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Time series analysis on precipitation with missing data using stochastic SARIMA

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सार – प्रस्तुत शोधपत्र में ईरान में वर्षा की मॉडलिंग के लिए बॉक्स जेनकिंस पद्धति के अनुप्रयोग को प्रस्तुत किया गया है। 44 वर्षों के मासिक वर्षा के आँकड़ों के लिए, रेखीय प्रसंभाव्य मॉडल जिसे मल्टीप्लिकेटिव सीजनल एरिमा (ARIMA) के नाम से जाना जाता है, तैयार किया गया। कुछ कारणों से बीच के 34 महीनों के आँकड़े अनुपलब्ध पाए गए। इस कमी को पूरा करने के लिए पहले 180 उपलब्ध प्रेक्षणों के आधार पर एक सीज़नल एरिमा (ARIMA) मॉडल तैयार किया गया और अगले 34 महीनों के अनुपलब्ध आँकड़ों को पूर्वानुमान द्वारा प्रतिस्थापित किया गया। फिर सम्पूर्ण आँकड़ों के लिए SARIMA मॉडल तैयार किया गया। परिणामों से ज्ञात हुआ कि यह मॉडल सम्पूर्ण आँकड़े ठीक से दिखाता है।

ABSTRACT. This paper presents an application of the Box-Jenkins methodology for modeling the precipitation in Iran. Linear stochastic model known as multiplicative seasonal ARIMA was used to model the monthly precipitation data for 44 years. Missing data occurred in between for 34 months for some reason. To fill the gap a SARIMA model was fitted based on the first 180 available observations and the missing observations were substituted by the forecasts for the next 34 months. Then a SARIMA model was fitted for the full data. The result showed that the fitted model represent the full data well.

Key words – Time series, Rainfall data, Missing data, ARIMA, SARIMA.

1. Introduction

There are many hydrological variables whose observations change with time and such observations constitute a time series. One of such variables is precipitation. In many instances, the pattern of changes in these kinds of observations can be ascribed to an obvious cause and is readily understood and explained, but if there are several causes for variation in the time series observations, it becomes difficult to identify the several individual effects. The definition of the function of this needs very careful consideration and may not be possible. The remaining hidden feature of the series is the random stochastic component which represents an irregular but continuing variation within the observed values and may have some persistence. It may be due to instrumental or observational sampling errors or it may come from random unexplainable fluctuations in a natural physical process.

A time series is said to be a random or stochastic process if it contains a stochastic component. Therefore, most hydrologic time series such as precipitation may be thought of as stochastic processes since they contain both deterministic and stochastic components. If a time series contains only random/stochastic component it is said to be a purely random or a white noise process.

A number of modeling studies have been carried out on rainfall, one of the hydrological variables, Amha and Sharma (2011) attempted to build a seasonal model of monthly rainfall data of Mekele station of Tigray region (Ethiopia) using Univariate Box-Jenkins's methodology. The method of estimation and diagnostic analysis results revealed that the model was adequately fitted to the historical data. Mohammed (2018) fitted SARIMA (2,1,0) $(0,1,1)_{12}$ to describe the rainfall of Addis Ababa (Ethiopia) based on monthly data for 18 years. In the current study time series analysis on monthly precipitation for 44 years with an approach for tackling the problem of missing data is presented.

2. Materials and method

The data on precipitation were provided by Sara K. from Iran because she was experiencing in the analysis of data. She requested from the first author but due to business the first author could not help her. Now we got time so we tried to analyze the data. The monthly data consist of the first 180 (15 years) observations followed by 34 monthly missing entries followed by 314 monthly observations. Before embarking on the analysis first a seasonal model was fitted on the first 180 available observations to estimate the missing ones by way of forecasting assuming that past patterns would continue in to the future (Table of the Appendix of this article for the full data). In order to analyze the time series for precipitation, linear stochastic model known as either Box-Jenkins or ARIMA was used. The MINITB software package was employed for data analysis.

