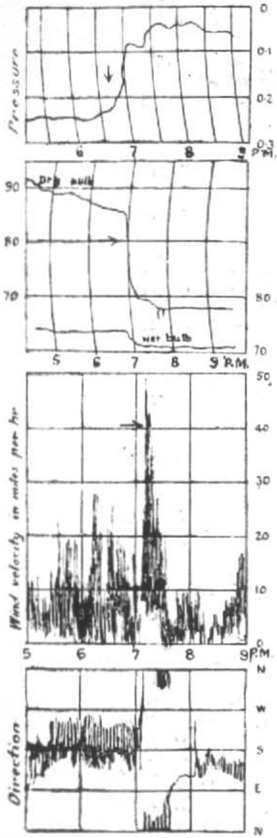
(*i*) Those which are associated with the outbreak of cold air at the surface and after the occurrence of which there is a marked change in the airmass at the surface or in other words "COLD FRONT THUNDERSTORMS."

(*ii*) Those which either occur in the same airmass or those which are associated with a surface of discontinuity at some higher level but by the occurrence of which no lasting change is produced in the airmass at surface. These thunderstorms depend upon thermodynamic causes for their production and may be termed as "INSTABILITY THUNDERSTORMS." The heat thunderstorms and the "NORWESTERS OF BENGAL" come under this category.

5. As the thunderstorms of the first type are invariably associated with a pressure rise and temperature fall of the persistent type Fig. (1) which is mainly due to static effect as a result of the replacement of warm air by cold air in lower levels, it is proposed to confine the discussion to the pressure changes in the thunderstorms of the second type.

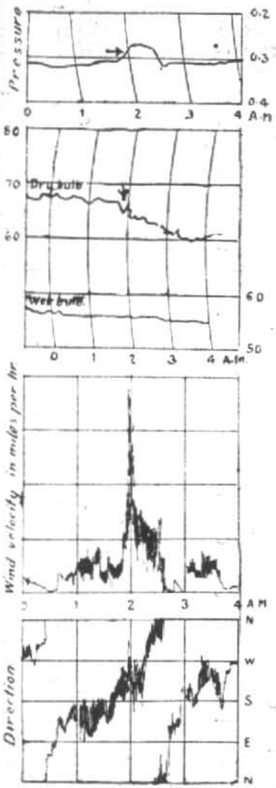
III—Important facts associated with Instability Thunderstorms.

6. As the nondiscontinuity thunderstorms popularly known as heat thunderstorms develop in the same airmass, it is natural to conclude that the variation of meteorological elements associated with them would be different from those associated with the "High level" discontinuity type and hence it would be easy to distinguish them from one another. Experience however shows that this is not easy and that the airmass or heat thunderstorms are accompanied at the ground by shifting winds, temperature and barometer changes which can hardly be distinguished from the customary signs of the "High level discontinuity" thunderstorms. In order, therefore, to be able to interpret the physical processes which give rise to these variations, it is absolutely essential to describe briefly the feature of the pressure curves and the variation of the other meteorological elements associated with them.



16-4-1946.

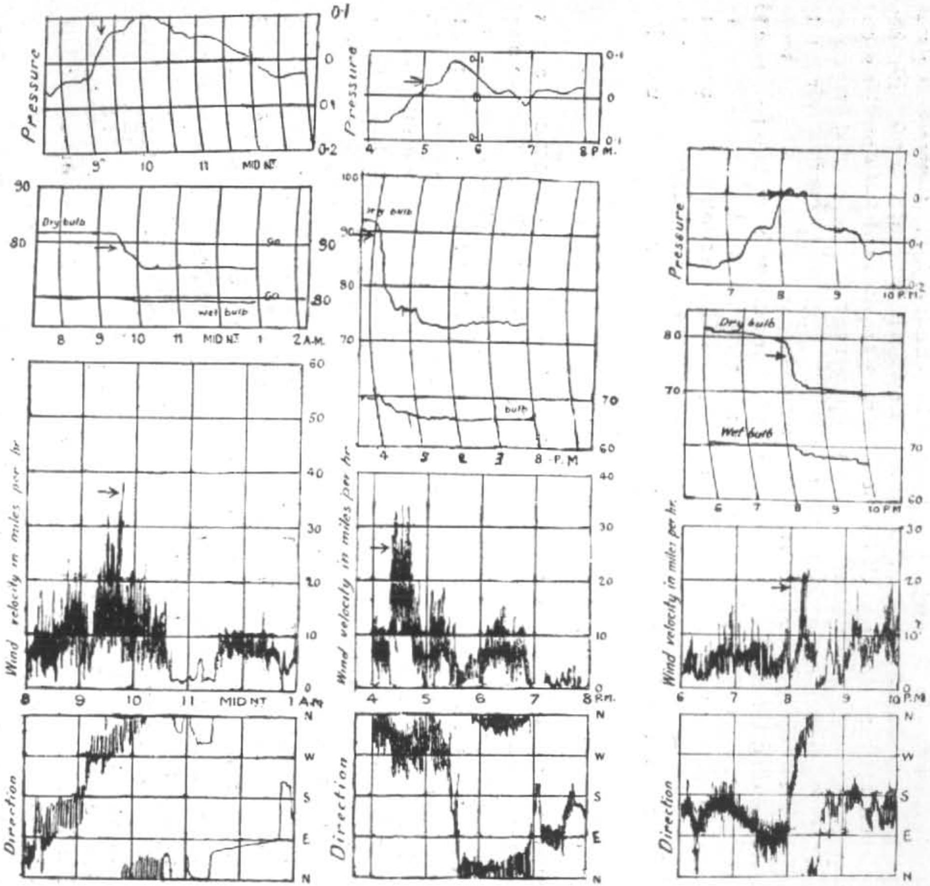
Fig 1



13-3-1941.

Pressure rise with no change in D.B.

Fig 2(a)

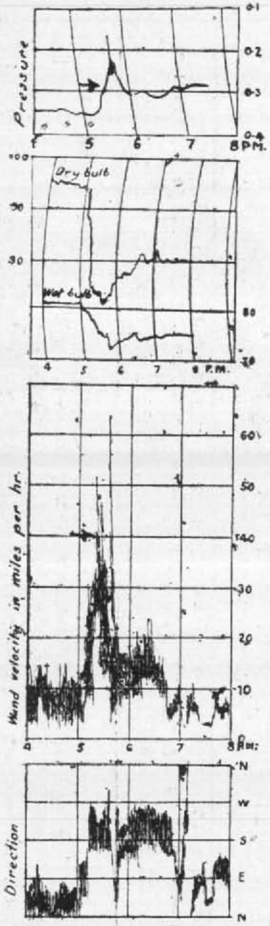


17-5-1946. 21-5-1941. 22-4-1945.

