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The change in the distribution of drops in a cloud

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ABSTRACT. The change in the drop-size spectrum, and the liquid water content has been computed for different types of distribution curves. The drop-size distribution was assumed to follow a Gaussian curve, and the change in the spectrum was computed numerically from an equation developed by Schumann. It is shown that the number of large drops in a cloud cannot increase by coalescence, unless the spectrum has a very large range or a sharp peak.

1. Introduction

In recent years much work has been done to study the growth of drops in a cloud by coalescence. The starting point in much of the work has been to assume a homogeneous cloud (a cloud consisting of drops of the same size), and, then, to inject a comparatively large drop in it. The rate of growth of the latter is next computed by a step-by-step method. This procedure is somewhat unrealistic because a cloud is seldom homogeneous; it consists of drops having all different sizes. For this reason it was considered worthwhile to find out in what manner does the drop-size spectrum change, both with respect to time and space, in a cloud. We begin, however, with the recent experimental evidence regarding observed drop-size spectra and liquid water content of clouds.

2. The observed drop-size spectrum and liquid water content of clouds

Squires and Gillespie (1952) developed a technique of measuring the size of cloud droplets from an aircraft. Their observations showed that the histogram of cloud droplets was similar to a Gaussian distribution, particularly in six of the ten slides shown in their paper. The diameter of drops observed by them varied from 10 to 60 μ, with a mean value of approximately 30μ, and the liquid water content was from 0·2 to 1·6 gm/m^3. Similar features were also noticed by Weickmann and Aufm. Kampe (1953) in another series of experiments using the impactor technique.

In the subsequent work, therefore, it will be assumed that the size-distribution of drops in a cloud is given by a Gaussian curve, namely,

$$(n)r = n_0 .\ \exp.\ \alpha.\ (r - \bar{r})^2 \qquad (2.1)$$

where, $n(r)\ dr$ is the number of drops between r and $r+dr$; and n_0, α are constants depending upon the liquid water content and the standard deviation of the radius (σ). The mean drop radius is given by $\bar{r}$ and n_0 is the number of drops corresponding to $\bar{r}$.

The total liquid water content of the cloud is obtained by the following equation—

$$w = \frac{4}{3} \cdot \pi .\ \rho .\int r^3 .\ n(r).\ dr \qquad (2.2)$$

where $N = \int n(r)\ dr$ is the total number of drops in the cloud, and ρ is the density of water. It thus follows that if the range of the drop-size spectrum, the standard deviation of the radius and the total number of drops in a cloud be known, then the liquid water content can be computed by using the equations (2·1) and (2·2).

In the present work four different types of distribution curves were studied. They are shown in Table 1 with their properties.

TABLE 1

Equation of curve	Standard deviation of radius in microns (σ)	Mean radius in microns	Total No. per c.c. (N)	Range (μ)
(a) $n(r) = 12 \text{ exp. } \left\{-0\cdot02\,(r-15)^2\right\}$	5·0	15·0	150·0	5–25
(b) $n(r) = 30 \text{ exp. } \left\{-0\cdot02\,(r-15)^2\right\}$	5·0	15·0	370·0	5–25
(c) $n(r) = 115 \text{ exp. } \left\{-0\cdot08\,(r-10)^2\right\}$	2·5	10·0	720·0	5–15
(d) $n(r) = 200 \text{ exp. } \left\{-0\cdot002\,(r-55)^2\right\}$	15·0	55·0	7500 per litre	10–100