The Box-Jenkins methodology [Box and Jenkins (1976)] assumes that the time series is stationary and serially correlated. Thus, before modeling, it is important to check whether the data understudy meet these assumptions. Let $X_1, X_2, X_3, ..., X_{t-1}, X_t, X_{t+1}, ..., X_n$ be a discrete time series measured at equal time intervals. A seasonal ARIMA model for w_t is written as (Vandaele, 1983).

$$\phi(\mathrm{B})\ \Phi(\mathrm{B}^s) w_t = \theta(\mathrm{B})\Theta(\mathrm{B}^s) a_t \qquad (1)$$

where,

$\phi(B) = 1- \phi_1 B- \phi_2 B^2 \ldots -\phi_p B^p$

$\Phi(B^s) = 1- \Phi B^s - \Phi B^{2s} \ldots -\Phi B^{Ps}$

$\theta(B) = 1- \theta B - \theta B^2 \ldots -\theta_q \mathrm{B}^q$

$\Theta(B^s) = 1- \Theta B^s - \Theta B^{2s} \ldots -\Theta B^{Qs}$

$w_t = (1 - B)^d (1 - B^s)^D X_t$

X_t is an observation at a time t;

t is discrete time;

s is seasonal length, equal to 12;

μ is mean level of the process, usually taken as the average of the wt series (if D + d > 0 often $\mu \equiv 0$);

At normally and independently distributed white noise residual with mean 0 and variance $\sigma^2 a$ (written as NID $(0, \sigma^2 a)$;

$\phi(B)$ non-seasonal autoregressive (AR) operator or polynomial of order p such that the roots of the characteristic equation $\phi(\mathrm{B}) = 0$ lie outside the unit circle for non-seasonal stationarity and the ϕ_t, i = 1, 2,..., p are the non-seasonal AR parameters;

$(1 - B)^d$ non-seasonal differencing operator of order d to produce non-seasonal stationarity of the d^{th} difference, usually d = 0, 1, or 2;

$\Phi(B^s)$ seasonal AR operator or order p such that the roots of $\Phi(B^s) = 0$ lie outside the unit circle for seasonal stationarity and Φ_t, i = 1, 2, . .. , p are the seasonal AR parameters;

$(1 - \mathrm{B}^s)^{\mathrm{D}}$ seasonal differencing operator of order D to produce seasonal stationarity of the D^{th} differenced data, usually D = 0, 1, or 2;

$w_t = (1 - B)^d (1 - B^s)^{\mathrm{D}} X_t$ stationary series formed by differencing X_t series ($n = N - d - sD$) is the number of terms in the w_t series);

$\theta(B)$ non-seasonal moving average (MA) operator or polynomial of order q such that roots of $\theta(B) = 0$ lie outside the unit circle for invertibility and θ_t, i = 1, 2,..., q;

$\Theta(B^s)$ seasonal MA operator of order Q such that the roots of $\Theta(B^s) = 0$ and B_s lie outside the unit circle for invertibility and Θ_t, i = 1, 2,..., Q are the seasonal MA parameters.

The notation (p, d, q) (P, D Q)s is used to represent the SARIMA model (1). The first set of brackets contains the order of the non-seasonal operators and second pair of brackets has the orders of the seasonal operators. If the model is non seasonal or an ARIMA, only the notation (p, d, q) is needed because the seasonal operators are not present.

3. Box and Jenkins approach to model building

Box and Jenkins (1976) recommend that the model development consist of three stages (identification, estimation and diagnostic check) when an ARIMA model is applied to a particular problem.

(*i*) *The identification stage* is intended to determine the differencing required producing stationarity and also the order of both the seasonal and non-seasonal autoregressive (AR) and moving average (MA) operators for a given series.

In general, for an MA (0, d, q) process, the auto correlation coefficient (r_k) with the order of k cuts off and is not significantly different from zero after lag q. If r_k tails off and does not truncate, this suggests that an AR term is needed to model the time series. When the process is an MA (0, d, q) (0, D, Q), r_k truncates and is not significantly different from zero after lag $q + sQ$. If r_k attenuates at lags that are multiples of s, this implies the presence of a seasonal AR component. For an AR (p, d, 0) process, the PACF (ϕ_{kk}) with the order of k truncates and is not significantly different from zero after lag p. If ϕ_{kk} tails off, this implies that an MA term is required. When the process is an SAR (p, d, 0) (P, D, 0), ϕ_{kk} cuts off and is not significantly different from zero after lag $p + sP$. If ϕ_{kk} damps out at lags that are multiples of s, this suggests the incorporation of a seasonal MA component into the model.

(*ii*) *The estimation stage* consists of using the data to estimate and to make inferences about values of the parameter estimates conditional on the tentatively identified model. In an ARIMA model, the residuals (a_t) are assumed to be independent, homoscedastic and usually normally distributed. However, if the constant variance and normality assumptions are not true, they are often made to meet these requirements when the observations are transformed by a Box-Cox transformation (Wei, 2006).

Box and Jenkins (1976) state that the model should be parsimonious and recommend the use of as few model parameters as possible so that the model fulfils all the diagnostic checks.