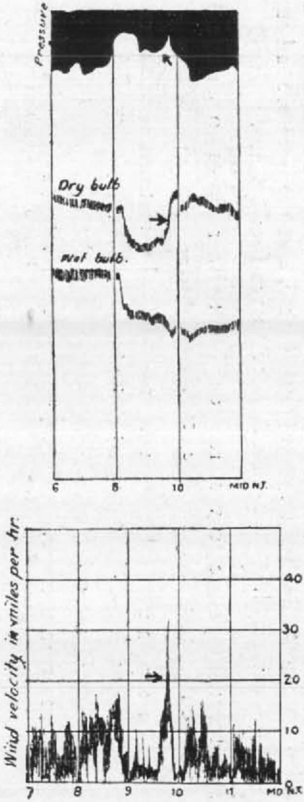
WITH SQUALL WITHOUT SQUALL

Pressure rise with persistant fall in D.B

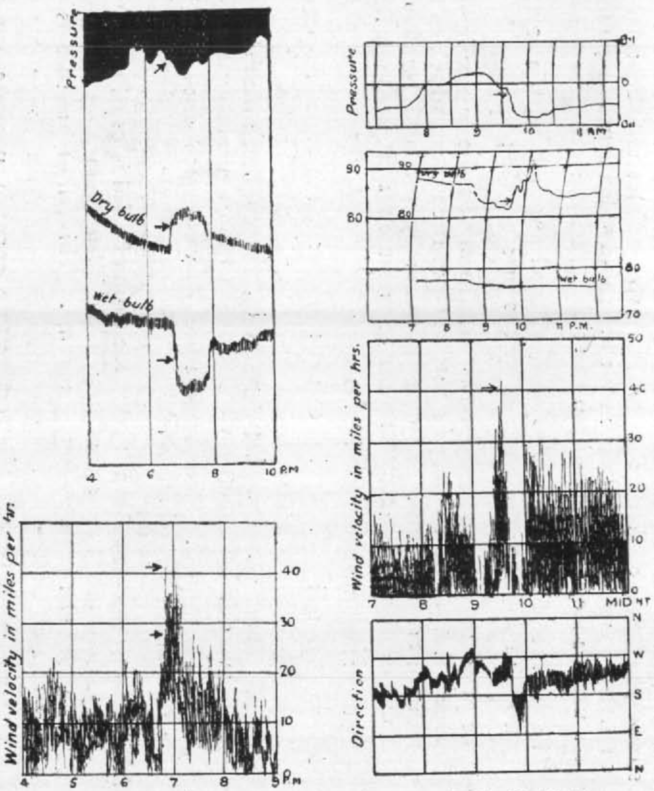
Fig. 2 (b)i.



21-8-1941.
Pressure rise with fall in D.B.
(dip type)
Fig. 2(b)ii.



26-5-1908.
Pressure rise with rise in D.B.
Fig. 2(c).



22-4-1915.
With squall.

5-5-1943.
With squall.

Pressure fall with rise in D.B. Temperature.
Fig 2 (d).

7. In Fig. (2) have been reproduced autographic charts for a number of thunderstorm days arranged according to the type of pressure variation. Examples under each pressure type have again been further sub-divided depending upon the wind and temperature change. It will be seen that:

(*i*) the passage of thunderstorms is associated not only with pressure rise, but also with no pressure change as well as pressure fall;

(*ii*) the pressure curve when it takes the form of a rise is usually of a "hump type" *i. e.*, in which the pressure having risen returns back to its original value after some time. This rise is generally preceded by slow slight fall going on for sometime. The rise of pressure may be accompanied by—

(*a*) no change in the D. B. (Fig. 2 (*a*));

(*b*) by a fall in the D. B. The temperature fall sometimes is of a persistent type (Fig. 2 (*b*) (*i*)) and

at other times of a "Dip type" (Fig. 2 (*b*) (*ii*)), in which the temperature having fallen comes back to more or less its original value.

(*c*) by a rise in D. B. The temperature curve in such cases is always of a "hump type" (Fig. 2 (*c*)).

(*iii*) the pressure curve when it takes the form of a fall is usually of a "Dip type" *i. e.* in which the pressure after it has fallen again rises to more or less the original value after sometime. This fall of pressure is sometimes associated with—

(*a*) temperature rise. The temperature curve in such cases is always of a "hump type" (Fig. 2 (*d*));

(*b*) no temperature change (Fig. 2 (*e*));

(*c*) temperature fall. The temperature curve in such cases is of a "Dip type". Such cases are usually rare, and the pressure fall associated is very small. (Rao—1938. See curves for 16-9-43).

(*iv*) when the pressure curves are such that it either shows no change or a negligent rise or fall, they are associated with—

(*a*) temperature fall. In such cases the fall sometimes is of a persistent type (Fig. 2 (*f*) (*i*)) and sometimes of a "Dip type" (Fig. 2 (*f*) (*ii*));

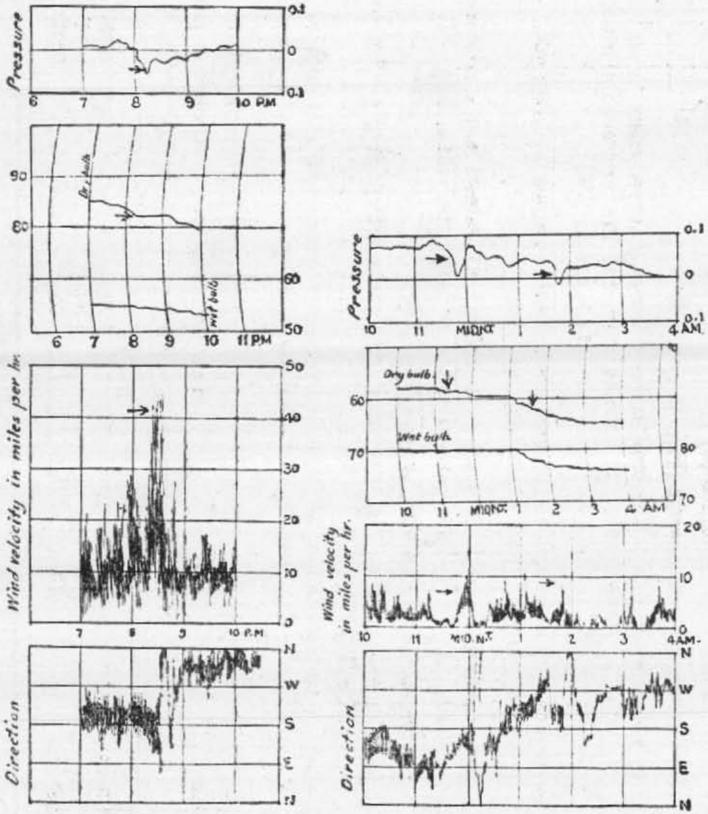
(*b*) negligent temperature change (Fig. 2 (*g*));

(*c*) temperature rise. In such cases the curve always is of a "hump type" (Fig. 2 (*h*)).

(*v*) there is no correlation between the pressure change and the velocity of the squall as will appear from the following table :—

Date.	Time (I. S T.)	Max. Wind Speed. (M. P. H.)	Change in pressure.
17-5-46	1600 hours.	74	0·02"
,,	2100 ,,	38	0·14"
21-8-41	1730 ,,	64	0·20"

(*vi*) the pressure variation is not directly related to temperature;

