Vertical diffusion of Horizontal Momentum by Turbulence in the lower atmosphere*

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1. Introduction

In the treatment of atmospheric turbulence the necessity of taking into consideration the joint effect of the vertical temperature distribution and the wind gradient instead of the latter alone, was first pointed out by Richardson (1920). It is the resultant effect which determines the growth or decay of turbulence in the atmosphere and not the individual effects. Thus he showed that if E' be the turbulent energy per unit volume of the atmosphere, its time rate of change is given by

$$\frac{\partial E'}{\partial t} = K_M\left(\frac{\partial \vec{V}}{\partial z}\right)^2 - \frac{gK_H}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right) \quad (1)$$

Where

K_M = co-efficient of kinematic viscosity

$\frac{\partial \vec{V}}{\partial z}$ = vertical shear of the horizontal wind velocity, $\vec{V}$

g = acceleration due to gravity

K_H = co-efficient of heat diffusivity

T = temperature

and Γ = adiabatic lapse rate of temperature

In eqn. (1) the first term represents the effect of the Reynolds stresses whereas the second represents the effect of gravity. The predominance of one term over the other decides the growth or the decay of turbulence. In the derivation of eqn. (1) Richardson assumed an atmosphere with 'just-no-turbulence' meaning the existence of very slight turbulence. His conclusion was that if the energy due to Reynolds stresses exceeded that due to gravity, turbulence in the medium would increase and *vice versa*. In other words, turbulence will increase or decrease according as

$$K_M\left(\frac{\partial \vec{V}}{\partial z}\right)^2 > \text{or} < \frac{gK_H}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right)$$

The critical value of Richardson's criterion of turbulence was, therefore, given by

$$Ri = \frac{\frac{gK_H}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right)}{K_M\left(\frac{\partial \vec{V}}{\partial z}\right)^2} \quad (2)$$

Drust (1933) brought forward experimental evidence in favour of a value of unity for the Richardson's criterion but Schlichting (1935) found a value of 0·04. Recently, Calder (1949) has re-examined some of the theoretical aspects of the Richardson's theory and proposed a form $Ri = 1 - \delta$ where $\delta > 0$ but no definite value was given to δ. Deacon (1949) has recently suggested a value 0·15 for atmosphere close to the earth's surface whereas for the open atmosphere Petterssen and Swinbank (1947) suggested $Ri = 0 \cdot 65$.

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It is possible for large gradients of wind velocity or even discontinuities in the winds to be maintained in the atmosphere as long as the vertical temperature distribution does not counteract them to the extent of breakdown of the layer into large-scale turbulence. The moment turbulence sets in, eddies churn up the atmosphere and as a result momentum is diffused vertically down the velocity gradient. It is the object of this note to study the turbulence diffusion of momentum in the atmosphere with particular regard to

(*a*) the vertical diffusion of a layer of wind discontinuity and

(*b*) the diffusion of higher momentum from upper layers to the ground.

2. The effect of thermal stratification on the variation of the wind with height

An expression which appears to represent fairly satisfactorily the effect of thermal stratification on the vertical distribution of the wind close to the ground is that given by Deacon (1949).

$$\frac{du}{dz} = a.z^{-\beta'} \tag{3}$$

where β' is less, equal or greater than 1 according as the atmosphere is thermally in a stable, neutral or unstable equilibrium, and a is a constant.

This expression in the case of neutral equilibrium reduces to the well known logarithmic relation of von Karman, *viz.*,

$$u = (u^*/k_0) \log (z/z_0'') \tag{4}$$

where u^* is the frictional velocity, k_0 the Karman constant, and z_0'' the roughness parameter.

The relation between the stability factor β' and the Richardson number Ri has been discussed by Deacon (*loc. cit*). According to equation (3), the horizontal component of the wind increases uniformly with height but at different rates depending upon the thermal stratification of the layer under consideration. The wind gradient is higher in a thermally stable layer than in a neutral or unstable layer. Ramdas (1944) has studied the diurnal variation of the wind structure over Poona in a layer of about 35 feet above the ground and shown the validity of a power-law of the variation of the wind with height. A large amount of substantiating data is assembled by him to prove that the gradient of wind velocity with height is appreciably greater in a thermally stable layer than in an unstable layer. This conclusion is in line with that derived by many other workers in this field.

