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Spells of abnormally cold and hot days at Poona

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ABSTRACT. A fifty year record of daily maximum temperature during April and May and daily minimum temperatures during the period December and January at Poona is examined with reference to spells of abnormally hot weather in summer and abnormally cold weather in winter. It is found that an empirical relationship of the form $S_x = Ka^x$ holds good, where S_x is the frequency of a particular spell lasting for x days and K and a are the parameters determinable from the data. This provides a means of assessing the chance of occurrence of a spell of particular length. The frequencies were also represented by the logarithmic series $S_x = \Sigma \beta r^x / x$ where S_x and x have the same notations as referred to above, β and r being determinable from the data. The results obtained by both the geometric series and the longarithmic series are compared and discussed in the paper.

1. Introduction

Weather sequences of certain types have been found to have a tendency to persist. Frequency distributions of wet and dry weather were discussed by Gold (1929) and Cochran (1938) by applying the 'theory of runs' to the combined data of wet and dry spells. The latest contribution on this subject is due to Williams (1952) who has shown that meteorological elements having the property of 'persistence' are such that the frequency of a spell is logarithmically related to the length of the spell. Spells of rain and rainless days were discussed by Ramabhadran (1954) applying the logarithmic series to the daily rainfall data of Poona. Thus, a good amount of work has been done on the 'run' of like events. The present paper deals with abnormally cold days during the winter months, December-January, and abnormally hot days in the summer months, April-May, at Poona.

2. Brief description of weather at Poona

The months, December and January, are characterised generally by bright weather with a mean minimum temperature of 53°F. The lowest temperature ever recorded at Poona so far is 35°F (on 17 January 1935). Temperature starts rising in March, April and May being the hottest months with mean maximum temperatures of 101° and 99°F respectively. On individual days the maximum has gone upto 110°F, the number of such occasions being very small. June to September is the monsoon season when over 75 per cent of the annual rainfall of 27″ is received. The monsoon withdraws in October and the withdrawal is associated with a slight warming up before the atmosphere cools down in November.

3. Temperature abonormalities

In the India Meteorological Department the criterion for specification of 'heat wave' is that the maximum temperature should be in excess of normal by 13°F or more. A moderate heat wave is indicated when the maximum temperature is in excess of normal by 9–12°F. Similarly, a cold wave and a moderate cold wave are indicated when the minimum temperature falls below normal by 13° and 9-12°F respectively. According to these criteria the number of 'cold waves' during the fifty winters (1901-1951) was 14 and of 'heat waves' during the same period nil. These criteria appear to have been fixed on an empirical basis for all areas. To get at suitable criteria objectively, some knowledge of distribution of temperature and characteristics of spells are necessary.

The means and standard deviations of the daily minimum and maximum temperature distributions during the periods December-January and April–May respectively are given in Table 1.

TABLE 1

Month	Mean (°F)	Standard Deviation
Minimum temperature		
December	53·3	5·4
January	53·6	4·9
December—January	53·4	5·2
Maximum temperature		
April	100·4	3·9
May	99·3	4·0
April—May	99·8	4·3

As the standard deviation is a measure of the dispersion of the distribution, it is considered worthwhile to examine the deviations which are the multiples of σ. We have defined a day as an abnormally cold day if that day's minimum temperature fell below the normal minimum by σ or more. Similarly an abnormally hot day is defined if that day's maximum temperature exceeded the mean maximum by σ or more. With these criteria the abnormal days have been marked out during the winter months, December—January, for the years 1901—1951 and during the hot weather period April—May for the years 1901—1950. During the fifty Decembers (1901—1950) there were 287 abnormally cold days (*i.e.*,$X_N \leqslant \overline{T}_N-\sigma$) out of a total of 1522 days giving the probability of ·19 for a day of abnormality on chance. Similar probabilities work out to be ·19 for January and ·16 for December and January combined. For the hot weather the values are ·22, ·22 and ·18 respectively for April, May and April—May respectively. The probabilities of temperature abnormalities exceeding $\pm 2\sigma$ and $\pm 3\sigma$ have also been computed and are shown in Table 2. Based on chance, therefore, we can expect on an average about 6 occasions of abnormally cold days each in December and January and a similar number each in April and May for abnormally hot days.