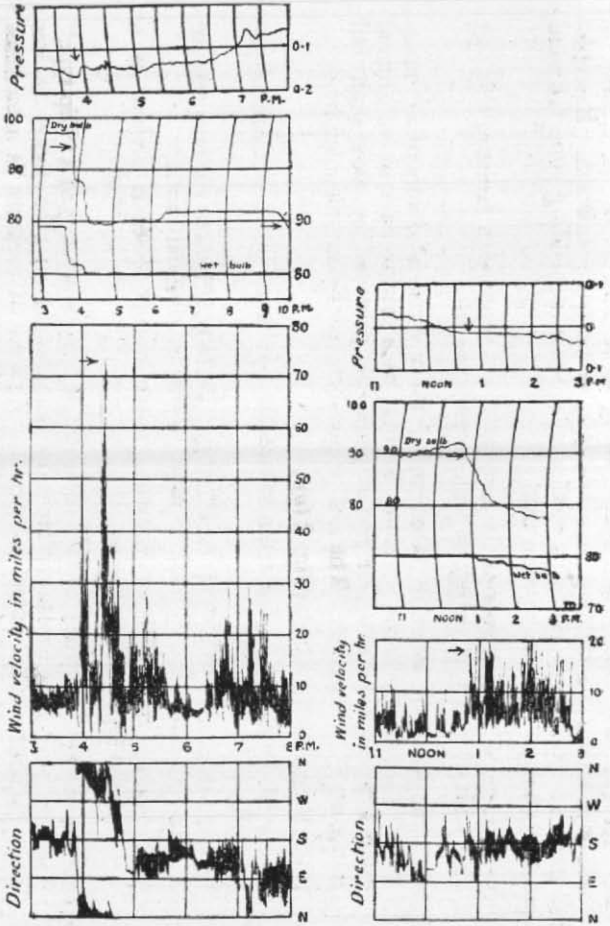


25-4-1946
With squall.

15-5-1945.
Without squall.

Pressure fall with no change in D.B.

Fig. 2(e).

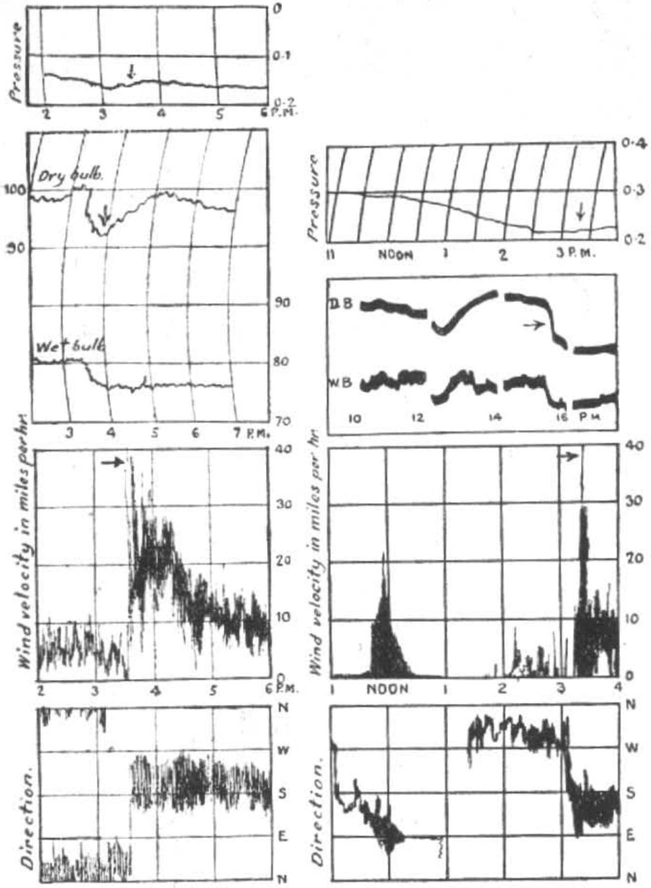


17-5-1946.
With squall.

18-5-1945.
Without squall.

No pressure change with fall in D.B. (persistant type).

Fig. 2(f)i.

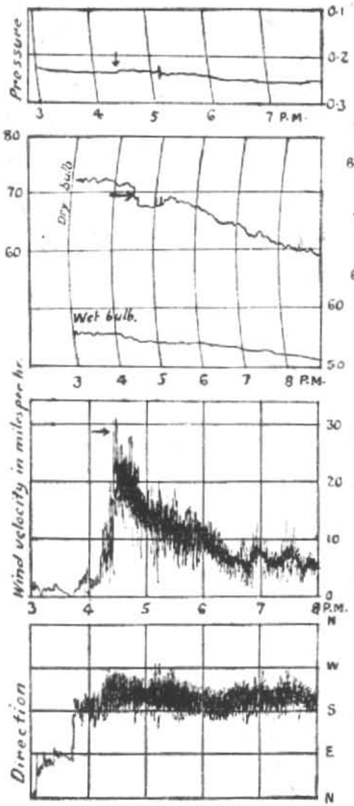


30-8-1941

11-9-1940.

No pressure change with fall in D.B. (dip type).

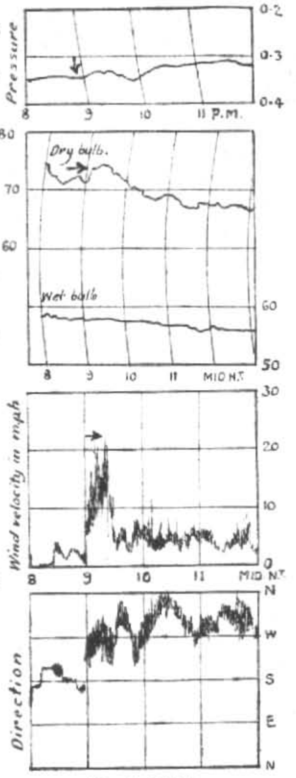
Fig. 2 (f) ii.



24-2-1941

No pressure change with negligible change in D.B. Temp.

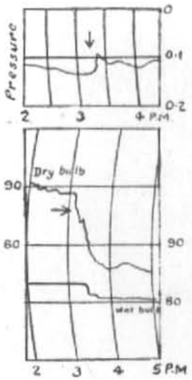
Fig. 2 (g).



12-3-1941

No pressure change with rise in D.B. Temp.

Fig. 2 (h).



27-9-1945.

Fig. 2 (i).

(*vii*) although 2·87″ of rain fell with the thunderstorm, the pressure change was negligible in spite of a marked temperature fall (*vide* autographic charts for 27-9-45 (Fig 2 (*i*)); or in other words the pressure variation is not directly related to rainfall.

8. In connection with an investigation on the polarity of clouds and the direction of field change resulting from a given lightning discharge, J. C. Jensen[2] installed 4 microbarographs, three in a north-south row 9½ miles long and the fourth four and a half miles south-west of the middle one with a view to get the size and velocity of the storm area. *The records obtained showed that pressure records at all the four stations had the same general feature except for the difference in the amount and time of variation.*

IV—Present ideas regarding the physical processes leading to thunderstorms.

9. In order to be able to discuss the implications of the various attempts so far made to explain the pressure variation associated with thunderstorms, it is necessary to state the present accepted ideas regarding the origin and development of thunderstorms in the atmosphere. As mentioned before, the discussion is confined to the thunderstorms of the "Instability type."

(*a*) *Airmass thunderstorms*

10. Airmass thunderstorms are thunderstorms which occur well within an airmass unaffected by activity at a surface of discontinuity. As they are initiated by convective currents caused by either surface heating or by the movement of air over a warmer surface, they may be said to be thermally produced. There are usually three stages of cloud development for such a thunderstorm. In the first stage on account of the setting up of a nearly dry adiabatic temperature gradient either due to local heating here and there or due to the transport of air over warmer regions, small areas of ascending currents are produced in the atmosphere. As the day advances or as the air progresses over hotter and hotter regions, ascending currents increase in intensity as well as area until each one results in the formation of a cumulus above the condensation level. The stream-lines of the uprising air in this stage according to the present accepted idea are shown in Fig. 3 (*a*). Although the uprising branches of the convection currents may be rapid, the descending current is relatively gentle as the condition essential to local rapid downflow does not exist.