(*iii*) The diagnostic check stage determines whether residuals are independent, homoscedastic and normally distributed. The residual autocorrelation function (RACF) should be obtained to determine whether residuals are white noise. There are two useful applications related to RACF for the independence of residuals. The first is the ACF drawn by plotting $r_k(a)$ against lag k. If some of the RACFs are significantly different from zero, this may mean that the present model is inadequate. The second is the $Q(k)$ statistic suggested by Ljung and Box (1978). A test of this hypothesis can be done for the model adequacy by choosing a level of significance and then comparing the value of the calculated χ^2 to the actual χ^2value from the table. If the calculated value is less than the actual χ^2 value, the present model is considered adequate on the basis of the available data. The $Q(k)$ statistic is calculated by :

$$Q(k) = n(n+2)\sum (n-k)^{-1} rk(a)^2 \tag{2}$$

where,

$r_k(a)$ = autocorrelation of residuals at lag k;

k = the lag number;

and n = number of observations or data.

There are many standard tests available to check whether the residuals are normally distributed.

Chow *et al*. (1988) stated that if historical data are normally distributed, the graph of the cumulative distribution for the data should appear as a straight line when plotted on normal probability paper.

The purpose of a stochastic model is to represent important statistical properties of one or more time series. Indeed, different types of stochastic models are often studied in terms of the statistical properties of time series they generate. Examples of these properties include: trend, serial correlation, covariance, cross-correlation, etc. If the statistics of the sample (mean, variance, covariance, etc.) are not functions of the timing or the length of the sample, then the time series is said to be weekly stationary or stationary in the broad sense. If the values of the statistics of the sample (mean, variance, covariance, etc.) are dependent on the timing or the length of the sample, that is, if a definite trend is observable in the series, then it is a non-stationary series. Similarly, periodicity in a series means that it is non-stationary. For a stationary time series, if the process is purely random and stochastically independent, the time series is called a white noise series (Akaike, 1974; Rediat, 2012).

Records of precipitation form suitable data sequences that can be studied by the methods of time series analysis. The tools of stochastic modeling provide valuable assistance to statisticians in solving problems involving the frequency of occurrences of major hydrological events. In particular, when only a relatively short data record is available, the formulation of a time series model of those data can enable long sequences of comparable data to be generated to provide the basis for better estimates of hydrological behavior. In addition, the time series analysis of precipitation and other sequential records of hydrological variables can assist in the evaluation of any irregularities in those records.

Basic to stochastic analysis is the assumption that the process is stationary. The modeling of a time series is much easier if it is stationary, so identification, quantification and removal of any non-stationary components in a data series is under-taken, leaving a stationary series to be modeled.

4. Results and discussion

4.1. *Modeling to forecast the missing observations*

Fig. 1 shows the time plot of all the monthly precipitation for the first 180 observations from Iran. The ACF (Fig. 2) drawn for the data reflect the seasonality of the data. Subsequently one degree of regular and one degree of seasonal differencing were applied to yield the plot in Fig. 3 for the differenced series.

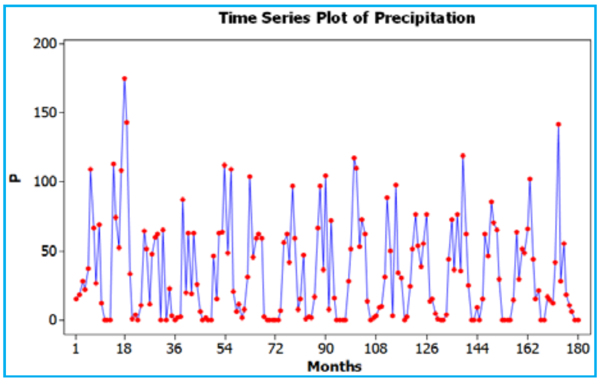

Fig. 1. Time series plot for precipitation of the first 180 precipitation data (the red dots are precipitation in mm against the months beginning with January of the first year and ending with December of the last year)

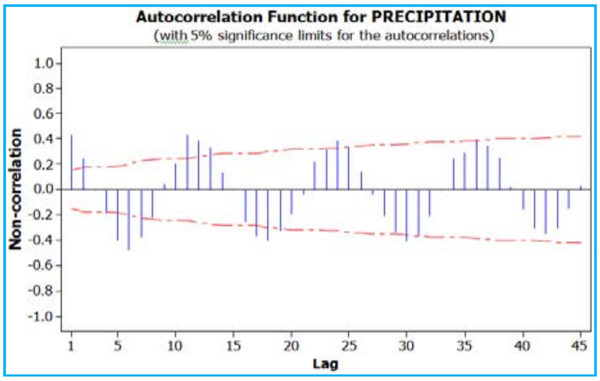

Fig. 2. ACF for the time series plot for precipitation of the first 180 precipitation data (where the dotted lines are the 2 standard error limits of the corresponding estimates here and also elsewhere for autocorrelations and partial autocorrelations)