TABLE 2

The probability of occurrence of the minimum temperature during December-January and the maximum temperature during April-May for some extreme values

	December	January	December–January
Winter season (December—January)			
$\leqslant \overline{T}_N-\sigma$	·19	·19	·16
$\leqslant \overline{T}_N-2\sigma$	·02	·03	·02
$\leqslant \overline{T}_N-3\sigma$	·00	·00	·00
Hot weather period (April—May)	April	May	April–May
$\geqslant \overline{T}_X+\sigma$	·22	·22	·18
$\geqslant \overline{T}_X+2\sigma$	·01	·03	·01
$\geqslant \overline{T}_X+3\sigma$	·00	·00	·00

Note—$\overline{T}_N$ and $\overline{T}_X$ are the normals for minimum and maximum temperatures respectively and σ the standard deviation in the respective periods considered

4. Persistence of temperature abnormalities

The aim of the present study is to see the degree of "persistence" in the abnormalities. Using the above criteria $T_N \pm \sigma$ for abnormalities, frequencies of cold spells and hot spells were determined from the data and are presented in Tables 3 (a) and 3(b) together with the frequencies cumulated from below and are graphed in Figs. 1 (a) and 1(b).

When these frequencies are graphed against x, the length of spell in days, an empirical relationship of the form $S_x = Ka^x$ suggests itself from the way in which the frequencies fall with increasing x. The chance of the abnormalities on any day if it is preceded by a spell of x days' duration can also be seen by calculating the probability of a certain type of spell after a spell of x days. This probability can be termed as the 'persistence probabilty' p_x and are given in Tables 4(a) and 4(b) and graphed in Figs. 2(a) and 2(b).

The behaviour of the 'persistence probability' [as seen from Tables 4(a) and 4(b)] are discussed for the individual months, December, January, April, May and for the respective periods. The failure of the 'persistence probability' for the months April and May to conform satisfactorily to a regular pattern is to be accounted for by the fact that these two months are always characterised by unstable conditions of the atmosphere over Poona.

TABLE 3(a)

Length of spell	December				January				December–January			
	A	Cu	G	L	A	Cu	G	L	A	Cu	G	L
1	28	85	25·5	36·6	38	103	37·0	48·7	59	160	51·0	78·9
2	17	57	17·9	15·9	25	65	23·7	20·2	33	101	34·7	33·9
3	10	40	12·5	9·2	15	40	15·2	11·2	19	68	23·7	19·4
4	8	30	8·7	6·0	8	25	9·7	7·0	15	49	16·1	12·5
5	6	22	6·1	4·2	5	17	6·2	4·6	10	34	10·5	7·2
6	3	16	4·2	3·1	4	12	4·0	3·2	5	24	7·1	5·3
7	2	13	2·9	2·4	2	8	2·6	2·3	3	19	4·8	3·9
8	4	11	2·1	1·7	2	6	1·7	1·7	6	16	3·3	2·9
9	3	7	1·4	1·4	1	4	1·1	1·2	3	10	2·2	2·2
10	2	4	1·0	1·2	1	3	0·7	0·9	3	7	1·9	1·9
11	2	2	0·7	0·8	..	2	0·5	0·7	..	4	1·3	1·5
12					2	2	0·4	0·6	2	4	0·9	1·1
13									2	2	0·6	0·9

TABLE 3(b)