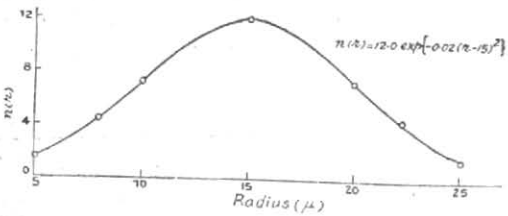

Fig. 1

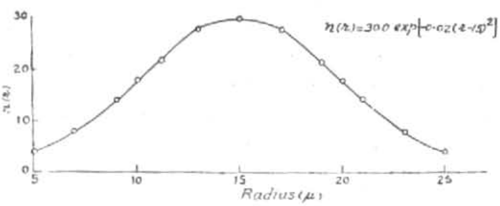

Fig. 2

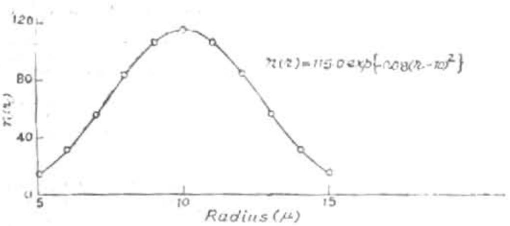

Fig. 3

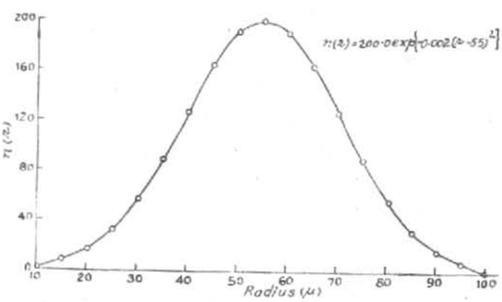

Fig. 4

TABLE 2

Distribution curve	Total No. of drops (N)	Liquid water content (w) (gm/m^3)
(a)	150 per c.c.	2·2
(b)	370 per c.c.	4·6
(c)	720 per c.c.	4·4
(d)	7500 per litre	5·0

These curves are shown graphically in Figs. 1-4. In the last curve the volume considered was one litre instead of 1 c.c., otherwise the number of extremely small or large drops in the two extremes of the spectrum would be less than unity. The range of drops in each case was so chosen that 95 per cent of the drops were included in the spectrum. The contribution of drops outside this range was not considered significant for computing the liquid water content. The distribution curves were also chosen in such a way (by adjusting the constants) that it would be possible to ascertain the effect of sharp peak and a narrow range (curve c), a large range and comparatively few drops (curve d) and a medium range with low peak value (curves a and b) on the overall change in the spectrum.

The liquid water content (w) for each of the above distribution curves was obtained by evaluating the integral in (2·2) numerically for different values of r spaced at appropriate intervals, and using Simpson's rule. The values obtained are shown in Table 2.

It is thus apparent that the above distributions refer to clouds of high liquid water content. These values are higher than those reported by Squires and Gillespie (1952) or by Weickmann and Aufm. Kampe (1953), but they are not necessarily unrealistic, because values as high as 5·0 gm/m^3 have also been reported (Lacey 1940).

3. Change in drop-size spectrum with time

To evaluate the rate of change in the drop-size spectrum with time, we consider the number of drops of radius between R and $R+dr$. The number of such drops is decreased by collision with other drops (both larger and smaller than R); at the same time there is an increase in the number due to collision of drops smaller than R, which coalesce to form bigger drops. Summing up for all the drops in the spectrum, we get,

$$\frac{dn(r)}{dt} = -n(R)\int_0^\infty \pi R^2\Big[V(R) \sim V(r)\Big]\times E(R,r)\times n(r)\,dr + \tfrac{1}{2}\int_0^R \pi r^2\Big[V(r)\sim V(r')\Big]\times E(r,r')n(r)\times n(r')\times R^2/r^2 \times dr'$$

$$= -I_1 + I_2 \qquad (3\cdot 1)$$

where, $n(R)$ is the number of drops between R and $R+dr$; $V(R)$, $V(r)$, $V(r')$ are the terminal velocities of drops of radius R, r, r' and $E(r,r')$ is the collection efficiency of a drop r falling through smaller drops r'. By r and r' we denote the radii of two small drops which coalesce to form a drop of radius R. The relation between r, r' and R is given by