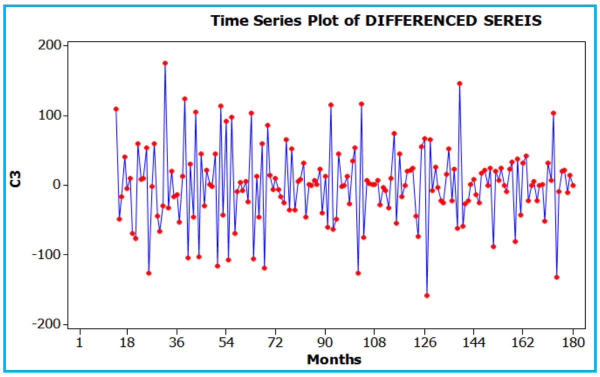

Fig. 3. Time series plot of differenced series of precipitation of the first 180 precipitation data

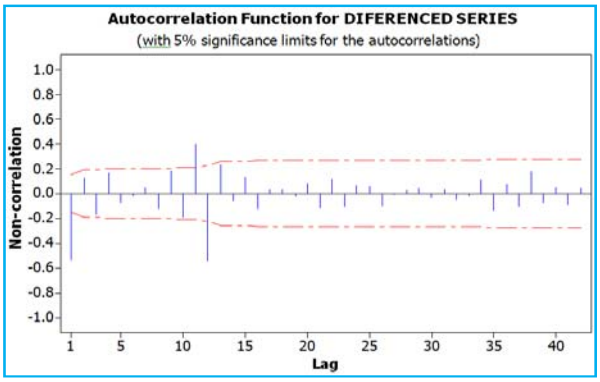

Fig. 4. ACF for the differenced series for precipitation of the first 180 precipitation data

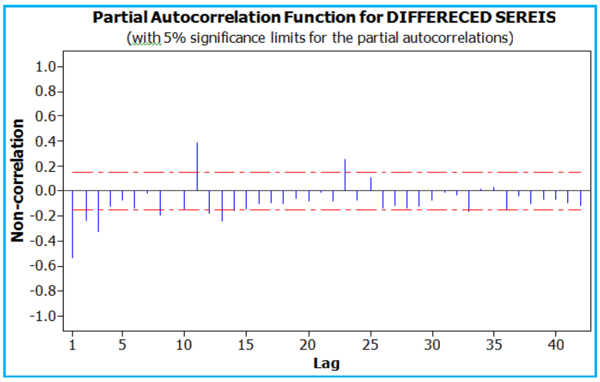

Fig. 5. Partial autocorrelation function for the differenced series time series plot for precipitation of the first 180 precipitation data

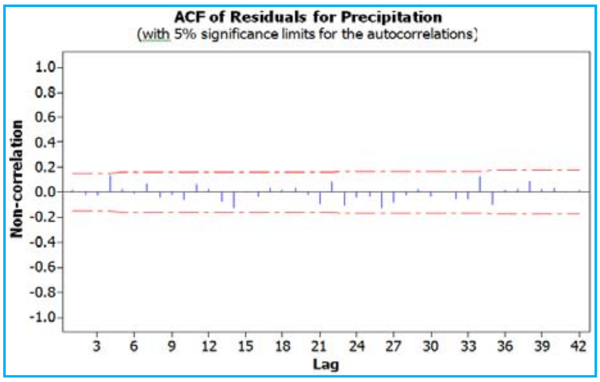

Fig. 6. ACF for the residual of the final model for precipitation of the first 180 monthly precipitation data

Then the ACF and the PACF (Figs. 4&5) for the differenced data were examined in order to identify the form of the ARIMA model. Visual inspections show that a SARIMA (0,1.1) $(1,1,1)_{12}$ may be considered as a tentative model. Then this model was fitted and plot of residuals produced (Fig. 6). The final estimates of the model parameters are given in Table 1. The plot for the residual ACF shows that all autocorrelations are zeros implying that the residuals are white noise. To check model adequacy the modified Box - Pierce (Ljung-Box)