Length of spell	April				May				April–May			
	A	Cu	G	L	A	Cu	G	L	A	Cu	G	L
1	55	136	57·1	68·7	55	135	55·3	68·1	103	231	97·0	116·4
2	39	81	33·0	27·2	35	80	32·6	27·2	59	128	56·2	45·4
3	17	42	19·1	14·5	21	45	19·2	14·6	29	69	32·6	23·6
4	11	25	11·0	8·9	9	24	11·3	8·9	16	40	18·9	13·9
5	4	14	6·4	5·6	6	15	6·7	5·6	8	24	11·0	8·6
6	1	10	3·7	2·9	3	9	4·0	3·2	3	16	6·4	5·6
7	5	9	2·1	1·6	3	6	2·4	2·5	5	13	3·7	3·7
8	1	4	1·2	0·9	..	3	1·4	1·8	3	8	2·1	2·5
9	2	3	0·7	0·5	..	3	0·9	1·3	..	5	1·3	1·8
10	1	1	0·4	0·3	1	3	0·5	1·1	2	5	0·7	1·2
11					..	2	0·3	0·8	1	3	0·4	0·9
12					1	2	0·2	0·6	1	2	0·2	0·6
13					..	1	0·1	0·4	1	1	0·1	0·4

A = Actual frequencies
Cu = Cumulated frequencies
G = Theoretical frequencies derived from geometric series
L = Theoretical frequencies derived from logarithmic series

TABLE 4(a)

Probability of abnormal day following a spell of x days

Length of spell (x)	December	January	December–January
1	·67	·63	·63
2	·70	·62	·67
3	·75	·63	·72
4	·73	·68	·69
5	·73	·70	·71
6	·81	·67	·79
7	·84	·75	·84
8	·64	·67	·63

TABLE 4(b)

Probability of abnormal day following a spell of x days

Length of spell (x)	April	May	April–May
1	·60	·61	·55
2	·52	·56	·54
3	·59	·53	·55
4	·56	·63	·60
5	·71	·60	·66
6	·90	·66	·81
7	·45	·50	·62
8	·75	1·00	·63

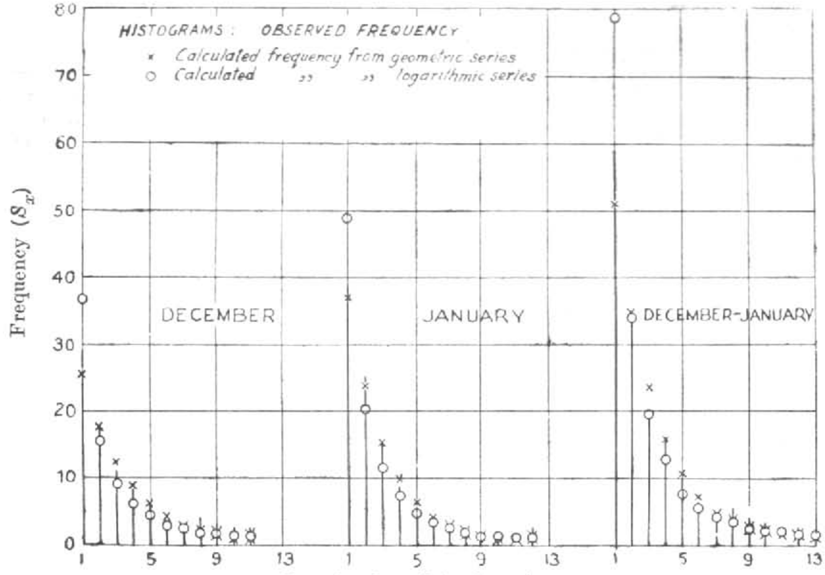

Fig. 1(a)