$$R^3 = r^3 + r'^3 \qquad (3\cdot 2)$$

Equation (3·1) was first given by Schumann (1940) in a slightly different form. In its present form the equation gives the rate of change of $n(R)$ due to large drops overtaking small ones; fluctuations in the velocity caused by turbulence are not considered. Schumann obtained solutions of (3·1) for two particular cases: for a constant collision frequency and, also, for a collision frequency varying linearly with drop radius. But, as we can see, there is no simple relation between the collision frequency and the radii of colliding drops. An attempt was made, therefore, to solve the equation by the approximate method of evaluating the integrals I_1 and I_2 numerically, by Simpson's rule (*vide* Appendix 1). This was done for different parts of the spectrum.

Two simplifying assumptions were made in the calculations. Firstly, it was assumed that the collection efficiency for all drops was unity. It was realized that this assumption was not likely to be valid in reality—particularly for very small drops, or drops of almost equal size—but it was felt that the order of magnitude of the final result will not be altered by this assumption. Secondly, it was assumed that the terminal velocity of all drops was given by Stoke's law, *i.e.*

$$V(r) = 1 \cdot 3 \times 10^6 \times r^2 \qquad (3 \cdot 3)$$

There might be some deviation from Stoke's law for the largest drops (of radius 100μ) considered in the last distribution curve (curve d), but for the others this assumption is not likely to cause much error.

4. Results obtained

Using the method outlined above, the rate of change in the drop-size spectrum was evaluated for different values of the radius in each of the four distribution curves. The range of the spectrum in each case was limited to $\pm 2\sigma$ (so as to include 95 per cent of the drops). The contribution of drops outside this range was neglected. The results obtained are summarised in Table 3.

The above values of $dn(r)/dt$ are also shown graphically in Figs. 5-8. These figures bring out a number of interesting features: it is noticed, for instance, that there is a well defined maximum value of $dn(r)/dt$, which generally coincides with the mean radius. It is also noticed that there is a tendency for broadening of the spectrum in all cases, and it is only when the spectrum has a sharp peak or a large range (as in c and d) that we get positive values of $dn(r)/dt$. This is also to be expected fιom a priori considerations, because unless the spectrum has a sharp peak or a large range, the integral I_2, which is the additive term in (3·1), cannot predominate. This fact is also of some interest in view of the recent observation of a few large drops (of radius 100 μ) in growing cumuli by Weickmann and Aufm. Kampe (1953). The number of such drops, if they exist, can

TABLE 3

Curve	$r(\mu)$	$n(r)$	$dn(r)/dt$. sec^{-1} per c.c.
	5	1·6	$-0 \cdot 05 \times 10^{-6}$
	10	7·3	$-0 \cdot 60$,,
(a) $n(r) = 12 \exp.\{-0 \cdot 02\,(r-15)^2\}$	15	12·0	$-1 \cdot 50$,,
	20	7·3	$-1 \cdot 40$,,
	25	1·6	$-0 \cdot 10$,,
	5	4·1	$-0 \cdot 003$,,
	10	18·2	$-0 \cdot 01$,,
	13	28·0	$-7 \cdot 60$,,
(b) $n(r) = 30 \exp.\{-0 \cdot 02\,(r-15)^2\}$	15	30·0	$-10 \cdot 40$,,
	20	18·2	$-9 \cdot 70$,,
	23	8·2	$-4 \cdot 90$,,
	25	4·1	$-0 \cdot 40$,,
	5	15·5	$-0 \cdot 87$,,
	8	83·5	$-6 \cdot 96$,,
(c) $n(r) = 115 \exp.\{-0 \cdot 08\,(r-10)^2\}$	10	115·0	$-7 \cdot 60$,,
	11	106·0	$-5 \cdot 50$,,
	12	83·5	$+1 \cdot 10$,,
	15	15·5	$+0 \cdot 40$,,
	10	1·0	$-0 \cdot 01$,,
	40	127·0	$-11 \cdot 50$,,
(d)* $n(r) = 200 \exp.\{-0 \cdot 002\,(r-55)^2\}$	55	200·0	$-23 \cdot 00$,,
	70	127·0	$-23 \cdot 00$,,
	85	33·0	$-4 \cdot 30$,,
	100	1·0	$+9 \cdot 00$,,