TABLE 1

Final estimates of parameters

Type	Coef	SE Coef	T	P
SAR 12	-0.2187	0.0833	-2.63	0.009
MA 1	0.9971	0.0015	671.37	0.000
SMA 12	0.9214	0.0497	18.54	0.000
Constant	0.008900	0.004440	2.00	0.047

TABLE 2

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	5.9	16.2	28.1	32.7
DF	8	20	32	44
P-value	0.663	0.707	0.665	0.895

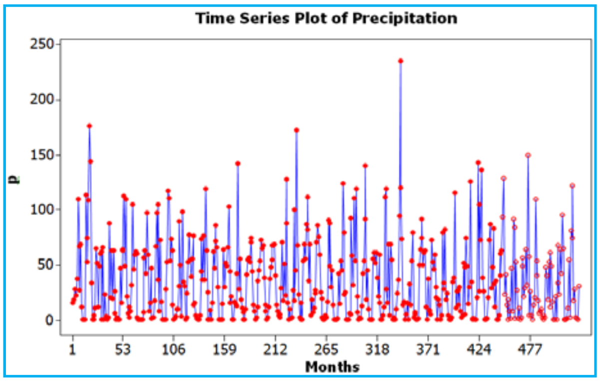

Fig. 7. Time series plot for precipitation data

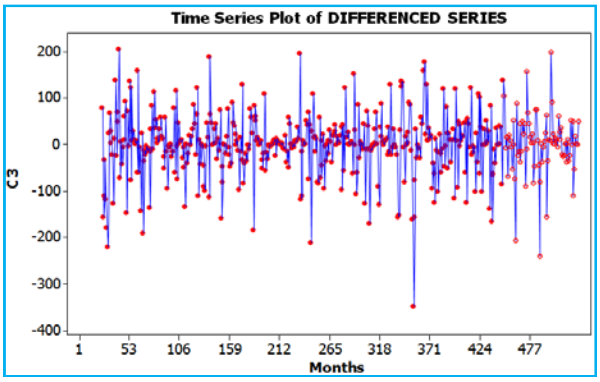

Fig. 8. Time series plot of differenced series (C3) of precipitation

Chi-Square statistics were computed and presented in Table 2 below. These statistics confirm that the model is adequate (*P*-values are all greater than or equal to 0.663).

TABLE 3

Forecasts from period 180

		95 Percent Limits	
Period	Forecast	Lower	Upper
181	9.550	-43.206	62.306
182	41.139	-11.618	93.895
183	53.853	1.096	106.609
184	56.516	3.759	109.273
185	51.966	-0.791	104.724
186	74.603	21.845	127.360
187	70.330	17.572	123.087
188	43.432	-9.326	96.190
189	13.354	-39.404	66.112
190	7.735	-45.024	60.493
191	-0.172	-52.930	52.587
192	2.272	-50.487	55.031
193	11.320	-41.933	64.573
194	35.528	-17.725	88.781
195	45.077	-8.176	98.330
196	53.509	0.256	106.763
197	71.941	18.687	125.194
198	64.706	11.452	117.960
199	67.434	14.180	120.688
200	38.183	-15.071	91.437
201	13.051	-40.203	66.305
202	7.795	-45.460	61.049
203	0.205	-53.050	53.459
204	2.123	-51.132	55.378
205	11.289	-42.290	64.869
206	37.121	-16.459	90.701
207	47.371	-6.210	100.951
208	54.550	0.970	108.131
209	67.965	14.384	121.546
210	67.271	13.691	120.852
211	68.477	14.896	122.058
212	39.750	-13.831	93.331
213	13.545	-40.037	67.126
214	8.218	-45.363	61.800

4.2. *Modeling the full data*

Fig. 7 shows the time plot of all the monthly precipitation data from Iran with the estimates (forecasts from the model fitted on the first 180 observations) in place for modeling (Table of the Appendix). This plot suggests that there is seasonal variation in these data. Then plots of the ACFs drawn for the data were examined in order to identify the form of the ARIMA model. Visual