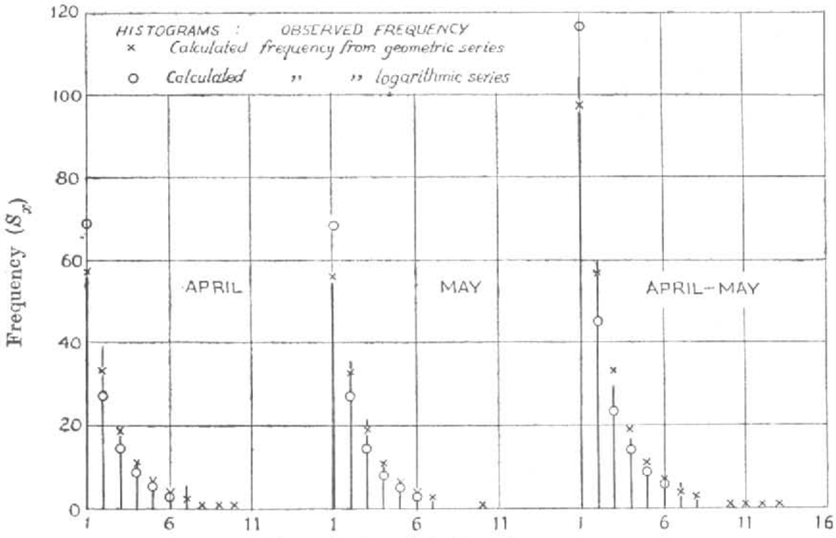

Fig. 1(b)

Abnormally cold days

December—We see from Table 4(a) that p_x remains more or less constant, *i.e.*, about ·67 to ·73 upto $x=5$ and, thereafter, suddenly increases to ·81 and ·84 for $x=$ 6 and 7 respectively and again falls down. We conclude from this that the chance of an abnormally cold day following a spell of such cold days remains more or less constant upto a spell of 5 days and then the chance increases to ·81 and ·84 if the spell persists beyond 5 days.

January— p_x remains constant upto $x=3$ and increases thereafter.

December-January— p_x varies from ·63 to ·72 upto $x=5$ and suddenly increases to about 0·8.

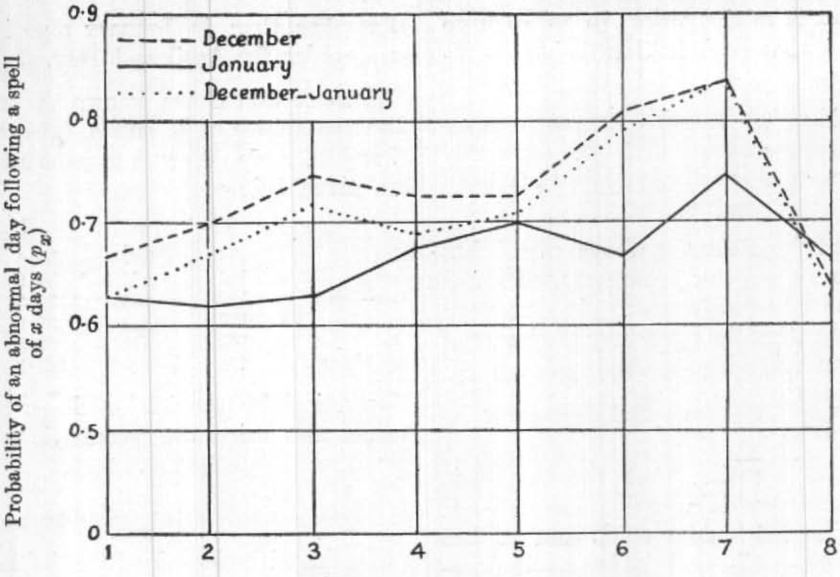

Fig. 2(a)