3. Examples of the diurnal variation of the vertical wind structure in relation to temperature distribution

In a study of low-level wind structure over the airfield at Ambala, India, by the method of tethered balloon flights (Saha 1954) on clear days, it was found that although the general distribution of the wind direction and velocity is in line with equation (3), there were some days on which the wind distribution above the ground was apparently at variance with that equation. For example, on a large number of mornings when there was well-marked inversion over the station, there was an appreciably thick layer of easterly wind close to the ground before the westerly winds were reached higher up. On most days the easterlies were light. Their strength appeared to decrease with height. After passing through a layer of transition, the winds became westerly with gradual increase in velocity with increasing height. On other mornings, the wind on the ground was light westerly and the momentum increased with height in accordance with Deacon's equation. Thus within the inversion layer, there were clearly observed two different types of wind distribution. In the first type the easterly layer closest to the ground was surmounted by the orthodox westerly layer and in the second there were westerlies throughout but the momentum increased with height.

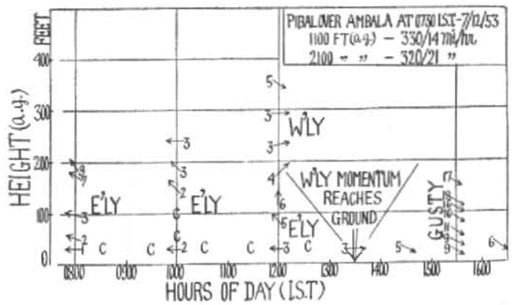

Fig. 1 (7 December 1953)

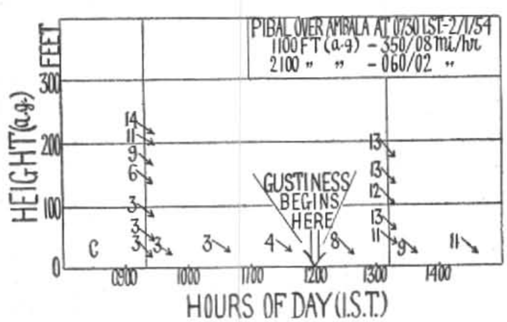

Fig. 2 (2 January 1954)

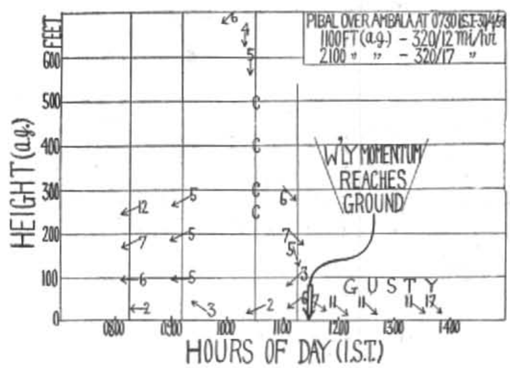

Fig. 3 (30 April 1954)

Figs. 1-3. Diurnal variation of the low-level wind structure and the downward diffusion of the upper momentum by turbulence at Ambala

In Figs. 1, 2 and 3 above — C-calm. The direction of arrows indicate the direction of winds. The figures at the beginning of arrows give the wind speed in mph

The diurnal variation in the first type was marked by a gradual lowering of the level of discontinuity till in the hot part of the day the upper westerlies had reached the ground and become turbulent. In the case of the second type, the direction of the wind remained unaltered but there was gradual diffusion of momentum downward till in the hot hours of the day the wind was highly turbulent. These characteristic diurnal variations are illustrated in Figs. 1 and 3 for the first type and in Fig. 2 for the second, with measured wind data on three days during the period 1953-54.

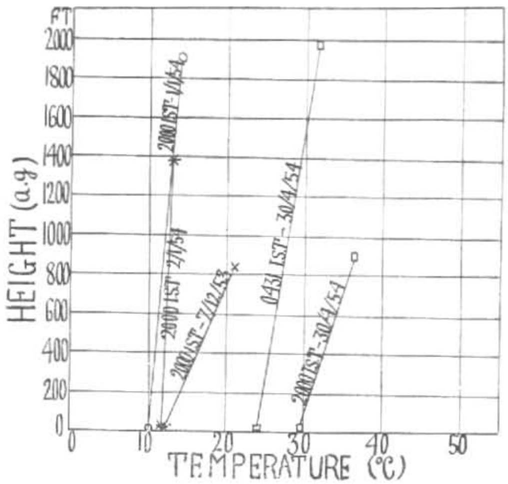

Fig. 4. Thickness and intensity of inversion layers over Delhi on days of wind measurement