*Values of $dn(r)/dt$ for this curve refer to one litre

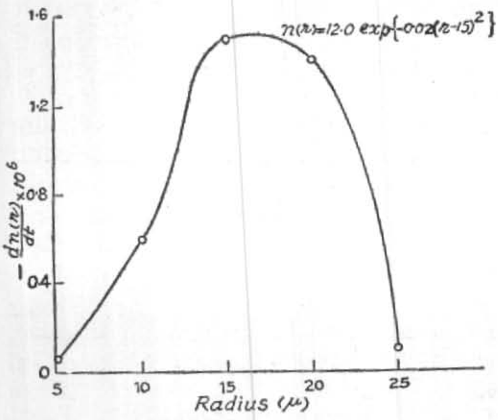

Fig. 5

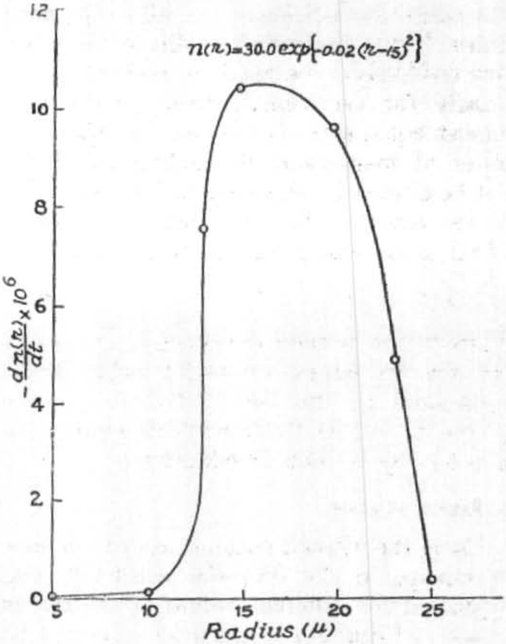

Fig. 6

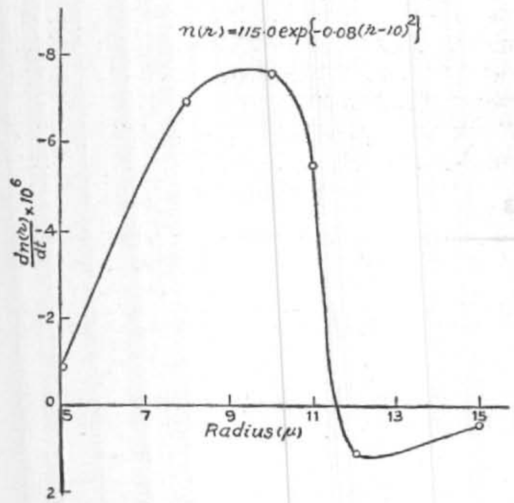

Fig. 7

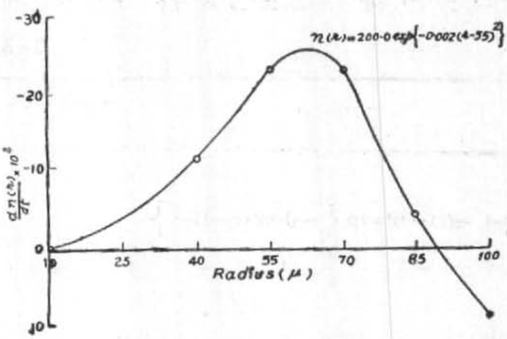

Fig. 8

increase by coalescence with smaller drops only if the drop-size spectrum has the characteristics mentioned above.