In the second stage the convection currents in the cumulus are accelerated on account of the release of latent heat and thus one or more of them get rapidly developed. *This is the stage prior to the beginning of rainfall.* Here even as the compensating downward current is relatively widespread compared to the concentrated core of rising air, the downward current is comparatively gentle. The stream lines of the uprising air at this stage of cloud development are given in Fig. 3(*b*).

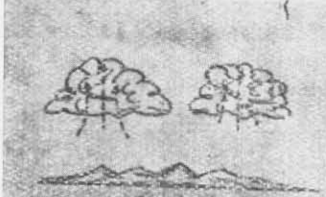


Fig. 3a

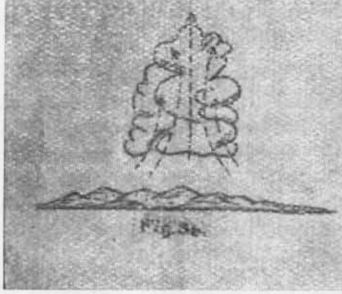


Fig. 3b

In the third stage of development, rain is formed at a considerable altitude as a result of the condensation induced by convective cooling. The air at this level is so cold that often hail is produced. The cold rain or hail as it falls, chills the air from the level of its formation all the way to the earth partly as a result of its initial low temperature and partly because of evaporation that takes place during its fall. Hence this continually chilled air partly because of the frictional drag of rain, but mainly because of the increase of its own density immediately and necessarily becomes a concentrated and vigorous return branch of the vertical circulation and acts as a sustaining cause of the storm circulation. The stream lines of the uprising and downcoming currents at this stage of the cloud are shown in Fig. 3 (*c*) and (*d*).

Fig. 3c.

Fig. 3(d).

11. The chief features of the above are :—

   (*i*) that the cloud development takes place as a result of insolation or in other words the instability which initiates convection is produced at the under-surface of the air column ;

   (*ii*) as the heating due to insolation is gradual but progressive, the cloud development is gradual and progressive ;

   (*iii*) until the rain mixed with hail starts falling and thus chilling the air by conduction and evaporation, no local concentrated cooling occurs in the cloud and hence the down compensating currents are usually weak until this stage is reached ;

   (*iv*) that the cloud represents one single vortex, and it is more or less symmetrical about a vertical through its centre so long as it is stationary, but when it has an appreciable velocity, most of the air entering the cloud does so only through its front under-surface, or in other words the cloud then has an asymmetric structure ;

   (*v*) the downward rush of the air because of the retardation of winds by surface friction is the chief cause of the rise in pressure.

(*b*) *High Level Discontinuity Thunderstorms.*

12. These are the thunderstorms of the Norwester type discussed by Mal and Desai[3] and not the frontal thunderstorms associated with either cold or warm front. In such cases warm dry air occurs above cold moist air. The three stages in the development of the thunderstoms are described below.

13. In the first stage, on account of the effect of insolation convection currents are set up irregularly and result in the formation of cumulus clouds as soon as they reach the condensation level in a way similar to that of the heat thunderstorm Fig. 4 (*a*).

In the second stage, with the advance of the day and greater heating, these currents increase progressively and go higher and higher resulting in the development of the towering cumulus here and there ; but in this stage the cloud tops still are below the surface of discontinuity.

In the third stage, on account of the greater and greater heating, ascending currents become so strong that the top of the towering cumulus reach the surface of discontinuity. When this happens the surface of discontinuity which was characterised by an inversion or stable gradient earlier gets completely wiped off and a layer with marked instability is set up with the consequence thatthe cumulus heads as soon as they reach this level shoot up with explosive violence causing a rapid downflow of the upper air thereby generating a severe type of thunderstorm called in Bengal as a Norwester. The structure of the cloud and the stream-line motion inside this is believed to be the same as that in the third stage of development of heat thunderstorms Fig. 4 (*c*) and (*d*).

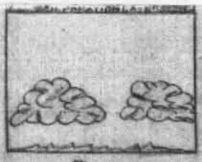


Fig. 4a.

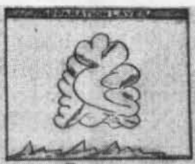


Fig. 4b.

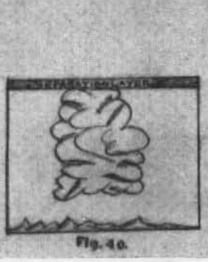


Fig. 4c.

Fig. 4d.

The main features of this type of formation are :—

(*i*) that the growth of instability in the earlier stages is gradual and takes place at the under-surface ; the cloud development therefore initially is gradual and occurs in a way similar to that of the heat-type thunderstorm ;

(*ii*) when however the cloud tops reach the boundary layer between the moist and dry air, the dry air which was relatively warmer than the moist air initially, suddenly becomes relatively cooler with reference to the moist air below and hence the devolopment of instability is sudden and occurs at the top-surface of the moist column ;

(*iii*) the main difference therefore between the physical processes leading to the formation of these thunderstorms and the heat thunderstorms is that whereas in the case of heat thunderstorms the only agency responsible for the growth of convection current is insolation and which influences the growth only from the under-surface, in the case of Norwesters the agency of insolation produces additional and sudden instability at the top of the column thus influencing the growth from the top surface of the moist column;

(*iv*) whereas in the case of heat thunderstorms the local cooling required for descending current was produced at a fairly advanced stage of cumulo-nimbus formation and was effective only when the water drops and hail could no more be supported by the ascending current, in the second case the intense local cooling was produced at the interface even at an early stage of the cumulus growth and being very sudden was responsible for the sudden growth of the cumulus to cumulonimbus and the simultaneous descending currents.

(*v*) as the trigger for the initiation of the down draught in both types of thunderstorm is the same, *i.e.*, intense local cooling, the variation of pressure, temperature, and wind associated with their passage will also be similar in both cases.

V—Review of the present explanations for pressure changes during Thunderstorms.

15. Till now most writers have believed that pressure rise is the only type of pressure change that can take place with the occurrence of a thunderstorm and as such all attempts have been made to formulate a structure of the cloud which would explain this pressure rise. As has however been shown before, there are not only cases of pressure rise, but cases with no pressure rise or even a pressure fall occur in association with the thunderstorms. As the passage of a thunderstorm is also associated with variation in D. B. and W. B. temperature and wind direction and wind velocity, it has been natural to look for the explanation of the observed pressure changes in variation of these other elements. A discussion of these in detail is given below.

(A) Temperature.

16. The passage of a thunderstorm in the majority of cases is associated with a temperature fall at the ground. The temperature fall is brought about as has been shown before by the descent of cold air from aloft. The descending cold air sinks down to the surface and maintaining its original velocity becomes a surface wind moving from the storm and spreading below the ascending warm air in front in the form of wedge as shown in Fig. 5.

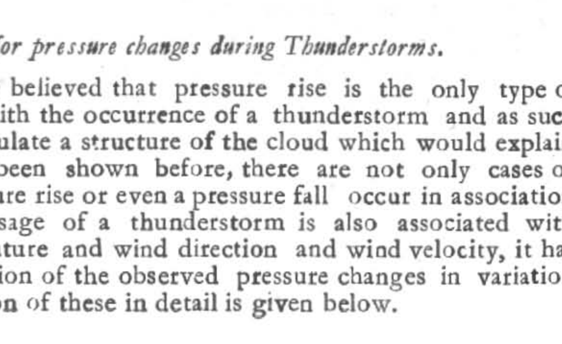


Fig. 5.