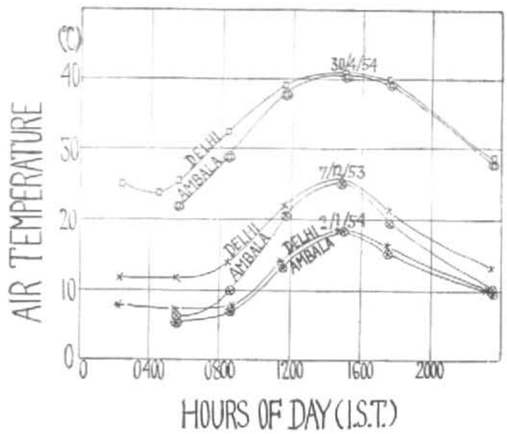

Fig. 5. Diurnal variation of surface air temperatures at Ambala and Delhi on days of wind measurement
(Symbols encircled refer to Ambala)

Corresponding temperature distributions at the times of wind measurements are not available owing to non-existence of any upper air sounding station at Ambala. No special device was adopted locally to measure upper air temperatures. The radiosonde station closest to Ambala is Delhi which is about 120 miles away to the south. Available temperature data over Delhi up to the height of the top of the inversion layer on the days of wind finding are presented in Fig. 4. These are mostly evening data except on 30 April 1954, when both morning and evening data were available. Although the data of temperature over Delhi may not quite truly represent the temperature state over Ambala, it was thought that owing to homogeneity of air mass over the area covering the two places during the period the general trends of temperature distribution in the vertical would be more or less similar in character over the two places. Available surface air temperatures over the two places plotted in Fig. 5 would seem to lend support to such a view. Local peculiarities like type of land surface, proximity to hills etc, would create some difference in temperature values in the morning when under steep temperature inversion the air layers are stratified and the local peculiarities are imposed into the air layers in contact with the surface but in the hot part of the day, these local peculiarities are washed away by continual turbulent mixing with the upper layers with the result that surface air conditions become more or less uniform over the two places. It should be noted that on 2 January 1954, when the night inversion was weak or almost isothermal conditions prevailed close to the ground owing to the approach of a western disturbance over the area, the easterlies could not develop and the higher momentum of the westerlies from the upper layers rapidly diffused to the ground.

4. Vertical diffusion of momentum and the onset of turbulence at the ground

The observations in Figs. 1, 2 and 3 show that during the course of a day, there is a gradual diffusion of westerly momentum from the upper layers of the lower atmosphere to the ground. Owing to the thermal stability, the Richardson number in the layer is high in the morning and there is little turbulence in the atmosphere. There is stratification of the momentum in the atmosphere with the result that a high wind gradient may remain in existence.

With the advance of the morning, the ground gets heated up on clear days and by noon the inversion of temperature is destroyed and replaced by a lapse of temperature. The result is growth of instability in the air layers close to the ground. At some stage during the heating of the surface layers of the atmosphere, the Richardson number of the layer falls below the critical limit for just-no-turbulence, and there is a breakdown of the quasi-laminar flow and turbulent mixing occurs. When this happens the easterlies weaken and disappear followed by the downward movement of the westerlies to replace the easterlies at the ground. As the westerly momentum gradually increases at the ground the wind becomes highly turbulent. In the same way, the higher momentum of the upper westerlies diffuse to the ground through the lower light westerlies as a result of turbulent mixing. When the atmospheric layer is well mixed, the vertical gradient of wind velocity becomes small.

It is, therefore, observed that though in the morning hours a discontinuity in the vertical wind distribution with the easterlies below and the westerlies above may be maintained by a temperature inversion, turbulent mixing in the hotter parts of the day leads to a diffusion of westerly momentum to the ground. This diffusion causes not only change of wind direction in some cases but great turbulence of the wind near the ground.

5. Importance of studies of momentum diffusion

In many airfields where a recommendation is to be given by the local meteorological staff regarding laying on the runway for the day, the meteorological officer will be well-advised to look for any discontinuity in the upper wind distribution. In case an easterly wind blows at the ground and the first reportable pilot balloon level shows a westerly wind in the upper air, it will be worthwhile to try to find out

(*i*) the depth and intensity of the inversion layer,

(*ii*) the thickness of the easterly layer, and

(*iii*) the gradient of the upper westerlies.

He should also estimate from the incoming solar radiation the approximate time of the day when the temperature lapse rate will cause a breakdown of the stability and the westerly momentum will diffuse to the ground. The change in the wind for the runway may be forecast in that way. The distribution of the wind, particularly the thickness of the westerly layer, may be explored by a tethered balloon of the type used by the author (Saha, *loc. cit.*) or by observation on a freely rising pilot balloon which drifts with the wind.

Observation shows that whatever the thickness of the layer of the easterly winds near the ground the westerlies diffuse to the ground much earlier in the day in summer than in winter. A study of turbulent diffusion of momentum is also important in such air operations as para-dropping, gun-firing, chemical warfare etc.

6. Acknowledgement

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