From the above values of $dn(r)/dt$ it is also possible to determine the change in the spectrum with distance of fall. We have,

$$dn(r)/ds = 1/V(r) \, . \, dn(r)/dt \qquad (4\cdot 1)$$

With the help of (4·1) the change in $n(r)$ was computed for a fall of 3 km for different values of r, but the change was found to be very small. It was less than 5 per cent of the original value of $n(r)$ in all cases. It may be concluded, therefore, that the drop-size spectrum is unlikely to change even after the drops have fallen through considerable distance.

5. Acknowledgement

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APPENDIX 1

The numerical calculation of $dn(r)/dt$

As an illustration the computation of $dn(r)dt$ for a particular value of r is shown below—

We consider the first distribution curve, *i.e.*,

$$n(r) = 12\cdot0 \text{ exp.}-0\cdot02\,(r-15)^2$$

and calculate $dn(r)/dt$ for $R=15\mu$. We have,

$$I_1 = \int_5^{25} n(R).\ \pi R^2.\ \left(V_R \sim V_r\right).\ E(R,r).\ n(r).\,dr$$

Putting $E(R,r) = 1\cdot0$;

$V(R) \sim V(r) = 1.3.(15^2-r^2).10^{-2}$;

and $n(R) = 12\cdot0$

we get after some simplification,

$$I_1 = 1\cdot3.\ 10^{-5}. \int_5^{25} (15^2-r^2).\ \text{exp.}-0\cdot02\,(r-15)^2.\ dr$$

Let $$f(r) = \Sigma y = \int_5^{25} (15^2-r^2)\,\text{exp.}-0\cdot02\,(r-15)^2.dr$$

which gives the following values of y for different r—

$r(\mu)$	y
5·0	y_0 = 27·1
7·0	y_1 = 48·0
9·0	y_2 = 71·5
11·0	y_3 = 77·0
13·0	y_4 = 51·7
15·0	y_5 = 0·0
17·0	y_6 = 59·1
19·0	y_7 = 100·7
21·0	y_8 = 107·3
23·0	y_9 = 82·9
25·0	y_{10} = 54·1

Summation by Simpson's rule gives—

$$f(r) = (y_0 + 4y_1 + 2y_2 + \ldots + y_{10})\ \Delta.\ r/3 = 0\cdot13$$

Hence, $I_1 = 1\cdot7\times10^{-6}$

To determine I_2, we first obtain a set of values of r and r' from the relation,

$$R^3 = r^3 + r'^3$$

The values obtained were as follows—

$r'(\mu)$	$r(\mu)$
5·0	14·8
7·0	14·5
9·0	13·8
11·0	12·7
13·0	10·5
14·8	5·0

Next, we have

$$I_2 = \tfrac{1}{2} \int_5^{15} \pi r^2 \left(V_r \sim V_{r'}\right) E\,(r,r').\ n(r).\ n\,(r').\ R^2/r^2.dr'$$

$$= 6\cdot6\times10^{-6} \int_5^{15} (r^2 \sim r'^2)\ \text{exp.}-0\cdot02\,(r-15)^2.\ \text{exp.}-0\cdot02\,(r'-15)^2.\ dr'$$

As before we put,

$$f(r,r') = \Sigma y = \int_5^{15} (r^2 \sim r'^2)\ \text{exp.}-0\cdot02\,(r-15)^2.\ \text{exp.}-0\cdot02\,(r'-15)^2\,dr'$$

which gives the following values of y for different r'

$r'(\mu)$	y
5·0	y_0 = 26·3
7·0	y_1 = 43·9
9·0	y_2 = 52·5
11·0	y_3 = 27·0
13·0	y_4 = 36·4
15·0	y_5 = 26·3

Summing up by Simpson's rule we get,

$f(r,r') = 3\cdot7\times10^{-6}$

Consequently, $I_2 = 0\cdot25\times10^{-6}$

so, $dn(r)/dt = -I_1 + I_2$

$= -1\cdot5\times10^{-6}$