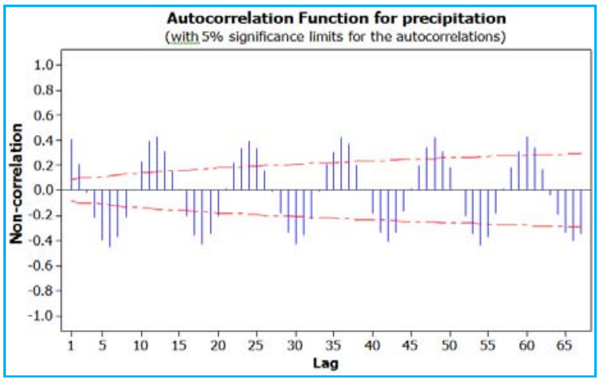

Fig. 9. ACF for precipitation

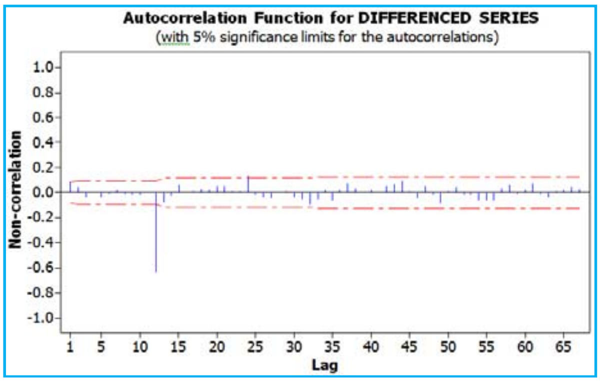

Fig. 10. ACF for the differenced series

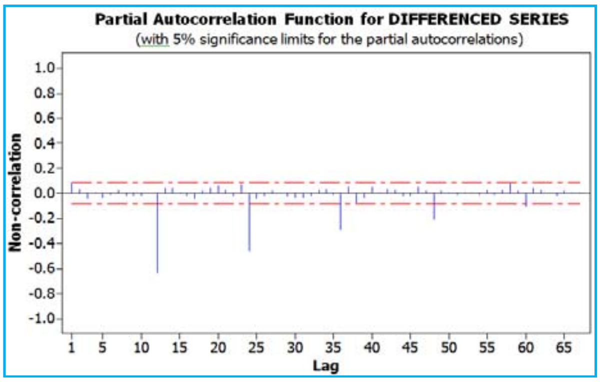

Fig. 11. Partial autocorrelation function for the differenced series

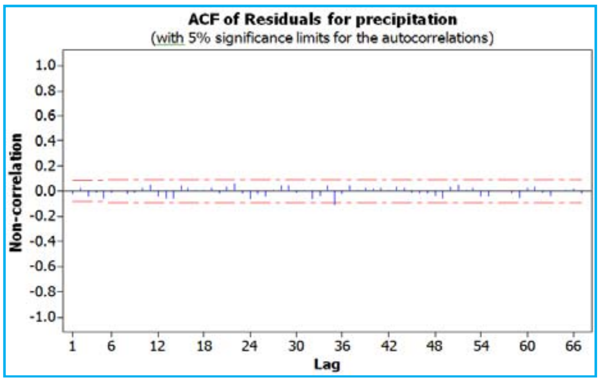

Fig. 12. ACF of residuals of the final model to precipitation

inspections show that the plot of original series and the ACF graph for the data (Fig. 9) reflect the periodicity of the data and possibly indicate the need for seasonal parameters in the model.

4.3. *Autocorrelation Function (ACF)*

Fig. 9 shows the ACF for the original precipitation data. This ACF clearly shows that there is marked seasonality.

The non-stationarity and seasonality were removed by apply in regular differencing of degree one and seasonal differencing of degree two (Fig. 8 for the plot of the differenced series). The ACFs and PACFs were estimated for the differenced data and depicted in Figs. 10&11.

The ACF (Fig. 4) cuts off after the twelfth lag. This may suggest the presence of a seasonal AR term. The PACF (Fig. 5) possess significant values at some seasonal lags and tails off. This may imply the presence of SMA terms. Moreover there are peaks on the graphs of the PACFs at lags that are multiples of 12; namely lags 12, 24, 36 and 48 that may suggest seasonal MA terms, but these peaks damp out. It appears that the non-seasonal lags have little or no effect on the model to be fitted. Consequently a seasonal ARIMA model of order (1,0,0) $(1,2,3)_{12}$ was identified as a tentative model and estimated based on the patterns of the ACF and PACF graphs from the differenced monthly data obtained for precipitation (Figs. 4&5).

Table 4 shows the parameter estimates for the tentative model. From these parameter estimates the constant is not significant even at the 10% level (*P*-value = 0.729). So the constant was dropped and the parameters were re-estimated to give the results shown in Table 5. This re-estimation raised the *P*-value for the only regular AR parameter estimate to 0.166 greater than the 10% level of significance. Yet it is retained as the multiplicative seasonal model theory requires both regular and seasonal parameters to appear in a SARIMA model.