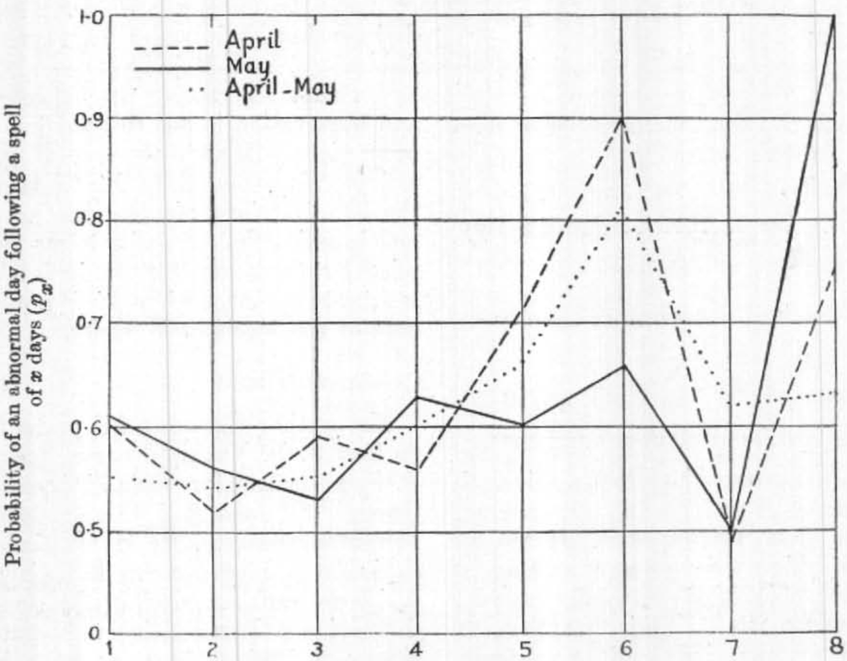

Fig. 2(b)

Abnormally hot days

April—From Table 4 (b), we see that p_x remains more or less of the same order upto $x=4$ and thereafter increases to ·71 and ·90 for $x=5$ and 6 respectively and the subsequent behaviour of p_x is rather irregular.

May— p_x is of the order ·5 to ·6 upto $x=7$ and subsequent behaviour being irregular, increases to 1·0 for $x=8$.

April-May— p_x remains almost constant of the order of ·6 upto $x=5$ and suddenly increases to ·8 at $x=6$ and again falls down to ·6 for $x=7$ and 8.

It is seen from all the above cases that the values of p_x for shorter spells, *i.e.*, less than 5 or 6 remains more or less constant thereby indicating the constancy of chance of abnormality following shorter spells.

5. Evaluation of the theoretical frequencies by geometric series

On the assumption that the geometric series represent our data we shall now proceed to determine the values of K and a. Let $S_x=Ka^x$ where S_x and x denote the frequency of spell of length x days and the length of spell in days respectively. Then $S=\Sigma S_x$, *i.e.*, the total number of sequences

$$=\Sigma Ka^x=\frac{Ka}{1-a}\cdot \text{ as } x \text{ tends to infinity.}$$

Let $N=\Sigma x\, S_x$, the total number of days belonging to this category

$=\Sigma Kxa^x=K(a+2a^2+3a^3 \ldots \text{upto infinity})$

$Na=K(a^2 \times 2a^3 \ldots\ldots\ldots\ldots \text{upto infinity})$

$\therefore N-Na=K(a+a^2+a^3\ldots\ldots)$

$=\dfrac{Ka}{1-a}$ in the limit as x tends to infinity

$$\therefore N=\frac{Ka}{(1-a)^2}, \text{ and } \frac{N}{S}=\frac{1}{1-a}, \textit{ i.e.,}$$

the average number of days per sequence.

It is seen from above that the quantity a is determined by S and N, *i.e.*, the average number of days per sequence. Hence for places for which the average number of days per sequence is the same the quantity a will remain constant. This quantity can be termed as the 'index of persistence. The 'index of persistence' is higher in December than in January and is of the same order for April and May.

Since N and S are known we can easily find the values of K and a. The values of K and a for the different months and seasons are given below

	S	N	K	a
Dec	85	287	36·4	0·70
Jan	103	287	57·9	0·64
Dec–Jan	160	498	75·3	0·68
Apr	136	325	98·6	0·58
May	135	330	93·8	0·59
Apr–May	232	545	167·3	0·58

Theoretical frequencies based on the above values and using the geometric series, are given in Tables 3(a) and 3(b).

6. Representation of data by logarithmic series

The logarithmic series $\Sigma\beta\, r^x/x$ have been used to represent the data. The successive terms of the series give the frequency of spells lasting for x days. Frequencies derived from this series are presented in Tables 3(a) and 3(b). It is seen from the tables that the agreement between theory and observation is not so good in the case of shorter spells as in the case of goemetric series. This is essentially due to the logic of the two functions. In the case of the geometric series the successive terms are multiplied by the same factors indicating constant rate of change while this is not so in the case of the logarithmic series.

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