17. If the above diagram represents the structure of the thunderstorm, it is easy to see that at the place where the thunder-cloud first breaks down the temperature fall and squall will occur simultaneously. But at later stages there are two possibilities. If the horizontal wind is so strong as not to allow the horizontally moving descending air to get affected by external effects, the temperature curve would show a steep fall, but it would be always in advance of the maximum squall, Fig. 6(a). If, however, the horizontally moving air is weak, the fall will be gradual and the maximum fall will coincide with the time of maximum squall Fig. 6(b). As the rear is occupied only by the cold air, the temperature fall will be of the type with a persistent fall in temperature in both the cases.

18. It is seen from the above discussion that the temperature fall is produced at the surface by purely horizontal movement. Hence when a thunderstorm of any of the two types passes over the station, provided temperature is the sole reason for the pressure variation, the pressure may be expected to rise since the air of higher temperature is replaced by the air of lower temperature. The pressure rise due to this static effect can easily be computed. Consider for instance Fig. 7, where AB represents the boundary between the cold and the warm air.

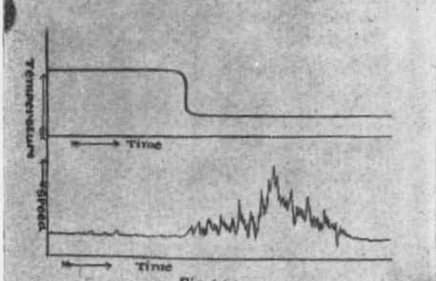


Fig. 6(a)

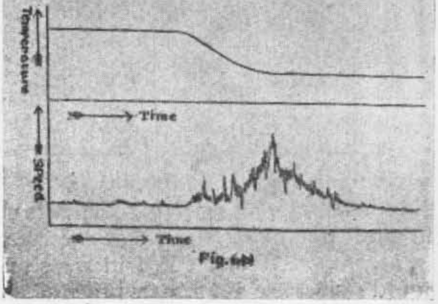


Fig. 6(b)

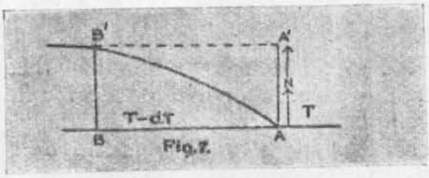


Fig. 7.

Let the mean temperature of the warm airmass be T and of the cold airmass T—dT. If the pressure at A is P_o and at A' is P, then $P_o = Pe^{gz/RT}$.

When the boundary has moved the distance AB, the warmer air column is no more above the point of observation at the surface as at A, but is now above the level B', the column below having been replaced by colder air. As a result, the surface pressure changes from P_o to $P_o + dP_o$ owing to the replacement of the warm air by cold air whereas the pressure at height z may remain constant since there has been no temperature change above that level. From logarithmic differentiation—

$$\frac{dP_o}{P_o} = \frac{-g}{R} \; \frac{z}{T^2} . \, dT.$$

whereas dP_o is the pressure variation during the time it takes the boundary to travel the distance AB.

19. As is seen from the above equation dP_o is directly roportional to z and dT. Hence the nature of the pressure curve will depend upon the temperature curve and slope of the boundary. If the temperature fall and the inclination of boundary are sharp, the pressure rise will be sharp. If, however, either both, *i.e.*, the slope of the boundary, as well as the temperature fall, or one of them is gradual, the pressure rise will also be gradual. In any case the maximum pressure rise will coincide with the maximum in dT or z whichever is attained later. As in the reat both dT and z after they have attained the maximum value are steady, the pressure curve will be one with persistent rise.

It will be seen that with the present accepted structure of the thunderstorm even assuming that temperature fall is the cause of pressure rise, one can only explain qualitatively those cases where records show temperature curves with persistent fall and a pressure curve with persistent rise.

20. That temperature is not the cause of pressure variations and that the above structure cannot explain all types of cases will be evident from the following :—

(*i*) That there are temperature curves in which the temperature after a fall again shows a rise or in other words the temperature curve is not always of a type with persistent fall but at times also a "dip" type curve.

(*ii*) That there are cases with large temperature fall, but no pressure rise.

(*iii*) Although the temperature curve is of a type with persistent fall, the pressure curve is sometimes of a type with persistent pressure rise and sometimes of a hump type.

(*iv*) That there are cases of pressure fall with either no change or with temperature rise.

(B)—Rapid downrush of air.

21. According to Bernoullis' theorem $P+\frac{1}{2}\rho U^2+\rho gz$ is a constant along the path of a parcel in an incompressible fluid. Hence where the velocity decreases there should be a corresponding rise of pressure. According to Humphreys[4] the downrush of air in a thunderstorm ceases at ground and hence should produce a vertically directed pressure on the surface of the earth in the same manner that a horizontal flow produces horizontally directed pressure against the side of an obstacle. Humphreys calculated the order of values and found that with a downward velocity of 60 Km per hour, the rise in pressure would be of the order of 1 mb. which is the right order of magnitude. That the rapid downrush of air cannot satisfactorily explain various types of pressure variation will be evident from the following :—

(*i*) According to the above reasoning, the downrush of air can only cause pressure rise and not pressure fall and hence according to the above hypothesis all mature thunderstorms, *i.e.,* thunderstorms in which a squall has been recorded at the ground, should only show curves with pressure rise and not pressure fall or no pressure change which is not the case.

(*ii*) Although it may be possible to explain (*a*) cases of no change of pressure on the assumption that the rapid downrush at the ground having become zero manifests itself completely as the horizontal veolocity and (*b*)the cases of pressure rise (only qualitatively) on the assumption that the downrush only partly manifests itself as a horizontal velocity, the cases of pressure curves with a fall are not understandable.

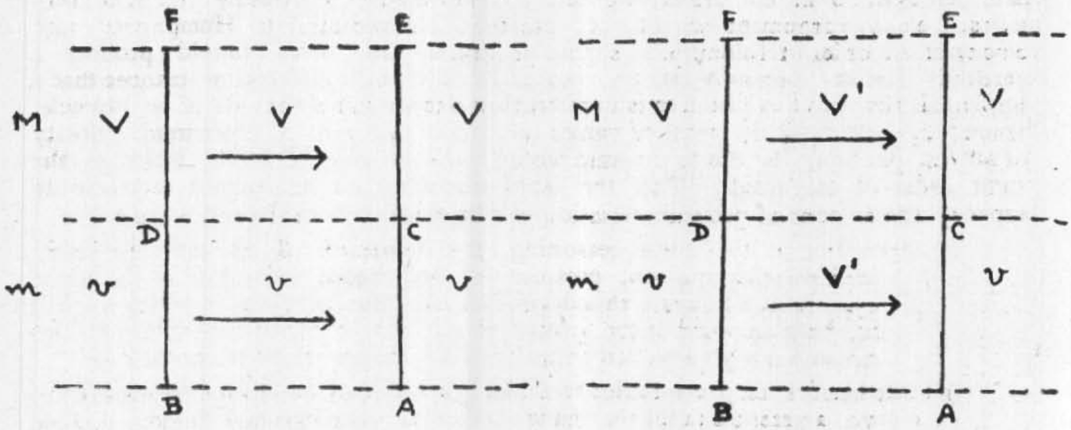
(C)—Interference to horizontal flow aloft caused by vertical circulation.