Additionally, the Ljung-Box Q statistics were estimated for lags 12, 24, 36, and 48 (Table 6). The $Q(k)$ statistics at these lags were obtained using equation (2) and are found out to be insignificant (the *P*-values are greater than or equal to 0.484 for all of them). This shows that the fitted model can be considered adequate to model the precipitation. Therefore, they emphasize that the

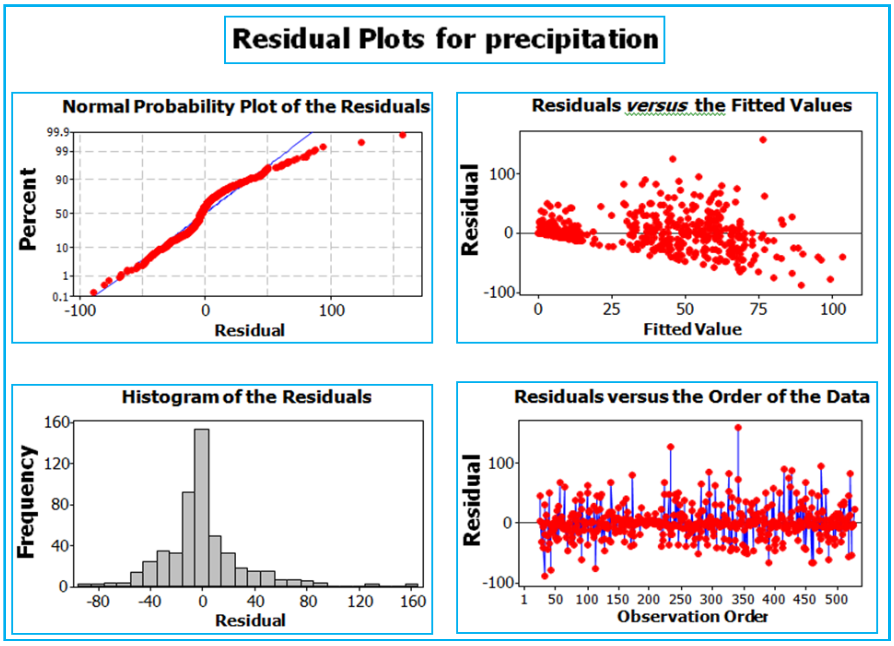

Fig. 13. Time series plot of residuals from the final model

TABLE 4

Estimates of parameters for the tentative model

Type	CoefSE	Coef	T	P
AR 1	0.0607	0.0449	1.35	0.177
SAR 12	-0.9667	0.0409	-23.63	0.000
SMA 12	0.9034	0.0447	20.22	0.000
SMA 24	0.9106	0.0825	11.03	0.000
SMA 36	-0.8312	0.0438	-18.98	0.000
Constant	-0.00968	0.02796	-0.35	0.729

TABLE 5

Estimates of parameters for the tentative model without constant

Type	Coef	SECoef	T	P
AR 1	0.0622	0.0448	1.39	0.166
SAR 12	-0.8875	0.0366	-24.28	0.000
SMA 12	0.9817	0.0094	104.65	0.000
SMA	240.7587	0.0598	12.68	0.000
SMA	36-0.7564	0.0526	-14.38	0.000

TABLE 6

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	5.7	16.3	30.7	34.7
DF	7	19	31	43
P-Value	0.580	0.636	0.484	0.812

ACF of residuals (Fig. 12) obtained from the final model are not different from zero. In fact a SARIMA model with four seasonal parameters was fitted to clear the suspicion that the ACF suggests but this parameter estimate was found out to be insignificant (*P*-value = 0.683).

Moreover, Fig. 13 shows various plots of residuals versus the order of data. Clearly there is no serious problem in the patterns revealed by these plots. Hence the residuals appear to be normal, random and homoscedastic. Moreover, diagnostic checks were applied in order to determine whether the residuals of the fitted model from the ACF and PACF graphs were independent, homoscedastic and normally distributed. These plots confirm that the residuals may be regarded as a purely random or a white noise process. In addition, the pattern of residuals for 44-years from the fitted model for precipitation shows that one can safely conclude that the fitted model has transformed the precipitation data into residuals that are a white noise process. Therefore the model to represent the precipitation can be considered to be SARIMA (1,0,0) $(1,2,3)_{12}$.

5. Conclusions

Based on the analysis to model precipitation by the seasonal multiplicative ARIMA the following conclusions were drawn : the SARIMA model application to the precipitation showed that predicted data preserved the basic statistical properties of the observed series. Unfortunately, as the data are old enough and the lead time is so long no attempt has been made to forecast precipitation and the subsequent analysis done although the main objective of time series modeling is making forecasts.

The content and views expressed in this research paper are the views of the authors and do not necessarily reflect the views of the organizations they belong to.

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Appendix

Monthly Precipitation Data for 44 years

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
15	18	28	22	37.5	109.5	67.1	26.5	68.9	12	0	0
0	113.3	74.7	52.1	108.3	175.8	143.8	33.8	0.6	3.5	0	10.4
64.5	51.6	11.2	48.2	60.4	62.2	0.2	65.7	0	23.2	3.1	0
1.8	2.4	87.2	19.8	63.2	19.1	63.2	25.8	6.2	0	1.7	0
0	46.7	15.6	63	63.8	112.3	49	109.4	20.6	6.2	11.6	1.9
7.6	31.1	104.3	45.8	59.4	62.3	59.6	2	0.2	0	0	0
0	7.2	55.9	62.7	41.5	96.9	59.3	7.8	15	47	1.2	2.2
1.8	17	66.9	97	36.2	104.9	7.9	72.3	16	0	0	0
0	28	52	117	110.1	52.9	73.3	62.6	13.8	0	1.5	2.8
9.5	9.8	31	89	50	3	98.3	34	30.5	0	2.2	24
52	77	54	39.1	55.7	76.5	13.6	15	4.4	0.8	0	0
3.5	44	73.3	36.5	76.5	36	119	62.5	25.3	0	0	9
0	15.5	62	46.5	86	71	65.5	29.5	0	0.2	0	0
14.5	64	29.5	51.5	48.5	66	102.5	44	15	21.5	0	0
16.5	14.3	12.5	41.5	142	28	55.7	18	10.5	6.5	0	0
9.55	41.14	53.85	56.5	51.97	74.6	70.33	43.43	13.354	7.735	0	2.272
11.32	35.53	45.08	53.5	71.94	64.71	67.43	38.18	13.051	7.795	0.205	2.123
11.29	37.12	47.37	54.6	67.97	67.27	68.48	39.75	13.545	8.218	4	4.5
2.5	28.4	70.3	17	50.1	21.7	127.4	87.3	15.3	0	9.4	1.1
1.9	27.2	100	17	45	172	67.5	23	2	0	15	0
53	69	55.5	87	82	111	35.5	68.5	3.5	18	6.5	11
23.5	27	70	86	75	59	25	19.5	2.5	0.5	0	3.5
2	15.9	90.5	48.2	87.7	29.8	44.5	13.5	0	0	0	0
1	42	14.5	53	44.5	123.7	79.3	22.5	25	0	0	0
5.7	4.5	92.2	58	31.9	110.8	53.8	118.9	21.5	6.5	0	2
1.2	12.5	42	53.8	91.7	139.5	18	45	9.5	0	0	0
0.5	55	61	51.5	60.5	38.5	21.5	59	15	0.5	0	4
11.5	111	118.5	28	15	68.5	3.5	68.5	54	0	0	2.5
0	10	23.5	36.2	94.2	235	119.5	73.5	13	16	0	5
15	0	14.5	32.5	13	53	79	2.5	6.5	4.5	0	0
1.5	49.5	63.5	74	91.5	49.5	60.5	62.5	11.5	0.5	37	9
8	4.5	72	52.5	46	59.5	30	20.5	0	18.5	13	0
4	79.3	33.5	27.5	82	18	23.5	4	0	0	0	2
34.5	38.5	115.5	10.5	25.9	27	13	29.8	8	2	4	51.5
4.5	48	74	49	14.5	30	125.5	34.5	0	1	1.5	0
0	20	142.5	25.5	105	72	135.5	38.5	25.5	0	1.5	0
1	20	72	87	29	48	83	32	11.5	35	0	0
4	60	40	62.5	93	128.5	22.9	41	13	0	18.5	8
1	47	8	91	83.5	1.5	52.5	27	1	10.5	0	2
48.5	56.5	21.5	28.8	64	31.5	149.5	57.5	4	25.5	3.5	0
13.5	20.5	109	40.5	53.5	17.5	6	7.5	9.2	2	0.5	4
2	47.5	40.5	18	52.7	11.5	61	48.5	16	0.5	0	21
0.5	68	33	23	63.5	42	95	64.5	0	0	0	1
10.5	54	2	80.5	74	121.5	17.5	28	2.5	1	0	30.5