22. According to this hypothesis, a thunderstorm on account of the strong convective mixing between the lower and the upper wind layers of the atmosphere effectively retards the faster moving air strata aloft thus obstructing the free air flow in the vicinity of the storm. In front of the advancing storm the reaction of the barrier to the free air results in a reduction of the pressure. When however the storm breaks the effect of both the squall and the arrival overhead of the main storm barrier to the free air flow aloft raises the atmospheric pressure suddenly to a high level. It is evident that the above hypothesis can only satisfactorily explain a hump type of pressure rise curve observed in thunderstorms and not the cases of no pressure change or pressure fall. Further, as the interference of horizontal flow due to convection is always present even in the case of well-developed cumulonimbus, the pressure rise should be shown even when cumulonimbus passes over the station. That the hump type of pressure rise is not the only type observed and that cumulonimbus clouds do not show rise of pressure, but probably show a slight pressure fall, is sufficient to

prove that the above hypothesis for the pressure variation in the case of thunderstorm is not satisfactory.

(D)—Interference to horizontal flow in the lower layers caused by vertical circulation and the consequent increased frictional drag.

23. According to Humphreys[4] the increased surface velocity following convection, a phenomenon marked in the course of a thunderstorm causes an increased frictional drag and therefore a greater or less decrease in the total flow thereby producing a rapid and marked increase in the barometric pressure. In order to be able to appreciate the above hypothesis, it would be interesting to put the whole argument mathematically. Let us consider that there are two layers of air, an upper one CDEF and the lower one ABCD, flowing parallel to each other. Let their respective masses per unit length in the direction of horizontal flow be M, m and their velocities V, v Fig. 8(*a*).



Before exchange of momentum. (*a*)

After exchange of momentum. (*b*)

Fig. 8.

Now if as a result of convection only, interchange of layers takes place, then as there will be no change in the vertical mass per unit length there will be no change in the pressure at the surface. Even if as a result of convection there is complete mixing of the two air currents and if the various layers ABCDEF assume a single common velocity V' Fig. 8(*b*) then according to the Law of Conservation of Momentum.

$$(M+m)V'=MV+mv.$$

The divergence at the face CE is proportional to $V-V'$ while at AC it is $v-V'$. The effect of this on the pressure at A will be proportional to $M(V-V')+m(v-V')$ or $(MV+mv)-(M+m)V'$ which is equal to zero. Similarly at B also there will be no pressure change due to momentum transfer.

24. Let us now take the case where the velocity in the lower half of the layer II is reduced to half of V' due to frictional retardation near the ground. The total

divergence at the face ACE is now

$$
\begin{aligned}
& \mathrm{M}\,(\mathrm{V}-\mathrm{V}')+\tfrac{1}{2}\,\mathrm{m}\,(\mathrm{v}-\mathrm{V}')+\tfrac{1}{2}\,\mathrm{m}\,(\mathrm{v}-\tfrac{1}{2}\mathrm{V}') \\
& =\mathrm{M}\,(\mathrm{V}-\mathrm{V}')+\mathrm{m}\,\mathrm{v}-\tfrac{3}{4}\mathrm{mV} \\
& =\tfrac{1}{4}\mathrm{mV}'
\end{aligned}
$$

Assuming that the wind at the higher level is initially greater than below, there is thus a net divergence at the face A C E and pressure at A would consequently show a fall. Similarly a pressure rise would occur at B or in other words in advance of the squally winds of a thunderstorm there would be a pressure fall and in its rear a rise of pressure. This agrees with the observed changes. Humphreys computed the magnitude of rise as 2·5 mbs. assuming that the velocity in a layer 25 metres thick was reduced to one fifth its value due to surface friction and as this happens to be more or less of the correct order of magnitude, he concluded that retardation of surface winds by friction must be an important contributing cause of the marked and rapid increase of the barometric pressure that accompanies the onset of a thunderstorm. As velocities of 30 m. p. h. are often experienced in thunderstorms at heights of 3 to 6 metres, the above argument would mean that at some greater height a horizontal velocity of the order of 150 m. p. h. should prevail.

25. If the above explanation is correct, then one should expect—

(*i*) In all thunderstorms accompanied by squall, the pressure curve would show a rise.

(*ii*) The amount of pressure rise should increase with increase in the surface velocity. In other words, the greater the velocity, the greater should be the pressure rise.

26. That this is not so, and consequently the above explanation is not satisfactory to account for all types of variations of pressure will be evident from the following :—

(*i*) Since besides the pressure rise there are pressure curves with no pressure rise or even a pressure fall.

(*ii*) The pressure rise does not seem to bear any relation to the wind squall ; there are many days when pressure rise is very small compared to the strength of the squall ; days with practically no wind but rise in pressure ; and days of heavy squall with no rise in pressure.

27. Besides it is extremely doubtful if the strengthening of winds at the surface during the thunderstorm is due to transfer of horizonal momentum from upper winds as will be seen from the case mentioned below :

At Peshawar on 25-9-'45 a thunderstorm associated with a squall occurred between 1315 hrs. and 1545 hrs. G. M. T. The maximum velocity was recorded at 1325 hrs. and reached a value of 56 m. p. h. The upper winds recorded at 1000 hrs. and 2100 hrs. G. M. T. of 25th at Peshawar are given in Table I and show that the winds were very weak up to very high levels on that day :

TABLE I.

	Wind Velocity in Knots at										
Time.	2000 ft.	3000 ft.	5000 ft.	7000 ft.	10000 ft.	12000 ft.	15000 ft.	18000 ft.	20000 ft.	25000 ft.	30000 ft.
10 G. M. T. 25th Oct. '45.	6	7	4	2	4	3	11	9	10	19	41
21 G. M. T. 25th Oct '45.	4	2	7	12	15	..	..	..	..	..	..

(E)—Due to development of flow pattern of vortex pair type in the atmosphere.

28. Aslam[5] studied in detail the directions, their variations and speed profiles of squalls associated with thunderstorm and on the basis of these results and the associated pressure variations suggested another model for the structure of thunderstorm. According to him vertically a thunderstorm field Fig. 3 (d) consists of :—

(*i*) Convective Sector (or Sectors) having an upward component of wind velocity and

(*ii*) Compensating subsidence Sector (or Sectors) with a downward component of velocity. On a horizontal cross-section the thunderstorm field consists of a vortex pair located in the general translatory field orientated at right angles to the line joining the centres of the vortex pair. According to him such a cell pair may move in any direction depending upon aerodynamical transformation taking place in the general flow pattern. The streamlines with such a flow are shown in Fig. 9. The streamlines can be taken as isobars.

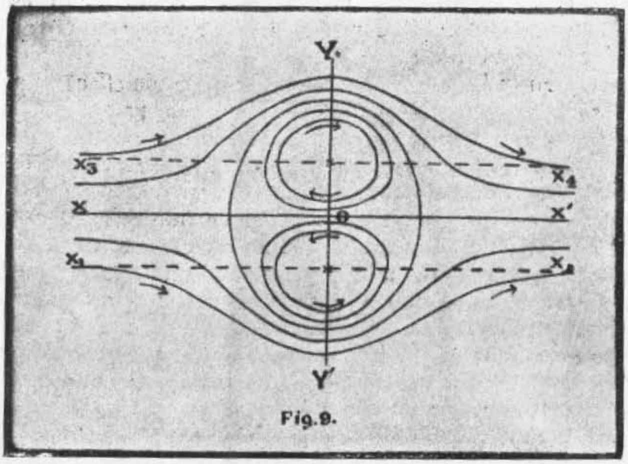
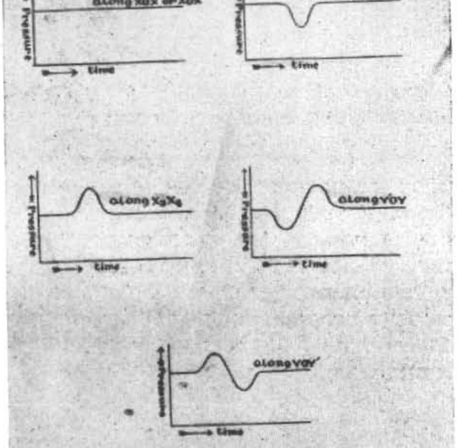


Fig. 9.

29. Aslam discusses the traces given by barographs moved in such a field along the lines XOX^1, YOY^1, etc. Some of the pressure profiles expected are reproduced in Fig. 10.

The curves are similar to those recorded in various thunderstorms. But there are certain fundamental objections to Aslam's assumption. His theory requires the vortex pair to be capable of moving in all directions. It is a well-known property of a vortex pair that it has translatory motion at right angles to the line joining the centres of the vortices and with the superposed translatory field being parallel to it there is only one direction in which the vortex pair can move. Also according to Aslam, for a given direction of motion of the vortex pair, a variety of pressure profiles may be recorded depending upon the relative location of the stations. This does not seem to be borne out by the pressure profiles recorded by Jensen[2] with barographs placed in four different parts of the thunderstorm which show definitely that the structure of the thunderstorm is symmetrical.



Pressure Profile

Fig. 10.

(F)—Vertical Acceleration.

30. Sücksdorff[6] studied shower storms in Germany and in the tropics by making detailed observations below the shower clouds with sensitive portable instruments. From these detailed observations he constructed a storm model combining the observed and some interpreted general characteristics. He made a particular note of the various types of pressure changes that accompanied these shower clouds and thought that the pressure rise observed in thunderstorms was brought about by some dynamic effect, but he did not elaborate his ideas.

31. More recently Levine[7] elaborated the dynamic effect suggested by Sücksdorff and tried to account for the symmetrical type of pressure change observed as a dynamic consequence of the vertical acceleration of the rising current in the thundercloud. Mal and Rao[8] have shown that Levine's explanation is untenable as it does not take into account the vertical deceleration in the upper portion of the thunder-cloud and taking into consideration this deceleration the pressure should show a fall below the cloud. Further the fact that even growing towering cumulus (from which no rain is falling) in which only the vertical acceleration and no deceleration exists does not show the symmetrical pressure rise, is a further proof of the untenability of Levine's explanation. Although the note of Mal and Rao has already appeared in Quarterly Journal of the Royal Meteorological Society, for the sake of completeness the discussion is reproduced below :—

If g represents the acceleration due to gravity, v the horizontal velocity in the west-east direction, ϕ the latitude, ω the angular velocity of the earth's rotation and $\frac{d^2z}{dt^2}$ the vertical acceleration, the change of pressure with height is given by equation

$$\frac{1}{\rho}\,\frac{\delta p}{\delta z} = -g + 2\,\omega\,v\cos\,\phi - \frac{d^2z}{dt^2} \quad \ldots\ldots\ldots\ldots(1).$$

The term $2\,\omega\,v\cos\phi$ is usually very small and may be neglected in comparison with other terms. Also writing f for $\frac{d^2z}{dt^2}$ we get

$$\frac{1}{p}\,\frac{\delta p}{\delta z} = -(g+f) \text{ or } \frac{1}{\rho}\,\frac{\delta p}{\delta z} = \frac{g+f}{RT} \quad \text{..........(2).}$$

If we now assume a parabolic variation of f with height, f is given by the relation :

$$f = Az\left(1-\frac{z}{h}\right) \quad \text{..........(3)}$$

where **A** is a constant. This relation means that the acceleration is zero at $z=o$ and $z=h$ and is positive between $z=0$ and $z=h$ and negative above the height h.

Levine averages f between $z=o$ and $z=h$ and substitutes this value in equation (2). This does not seem justified. The thunder-cloud extends not only through the region of positive acceleration (energy producing environment), but extends further upward ; the momentum of the rising air at $z=h$ (this being the height where the velocity is maximum) carries it further upwards through the energy-consuming environment. This process continues until a height is reached where the vertical velocity becomes zero. The averaging of f should be made over the entire cloud column, *i.e.*, between $z=o$ and the top of the cloud where the vertical velocity becomes zero.

Writing $w\frac{\delta w}{\delta z}$ for f we get

$$w\,\frac{\delta w}{\delta z} = Az\left(1-\frac{z}{h}\right) \quad \text{..........(4).}$$

Integrating,

$$\frac{1}{2}\,w^2 = A\left(\frac{z^2}{2}-\frac{z^3}{3h}\right)+k$$

Neglecting for the moment the constant k, the vertical velocity becomes zero at $z=3h/2$. Hence the averaging should be carried out over the range $z=3h/2$ and $z=o$ and not between $z=h$ and $z=o$ as was done by Levine. Averaging between the limits $z=3h/2$ and $z=o$ we get

$$f = \frac{2}{3h}\int_{o}^{3h/2} Az\left(1-\frac{z}{h}\right)dz = o.$$

Levine averaging between $z=h$ and $z=0$ gets an average value of $Ah/6$ as against the value of zero when the averaging is done between the limits $z=3h/2$ and $z=0$. This means that the acceleration inside the thundercloud should not produce any effect on the pressure below the cloud level when the entire cloud column is taken into account. If k is not taken to be zero, *i.e.*, air entering the cloud base has an initial upward velocity w would become zero at a height greater than $3h/2$, say at $3h/2 + \Delta h$. If f is averaged between $z=0$ and $z=3h/2+\Delta h$ we get a negative value for f. This would cause a pressure fall below the cloud rather than a rise. As the effect of k is not to cause a pressure rise, the value of k has been taken as zero in our further discussion in order to simplify the treatment.

Let us now examine Buell's[8] discussion which is more rigorous. If p is the pressure at height z and p_s at $z=0$ and if p' and p'_s are the corresponding values for the static column, when $f = 0$, then

$$\text{Log}\ \frac{p_s}{p} = \int_0^z \frac{g+f}{RT}\,dz\,;\ \text{Log}\ \frac{p'_s}{p'} = \int_0^z \frac{g}{RT}\,dz$$

subtracting, we get

$$\int_0^z \frac{f dz}{RT} = \text{Log}\,\frac{p_s}{p} - \text{Log}\,\frac{p'_s}{p'} = \text{Log}\ \frac{p_s}{p'_s} - \text{Log}\ \frac{p}{p'} \qquad \dots (5).$$

Writing Δ for the difference between corresponding values in the accelerated and static columns, and neglecting second order and higher terms, we get

$$\int_0^z \frac{f dz}{RT} = \frac{\Delta p_s}{p_s} - \frac{\Delta p}{p} \qquad \dots (6).$$

If the lapse rate in the atmosphere is λ and Ts is the temperature at $z = 0$

$$\frac{\Delta p_s}{p_s} - \frac{\Delta p}{p} = \int_0^z \frac{f}{R\,(T_s - \lambda z)}\,dz \qquad \dots (7).$$

The maximum vertical velocity v occurs at a height h and is
given by $v^2 = \frac{1}{3}\,Ah^2$ ... (8).

$$\text{or } A = \frac{3\,v^2}{h^2}$$

$$\therefore f = Az\,(1 - \frac{z}{h}) = \frac{3v^2}{h^2}\,z\,(1 - \frac{z}{h})$$

$$\therefore \frac{\Delta p_s}{p_s} = \frac{\Delta p}{p} = \frac{3v^2}{h^2}\int_0^z \frac{(z - \frac{z^2}{h})}{R\,(T_s - \lambda z)}\,dz = \frac{3v^2}{RT_s h^2}\int_0^z \frac{(z - \frac{z^2}{h})}{(1 - \frac{\lambda z}{T_s})}\,dz$$

$$\frac{\Delta p_s}{p_s} - \frac{\Delta p}{p} = -\frac{3v^2}{Rh^2\lambda}\int_0^z \left(z - \frac{z^2}{h}\right) d\log\,(1 - \frac{\lambda z}{T_s}) \qquad \dots (9).$$

$$\int_0^z (z - \frac{z^2}{h})\,d\log\,(1 - \frac{\lambda z}{T_s}) = \left[\,(z - \frac{z^2}{h})\log\,(1 - \frac{\lambda z}{T_s})\right]_0^z - \int_0^z \log\,(1 - \frac{\lambda z}{T_s})\,(1 - \frac{2z}{h})dz$$

Expanding $\log\,(1 - \frac{\lambda z}{T_s})$ as $-\frac{\lambda z}{T_s} - \frac{1}{2}(\frac{\lambda z}{T_s})^2 - \frac{1}{3}(\frac{\lambda z}{T_s})^3$

and taking only the first two terms of the expansion ($\frac{\lambda z}{T_s}$ being of the order of 0·1), we get

$$\int_0^z (z - \frac{z^2}{h})\,d\log\,(1 - \frac{\lambda z}{T_s}) = \left\{ -\frac{\lambda z^2}{2T_s} + \frac{1}{3}\frac{\lambda z^3}{hT_s} - \frac{1}{3}\frac{\lambda^2 z^3}{T_s^2} + \frac{1}{4}\frac{\lambda^2 z^4}{hT_s^2} \right\}$$

$$\frac{\Delta p_s}{p_s} - \frac{\Delta p}{p} = \frac{3v^2 z^2}{Rh^2 T_s}\left\{ \frac{1}{2} - \frac{1}{3}\frac{z}{h} + \frac{\lambda}{3}\frac{z}{T_s} - \frac{1}{4}\frac{\lambda z^2}{hT_s} \right\} \qquad \dots (10).$$

Buell[9] has obviously neglected the last terms within the brackets. This however is not correct as the fourth term becomes comparable with the third term when z is equal to h. Substituting $z = 3h/2$ and assuming that $\Delta p/p$ is zero at $z = 3h/2$ or that the pressure at the top of the cloud is unaffected, we get

$$\frac{\Delta p_s}{p_s} = - \frac{27}{64}\frac{v^2 \lambda h}{RT_s^2} \qquad \dots (11).$$

which means the effect of acceleration is to cause a pressure fall below the cloud instead of a pressure rise. Buell got for the corresponding case a pressure rise because he neglected the last term in brackets in equation (10) and obtained the expression,

$$\frac{\Delta p_s}{p_s} = + \frac{27}{8}\frac{V^2 \lambda h}{RT_s^2} \qquad \dots\dots (12)$$

for the pressure change. The above reasoning shows that the rise of pressure accompanying the passage of a thunderstorm cannot be explained as due to the effect of the vertical acceleration, at least not as suggested by Levine and Buell.

32. Besides, the derivation of equation (5) is fallacious. It implicitly assumes that the temperature at any horizontal level is the same inside and outside the cloud which is hardly correct.

Schaffer[10] has again discussed the problem and claims to have shown that the hump type of pressure rise in thunderstorm may be explained as the effect of vertical acceleration. He says that "the negative result found by Mal and Rao, using an approximate method, is not possible." It is, however, shown below that the negative pressure change obtained by us was not due to any approximation in integration.

Substituting $f = w\frac{dw}{dz}$ in equation (6), we get

$$\frac{\Delta p_s}{p_s} - \frac{\Delta p}{p} = \int_o^z \frac{1}{RT}\, w \,\frac{\delta w}{\delta z}\, dz.$$

$$= \int_o^z \frac{1}{2RT}\,\frac{\delta}{\delta z}(w^2).$$

$$= \left[\frac{w^2}{RT}\right]_o - \int_o^z \frac{w^2}{2R}\,\frac{\delta\left(\frac{1}{T}\right)}{\delta z} \qquad \dots\dots (13).$$

As w is zero at the heights o and z, the first term of equation (13) is zero. Temperature T decreases with the height or 1/T increases with height. Hence $\frac{\delta\,(1/T)}{\delta z}$ is positive. Also w^2 is positive between the heights o and z. Looking upon integration as a process of summation, it is easy to see that $\int_o^z \frac{w^2}{2R} \cdot \frac{\delta\,(1/T)}{\delta z} \cdot \delta z$ is positive. Hence $\frac{\Delta ps}{p} - \frac{\Delta p}{p}$ is negative. We may assume that the pressure difference between the accelerated and static columns at the top is zero or $\Delta p = o$ or Δp_s is negative or below the accelerated column there is a pressue fall. This derivation is based on the assumption that the temperatures inside and outside the cloud at any horizontal level are the same, which is not justified since all theories of convection maintain that convection is due to difference in temperature. This assumption was implicit in the derivation of all Levine and Buell.

Schaffer defines the symbol P as the "excess of pressure over the equilibrium hydrostatic pressure, *i.e.* P indicates the pressure associated with a vertical component of acceleration—a kind of dynamic pressure." He then writes the equation

$$-\frac{1}{\rho w}\;\frac{\delta P}{\delta z}\Delta z=\Delta w.$$

where p is the density of the air, and integrates for Δp, the pressure difference between the base and top of the cloud and shows it to be positive. This method of computing the dynamic effect cannot explain for the observed pressure rise in thunderstorms. The problem is therefore enunciated in the correct form below.

Let p and ρ, and p' and ρ' respectively represent the pressure and density at any horizontal level inside and outside the cloud. Assuming that vertical accelerations are negligible outside the cloud, we get

$$\frac{\delta p}{\delta z}=-\rho(g+f)$$

$$\frac{\delta p^{1}}{\delta z}=-\rho' g.$$

f is the vertical acceleration. Let the top of the thundercloud be at z and p_0 and p_z denote the pressures in the cloud column at the ground and at the height z. Then,

$$p_0=p_z+\int_0^z \rho(g+f)dz.$$

$$p_0'=p'_z+\int_0^z \rho' g dz.$$

It may be reasonably assumed that $p_z=p'_z$.

$$p_0-p_0'=\int_0^z \rho(g+f)dz-\int_0^z \rho' g dz.$$

It is to be shown that $p_0>p_0'$ and not merely that $\int_0^z \rho f dz$ is positive. For, the pressure recorded below the thundercloud is the resultant of the 'static pressure' ($\int_0^z \rho g dz$) and the "dynamic effect" $\int_0^z \rho f dz$. The pressure rise recorded with the thunderstorm shows that $p_0>p_0'$. Schaffer's computation of the dynamic pressure is equivalent to assuming $\int \rho g dz=\int \rho' g dz$. This is to be proved and not merely assumed.

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